<u>Warning</u>: this rule works for polynomials only, not exponentials.

$f(x) = x^2$ power of x (also x^{100}, $x^{\sqrt{2}}$, x^{π})

$f(x) = 2^x$ exponential <u>not</u> a polynomial.

<u>Other useful rules</u>

<u>Thm</u> (Linearity) If f and g are differentiable functions, then $f+g$ is differentiable and $(f+g)' = f' + g'$

$$\Leftrightarrow \frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

also if k is a constant $(kf)' = kf'$

$$\Leftrightarrow \frac{d}{dx}(kf) = k\frac{d}{dx}(f).$$

<u>Proof</u> (follows from limit laws)

$$(f+g)'(x) = \lim_{h\to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}.$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = f'(x) + g'(x)$$

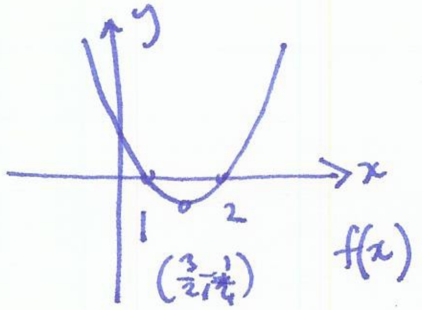
$$(kf)'(x) = \lim_{h\to 0} \frac{kf(x+h) - kf(x)}{h} = k \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = kf'(x). \quad \square.$$

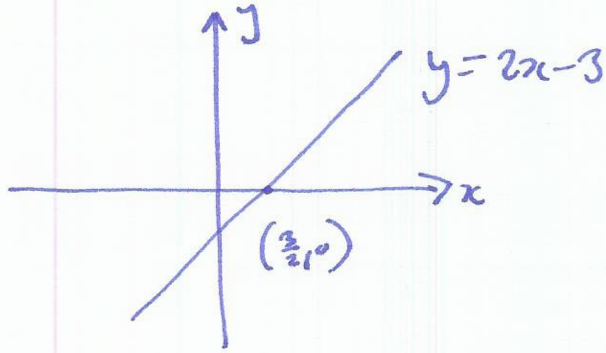
<u>Example</u> $f(x) = x^2 - 3x + 2$ find $f'(x)$

$$\frac{df}{dx} = \frac{d}{dx}(x^2 - 3x + 2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(2)$$

$$= \frac{d}{dx}(x^2) - 3\frac{d}{dx}(x) + \frac{d}{dx}(2) = 2x - 3 + 0.$$

graphs $f(x) = x^2 - 3x - 2 = (x-2)(x-1)$





Derivative of e^x

more generally, consider $f(x) = b^x$, $b > 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h \to 0} b^x \left(\frac{b^h - 1}{h} \right)$$

$$= b^x \underbrace{\lim_{h \to 0} \frac{b^h - 1}{h}}$$

doesn't depend on x!
assume this limit exists and call it m_b.

we have shown: for exponential functions derivative is <u>proportional</u> to value of function, i.e. if $f(x) = b^x$, $f'(x) = m_b b^x$

in particular, slope at $x = 0$ is $f'(0) = m_b b^0 = m_b$

<u>recall</u>: e is defined to be the special number such that the slope of e^x at $x = 0$ is equal to 1. Therefore if

$$\boxed{f(x) = e^x \text{ then } f'(x) = e^x} \Leftrightarrow \boxed{\frac{d}{dx}(e^x) = e^x}$$

<u>Example</u> differentiate $7e^x + 4x^2$

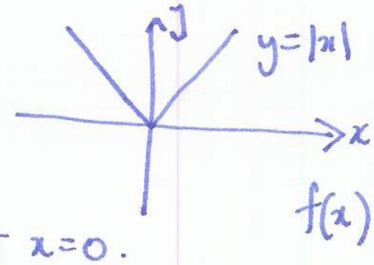
answer $7e^x + 8x$

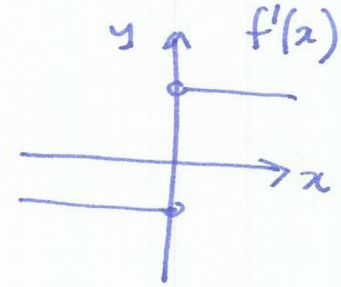
<u>Observation</u>: this shows e^x is <u>not</u> a polynomial! $\frac{d}{dx}(p(x)) \leftarrow$ degree goes down each time eventually get zero.

<u>Thm</u> Differentiable $\Rightarrow$ continuous.

<u>Warning</u> continuous $\not\Rightarrow$ differentiable

<u>Example</u> $f(x) = |x|$





continuous ✓

<u>claim</u>: not differentiable at $x=0$.

<u>check</u>: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{|x+h|-|x|}{h} = \lim_{h \to 0} \frac{|h|}{h}$ DNE

$\left(\lim_{h\to 0^+} \frac{|h|}{h} = 1, \lim_{h\to 0^-} \frac{|h|}{h} = -1 \right)$.

<u>Local picture</u>: if $f(x)$ differentiable at $x=c$, then if you look closely enough, the graph looks like a straight line.

<u>Proof</u> (differentiable $\Rightarrow$ cts)

$f(x)$ differentiable at $x=c$ means $\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ exists

(want to show $\lim_{h\to 0} f(c+h) = f(c)$)

consider $f(c+h) - f(c) = \frac{h\left(f(c+h)-f(c)\right)}{h}$

so $\lim_{h\to 0} f(c+h) - f(c) = \lim_{h\to 0} h \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ (product rule).

$= 0 \cdot f'(c) = 0$. $\square$

<u>§ 3.3 Product and quotient rules</u>

new functions from old: $f(x)g(x)$ $\frac{f(x)}{g(x)}$

product   quotient