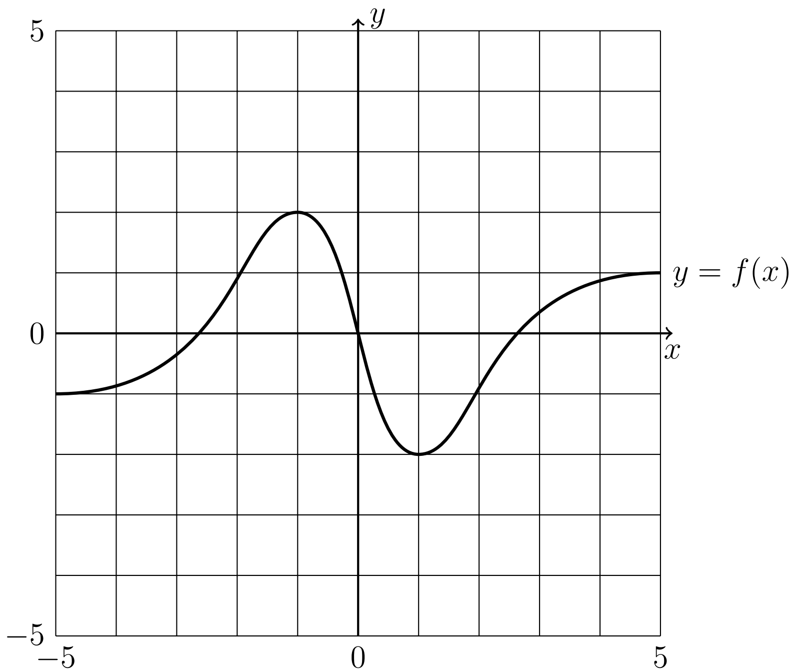
Math 231 Calculus 1 Fall 13 Sample Midterm 3

(1) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) What is $\lim_{x\to\infty} f'(x)$?
(d) What is $\lim_{x\to-\infty} f''(x)$?
(e) Sketch a graph of $f'(x)$ on the figure.
(f) Label the approximate locations of all points of inflection.

(2) Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = -1$ as its only critical point.

$$\begin{aligned} f(-1) &= 4 \\ f'(-1) &= 0 \\ f'(x) &> 0 \text{ for } x < -1 \\ f'(x) &< 0 \text{ for } x > -1 \\ \lim_{x\to\infty} f(x) &= \lim_{x\to-\infty} f(x) = -1 \end{aligned}$$

(3) Consider the function

$$f(x) = \frac{x}{27 - x^3}$$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.
(d) Use the 2nd derivative test to attempt to identify all local maxima and minima.
(e) Sketch the function and label all relative maxima and minima.

(4) Consider the following function:

$$g(x) = (x^2 - 2x)e^x$$

(a) Find, if they exist, the coordinates of all relative maxima and minima.
(b) Determine the interval(s) where g is increasing and those where g is decreasing.
(c) Find, if they exist, the coordinates of all points of inflection.
(d) Determine the intervals where g is concave up and those where g is concave down.
(e) Sketch the curve as accurately as possible.

(5) A function $f(x)$ has derivative

$$f'(x) = \frac{1}{e^{-2x} + 1}.$$

Where on the interval $[1, 4]$ does it take its maximum value?

(6) Take a circular piece of paper, and remove a sector of angle θ, and fold the remainder into a cone shape. Which angle θ gives the largest volume?

(7) Compute the following limits. Show all work.

(a)
$$\lim_{x\to-\infty} \frac{6x+2}{\sqrt{2x-4}}$$

(b)
$$\lim_{x\to 0+} \sqrt{x}\ln(x)$$

(c)
$$\lim_{x\to 0} \left(\frac{e^{2x}}{e^{2x}-1} - \frac{1}{2x} \right)$$

(d)
$$\lim_{x\to 0} \frac{3\sin x - \sin 3x}{\sin x - x\cos 3x}$$

(8) Evaluate the following

(a)
$$\int \frac{x^2-2x+1}{x} dx$$

(b)
$$\int 2e^x - 4\cos(x)dx$$

(c)
$$\int_1^2 3\sqrt[3]{x}dx$$

(d)
$$\int_0^t \frac{1}{x+1} dx$$

(9) Approximate the area under the graph of $y = e^{-x}$ between 0 and 2 using four rectangles. Use the right hand endpoints to find the heights of the rectangles. Can you say whether this is an under- or over-estimate?

(10) A particle starting at the origin at time $t = 0$ moves along the x-axis with velocity $v(t) = (t+1)^{-2}$. Will the particle ever reach $x = 1$?