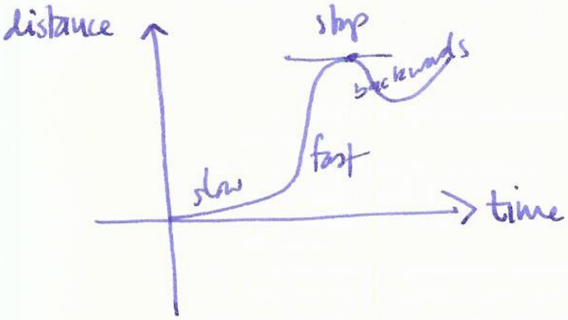
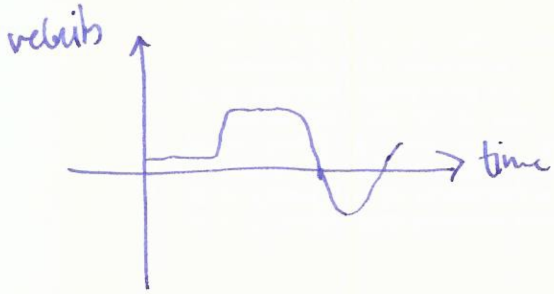
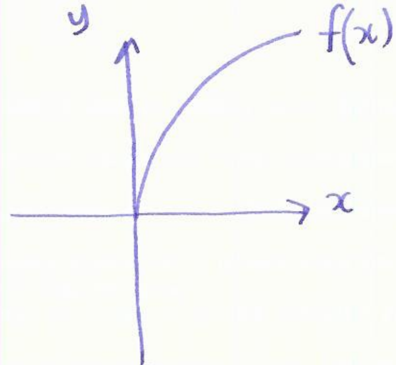
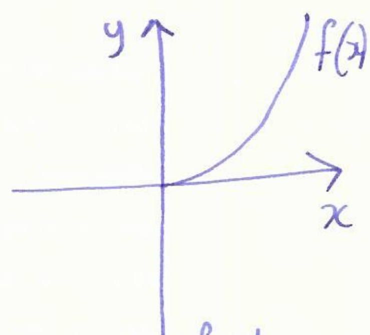
On the interpretation of graphs



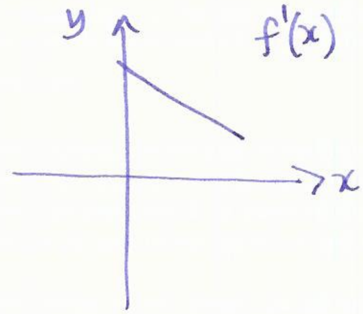


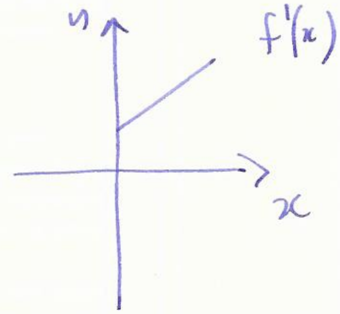


rate of change decreasing

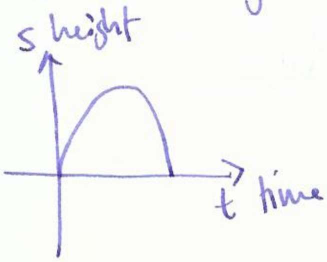


rate of change increasing





Motion under gravity



$$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$$

$$s'(t) = v(t) = v_0 - gt$$

$$s''(t) = v'(t) = a(t) = -g \text{ (constant) } g = 9.8 \text{ m/s}, \ 32 \text{ ft/s}.$$

$s_0 = s(0) =$ height at $t=0$

$v_0 = v(0) =$ speed at $t=0$

Q: ~~what~~ when is the max height?

A: when $v(t) = 0$: $v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$ so $s\left(\frac{v_0}{g}\right) = s_0 + \frac{v_0^2}{g} - \frac{1}{2} g \frac{v_0^2}{g^2}$

$$= s_0 + \frac{1}{2}\frac{v_0^2}{g}.$$

Example throw a stone upwards at 10 m/s from a height of 2 m. What is the max height?

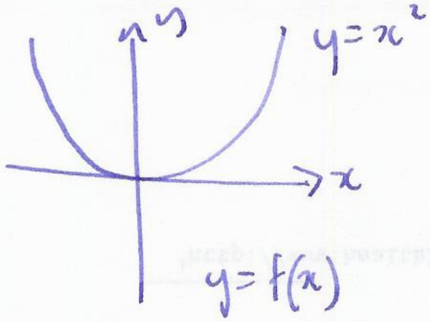
$s(t) = 2 + 10t - \frac{1}{2} g t^2$

$v(t) = 10 - gt$

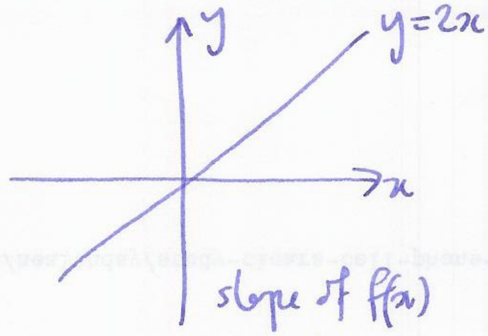
$v(t) = 0 \Rightarrow t = \frac{10}{g} \approx 1$ $\quad s(1) = 2 + 10 - 5 = 7\text{m}.$

Q: if I can throw a stone 10m high, how fast can I throw it?

