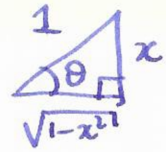
recall spose $\sin^{-1}(x) = \theta \iff x = \sin\theta$



want $\cos\theta = \sqrt{1-x^2}$

so $$\frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-x^2}} \quad \square.$$

Thm $$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{x^2+1} \qquad \frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{x^2+1}$$

$$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(\csc^{-1}(x)\right) = \frac{-1}{|x|\sqrt{x^2-1}}.$$

Example $$\frac{d}{dx}\left(\tan^{-1}\left(e^{2x}\right)\right) = \frac{1}{1+\left(e^{2x}\right)^2}\cdot 2e^{2x}.$$

§3.9 Derivatives of exponentials and logs

recall: $f(x) = b^x$ then $f'(x) = m_b\, b^x$

$f(x) = e^x$ then $f'(x) = e^x$

Thm $f(x) = b^x$ then $f'(x) = \ln(b)\, b^x$

Proof $f(x) = b^x = e^{x\ln(b)}$

$$f'(x) = e^{x\ln(b)} \cdot \ln(b) \quad \text{(chain rule)}$$

$$= \ln(b)\, b^x \quad \square$$

Example $f(x) = 3^{4x}$ $\quad f'(x) = 4\ln(3)\, 3^{4x}$.

Thm $f(x) = \ln(x)$ then $f'(x) = \frac{1}{x}$

<u>Proof</u> $\cancel{y = \ln(x)}$ $f(x) = e^x$ $\quad f^{-1}(x) = \ln(x) = g(x)$

$f'(x) = e^x$

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{e^{\ln(x)}} = \frac{1}{x} \quad \square.$$

<u>Example</u> ① $f(x) = \log_b(x) = \dfrac{\ln(x)}{\ln(b)}$

$$f'(x) = \frac{1}{x\ln(b)}.$$

② $f(x) = x\ln(x)$

$$f'(x) = \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1 .$$

<u>Hyperbolic trig functions</u> $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$ $\quad \cosh(x) = \dfrac{e^x + e^{-x}}{2}$

$\dfrac{d}{dx}\left(\sinh(x)\right) = \cosh(x)$ $\quad \dfrac{d}{dx}\left(\cosh(x)\right) = \sinh(x).$ $\quad \boxed{\cosh^2 x - \sinh^2 x = 1}$

check: $\left(\sinh(x)\right)' = \left(\dfrac{e^x - e^{-x}}{2}\right)' = \dfrac{e^x + e^{-x}}{2}.$

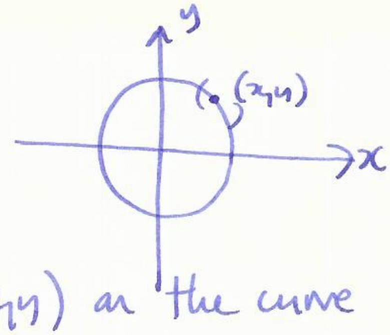
$\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$ so $\dfrac{d}{dx}\left(\tanh(x)\right) = \dfrac{1}{\cosh^2(x)}$ etc...

<u>inverse functions</u>: $\dfrac{d}{dx}\left(\sinh^{-1}(x)\right) = \dfrac{1}{\sqrt{x^2+1}}$

$$\frac{d}{dx}\left(\cosh^{-1}(x)\right) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\tanh^{-1}(x)\right) = \frac{1}{1-x^2}$$

§3.10 Implicit differentiation



explicit function: $f(x) = x^2 + x + 1$

implicit "function": $x^2 + y^2 = 1$

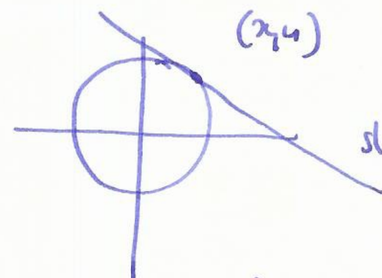
we get a "local" function near a point (x,y) on the curve we can think of as a function of x, i.e. $(x, f(x))$ or $(x, y(x))$

differentiate: $x^2 + y^2 = 1$ wrt x

$x^2 + (y(x))^2 = 1$ ← consider y as $y(x)$ and use chain rule.

$$\frac{d}{dx}\left(x^2 + (y(x))^2\right) = \frac{d}{dx}(1)$$

$$2x + 2(y(x)).y'(x) = 0$$



$$2x + 2y.y' = 0$$

$$y' = -\frac{2x}{2y} = -\frac{x}{y}$$

Example ① find tangent line to circle $x^2+y^2=1$ at $(3/5, 4/5)$

$\frac{dy}{dx} = -\frac{x}{y}$ $\quad \frac{-3/5}{4/5} = -3/4 \quad$ $y - \frac{4}{5} = -\frac{3}{4}\left(x - \frac{3}{5}\right)$

② find tangent line to ~~circle~~ $y^4 + xy = x^3 - x + 2$ at $(1,1)$

implicit differentiation: $4yy' + y + xy' = 3x^2 - 1$

at $x=1, y=1$: $\quad 4y' + 1 + y' = 2 \quad$ $5y' = 2 \quad y' = \frac{2}{5}$

$y - 1 = \frac{2}{5}(x-1)$.

Inverse functions $\quad y = f^{-1}(x) \Leftrightarrow f(y(x)) = 1$

$$f'(y(x)).y'(x) = 1$$

$$y'(x) = 1/f'(y(x))$$