Math 231 Calculus 1 Fall 13 Midterm 3b

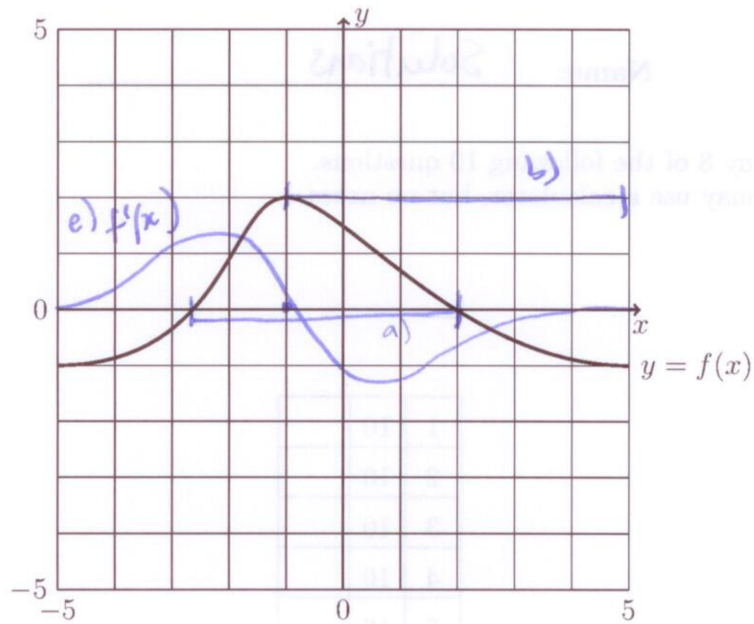
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f(x) > 0$.

(b) Label all regions where $f'(x) < 0$.

(c) What is $\lim_{x\to\infty} f(x)$? -1

(d) What is $\lim_{x\to\infty} f'(x)$? 0

(e) Sketch a graph of $f'(x)$ on the figure.

(2) (10 points) Consider the function $f(x) = \dfrac{1}{4-x^2}$.

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes: $4-x^2=0 \iff x = \pm 2$

horizontal asymptotes: $\lim\limits_{x\to\pm\infty} \dfrac{1}{4-x^2} = 0$

b) $f'(x) = -(4-x^2)^{-2} \cdot -2x = \dfrac{2x}{(4-x^2)^2}$

solve $f'(x)=0$: $x=0$

c)

$-$		$-$		$+$		$+$	$f'(x)$
	-2		0		2		

increasing $f'(x) > 0$ on $(0,2) \cup (2,\infty)$

decreasing $f'(x) < 0$ on $(-\infty,-2) \cup (-2,0)$

(3) (10 points) Consider the function $f(x) = xe^{3x}$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them.

a) $f'(x) = e^{3x} + x\,3e^{3x}$

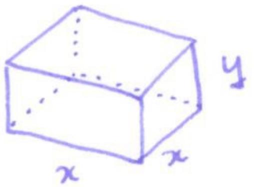
solve $f'(x) = 0$: $e^{3x}(1+3x) = 0 \Rightarrow x = -\frac{1}{3}$

b) $f''(x) = 3e^{3x} + 3e^{3x} + x \cdot 9e^{3x}$

$= e^{3x}(6+9x)$

$f''(-\frac{1}{3}) > 0 \Rightarrow$ local min

(4) (10 points) A cardboard box has a square base with sides of length x, and four vertical sides of height y, and no top. Find the dimensions of the box of volume 2m^3 with smallest surface area.