§3.5 Higher derivatives



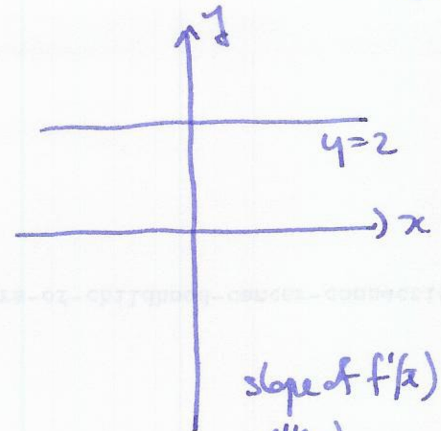
$y = f(x)$



slope of $f(x)$

$f'(x)$

"velocity"



slope of $f'(x)$

$f''(x)$

"acceleration".

Example $f(x) = xe^x$

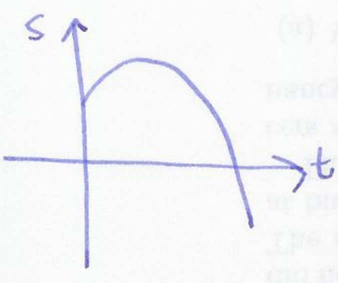
$f'(x) = xe^x + e^x$

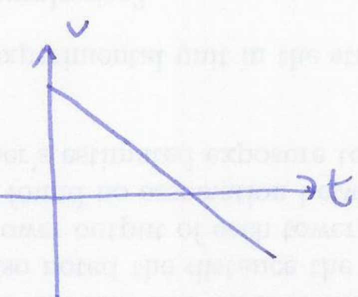
$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$

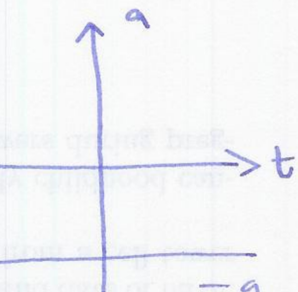
$f'''(x) = f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$ ok.

Example acceleration due to gravity, constant $-g\ m/s^2$

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$







§3.6 Trigonometric functions

Thm $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

Proof (for $\sin(x)$)

$$\frac{d}{dx}(\sin(x)) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

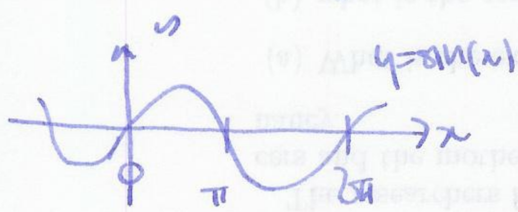
recall: $\sin(x+h) = \sin x \cos h + \cos x \sin h$

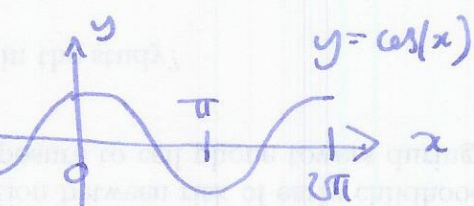
$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h} = \sin x \underbrace{\lim_{h\to 0} \frac{\cos(h)-1}{h}}_{=0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{=1}$$

$$= \cos(x) \quad \square$$

Q can this be right?





Example $f(x) = x \sin x$

$f'(x) = x\cos(x) + \sin(x)$

Thm $\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \qquad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \tan x$

Proof (of $\tan x$) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x - \sin x \cdot \overset{(-\sin x)}{\cancel{\sin x}}}{\cos^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x \quad \square$

Example $\frac{d}{dx}(e^x \cos x) = e^x(-\sin x) + e^x \cos x$.

§3.7 The chain rule

new functions from old: $f+g \quad fg \quad f/g$

what about composition: $f(g(x)) = (f\circ g)(x)$

<u>Examples</u> e^{4x}, $\sin^2(x)$, etc.

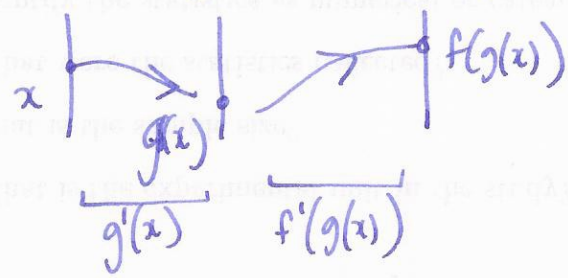
<u>Thm</u> <u>Chain Rule</u> If f and g are differentiable, then $f\circ g$ is differentiable and

$$\boxed{\left[f(g(x))\right]' = f'(g(x))\cdot g'(x)}$$

mnemonic: $\left[f(g(x))\right]' =$ (outside)'(inside) inside'

<u>Note</u> $f(g(x))$: $\mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$

$x \longmapsto g(x) \longmapsto f(g(x))$



<u>Examples</u>

① $e^{4x} = f(g(x))$ where $f(x) = e^x \quad f'(x) = e^x$

$g(x) = 4x \quad g'(x) = 4$

so $(e^{4x})' = f'(g(x))\,g'(x) = e^{4x}\cdot 4$

② $\sin^2(x) = f(g(x))$ where $f(x) = x^2 \quad f'(x) = 2x$

$g(x) = \sin(x) \quad g'(x) = \cos(x)$.

so $(\sin^2(x))' = f'(g(x))\,g'(x) = 2\sin(x)\cos(x)$.

③ $\sqrt{x^3+1}$, etc.