Sample midterm 2 Solutions

Q1 a) $e^{\sqrt{2x+1}} + x\, e^{\sqrt{2x+1}} \cdot \frac{1}{2}(2x+1)^{-1/2} \cdot 2$

b) $$\frac{(2\cos(3x)-2) - x(-2\sin(3x)\cdot 3)}{(2\cos(3x)-2)^2} = \frac{2\cos(3x) - 2 + 6x\sin(3x)}{(2\cos(3x)-2)^2}$$

c) $x^x = e^{\ln(x)\cdot x}$, so: $e^{x\ln(x)}(\ln(x)+1) + \frac{1}{\tan(x)}\cdot \sec^2(x)$

d) $$\frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot -x^{-2} = \frac{-1}{x^2+1} = -(x^2+1)^{-1}$$

Q2 a) $e^{\sqrt{2x+1}}(2x+1)^{-1/2} + \left(x e^{\sqrt{2x+1}}\right)'(2x+1)^{-1/2} + x e^{\sqrt{2x+1}}\left((2x+1)^{-1/2}\right)'$

$$= \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}} + \left(e^{\sqrt{2x+1}} + \frac{x e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\right)\frac{1}{\sqrt{2x+1}} + x e^{\sqrt{2x+1}} \cdot -\frac{1}{2}(2x+1)^{-3/2}\cdot 2$$

b) $$\frac{(2\cos(3x)-2)^2\left[-6\sin(3x) + 6x\cos(3x)\cdot 3 + 6\sin(3x)\right] - \left[2\cos(3x) - 2 + 6x\sin(3x)\right]\cdot 2(2\cos(3x)-2)(-6\sin(3x))}{(2\cos(3x)-2)^4}$$

c) $$\frac{\sec^2(x)}{\tan(x)} = \frac{\cos(x)}{\cos^2(x)\sin(x)} = \frac{1}{\sin(x)\cos(x)} = \frac{2}{\sin(2x)} = 2(\sin(2x))^{-1}$$

so: $e^{x\ln(x)}(\ln(x)+1)^2 + e^{x\ln(x)}\cdot\frac{1}{x} - 2(\sin(2x))^{-2}\cdot\cos(2x)\cdot 2$

d) $(x^2+1)^{-2}\cdot 2x$

Q3 a) $h'(x) = f'(x)g(x) + f(x)g'(x)$

$$h'(4) = \underbrace{f'(4)}_{1/2}\underbrace{g(4)}_{-2} + \underbrace{f(4)}_{3/2}g'(4) = -1 - \frac{3}{2} = -\frac{5}{2}$$

b) $h'(x) = f'(g(x))\,g'(x)$

$$h'(4) = f'(\underbrace{g(4)}_{-2})\cdot\underbrace{g'(4)}_{-1} = -f'(-2) = -\frac{1}{2}.$$

Q4 $4x^2 + y^2 = 8$ implicitly differentiate wrt x

$8x + 2y\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = -\frac{4x}{y} \qquad$ at $(-1,2)$ $\quad \frac{dy}{dx} = 2$

tangent line: $\quad y - 2 = 2(x+1)$

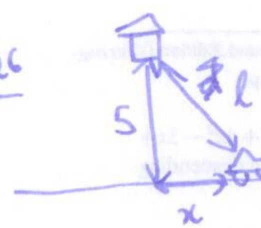
Q5 $x^3y^2 + 2xy^2 = x + y$ implicitly diff wrt x

$$3x^2y^2 + x^3 2y\frac{dy}{dx} + 2y^2 + 2x2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(2x^3y + 4xy - 1\right) = 1 - 3x^2y^2 - 2y^2.$$

$$\frac{dy}{dx} = \frac{1 - 3x^2y^2 - 2y^2}{2x^3y + 4xy - 1}$$

Q6



$\frac{dx}{dt} = 50 \qquad x^2 + 25 = l^2$

implicitly diff wrt t: $\quad 2x\frac{dx}{dt} = 2l\frac{dl}{dt}$

when $x = 2$, $l = \sqrt{29}$ $\quad \frac{dl}{dt} = \frac{2}{\sqrt{29}} \cdot 50.$

Q7 $124 = 5^3 - 1$ consider $f(x) = \sqrt[3]{x} = x^{1/3} \qquad f'(x) = \frac{1}{3}x^{-2/3}$

at $x = 125$ $\quad f(125) = 5 \quad f'(125) = \frac{1}{3}\cdot\frac{1}{25}$

percentage error $\left|\frac{5 - 1/75 - \sqrt[3]{124}}{\sqrt[3]{124}}\right| \times 100$

linear approx: $\quad f(x + \Delta x) \simeq f(x) + \Delta x f'(x)$

$f(124) \simeq 5 + (-1)\cdot\frac{1}{75}$

$\simeq 0.0007\%$

Q8 $f(x) = 2x^2 + 4x - 2$ $\quad$ critical points $f'(x) = 0$

$f'(x) = 4x + 4 \qquad 4x + 4 = 0 \Rightarrow x = -1$

no critical points in interval, so just check endpoints.

$f(1) = 2 + 4 - 2 = 4$ min

$f(5) = 50 + 20 - 2 = 68$ max.