$$V = x^2 y = 2 \qquad y = \frac{2}{x^2}$$

$$A = x^2 + 4xy \qquad A = x^2 + \frac{4x \cdot 2}{x^2} = x^2 + \frac{8}{x}$$

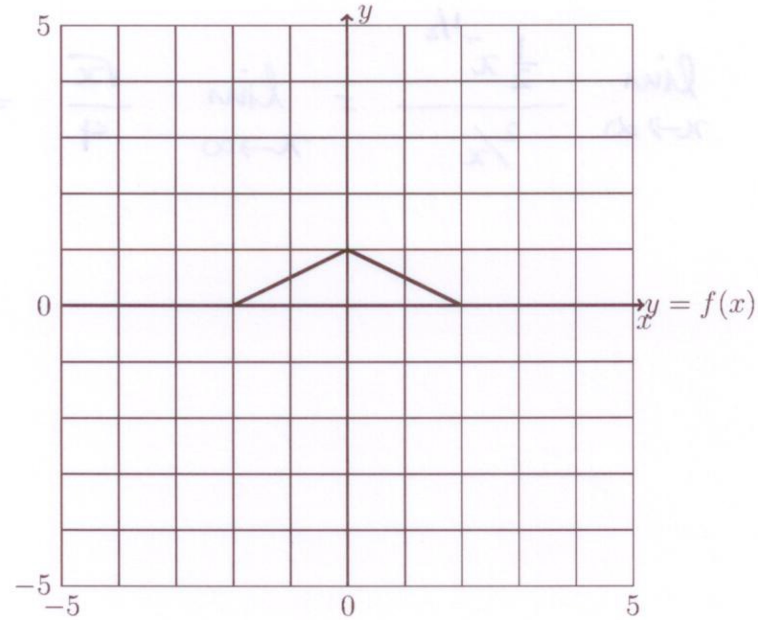
$$\frac{dA}{dx} = 2x - \frac{8}{x^2} \qquad \text{solve } \frac{dA}{dx} = 0 : \quad x^3 = 4 \qquad x = \sqrt[3]{4}.$$

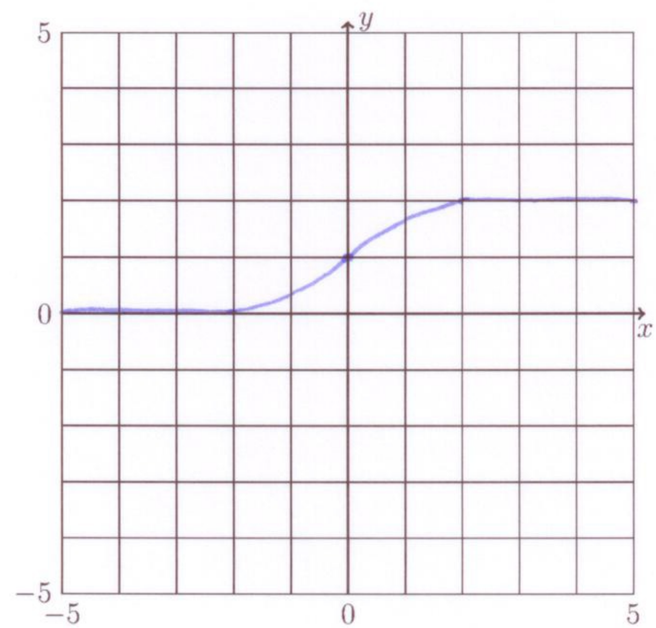
$$y = \frac{2}{4^{2/3}} = \frac{1}{2}$$

(5) (10 points) Find
$$\lim_{x\to 0} \frac{e^{2x}-1}{\sin x}.$$

l'H: $= \lim_{x\to 0} \frac{2e^{2x}}{\cos(x)} = 2$

(6) (10 points) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.





(7) (10 points) Which function grows faster, $\sqrt{x}$ or $\ln(x^2)$? Justify your answer. (Hint: take a limit.)

$$\lim_{x\to\infty} \frac{\sqrt{x}}{2\ln(x)} \underset{l'H}{=} \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{2/x} = \lim_{x\to\infty} \frac{\sqrt{x}}{4} = \infty$$

so $\sqrt{x}$ grows faster

(8) (10 points) Find the indefinite integral

$$\int 2\cos(x) - e^x \, dx.$$

$-2\sin x + e^x + C$

(9) (10 points) Evaluate the definite integral

$$\int_1^4 \frac{\sqrt{x}-1}{x}\,dx.$$

$$\int_1^4 x^{-1/2} - x^{-1}\,dx = \left[2x^{1/2} - \ln|x|\right]_1^4 = 4 - \ln 4 - 2$$

$$= 2 - \ln 4$$

(10) Find the area under the graph $y = 2x^2 + x$ between $x = 0$ and $x = 2$.

$$\int_0^2 2x^2 + x \, dx = \left[\frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^2 = \frac{16}{3} + 2 = \frac{22}{3}$$