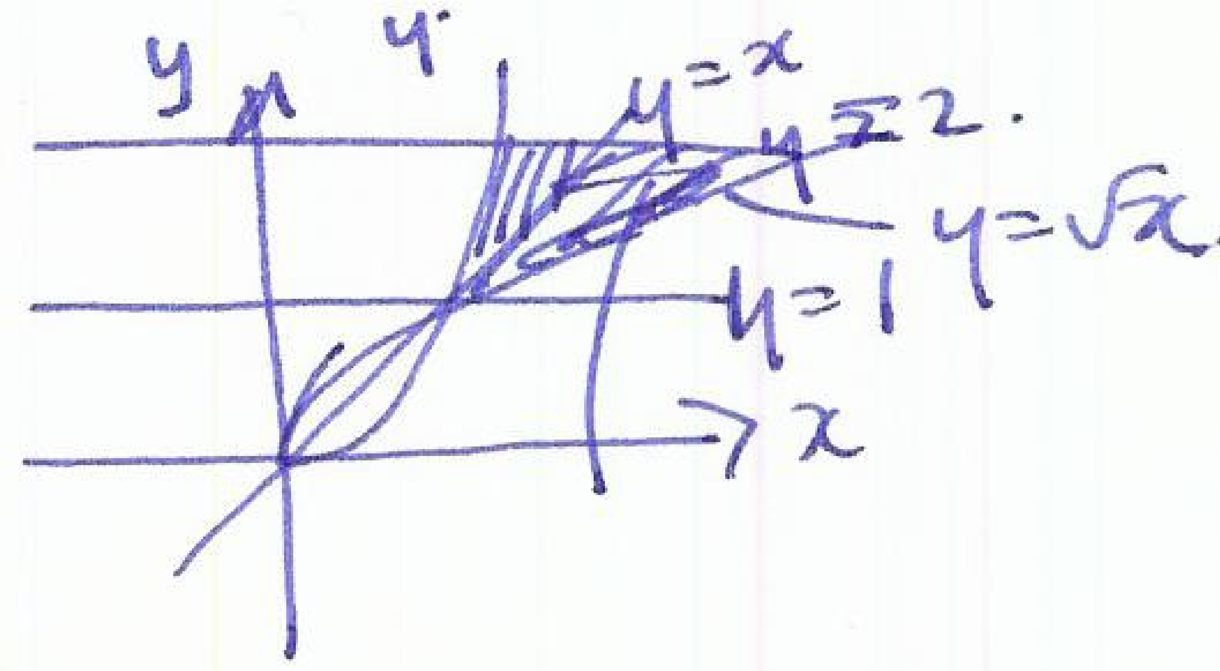
$$\underbrace{\int_1^2 \underbrace{\int_y^{y^2}}\, xy\, dx\, dy} = \int_1^2 \tfrac{1}{2}y^5 - \tfrac{1}{2}y^3\, dy = \left[\tfrac{1}{12}y^6 - \tfrac{1}{8}y^4\right]_1^2 = \text{some number.}$$

what do these limits mean? $1 \leq y \leq 2$

$y \leq x \leq y^2$

They describe a region in x-y-plane.



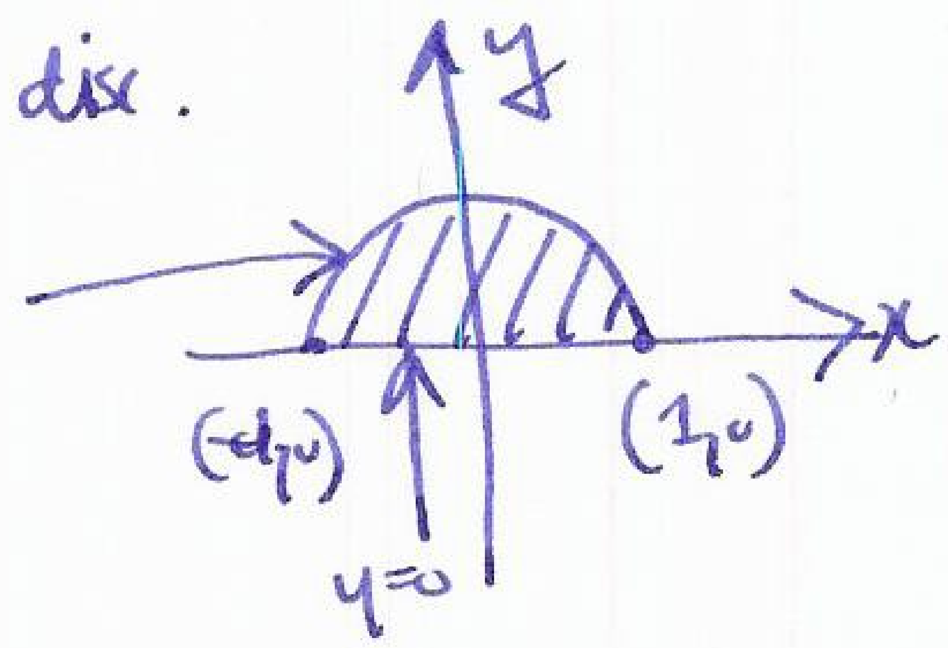
__notation.__ $\iint_D f(x,y)\, dy\, dx \qquad \iint_D f(x,y)\, dA$

__Example__ D = upper half disc.

$x^2+y^2=1$

$y=\sqrt{1-x^2}$

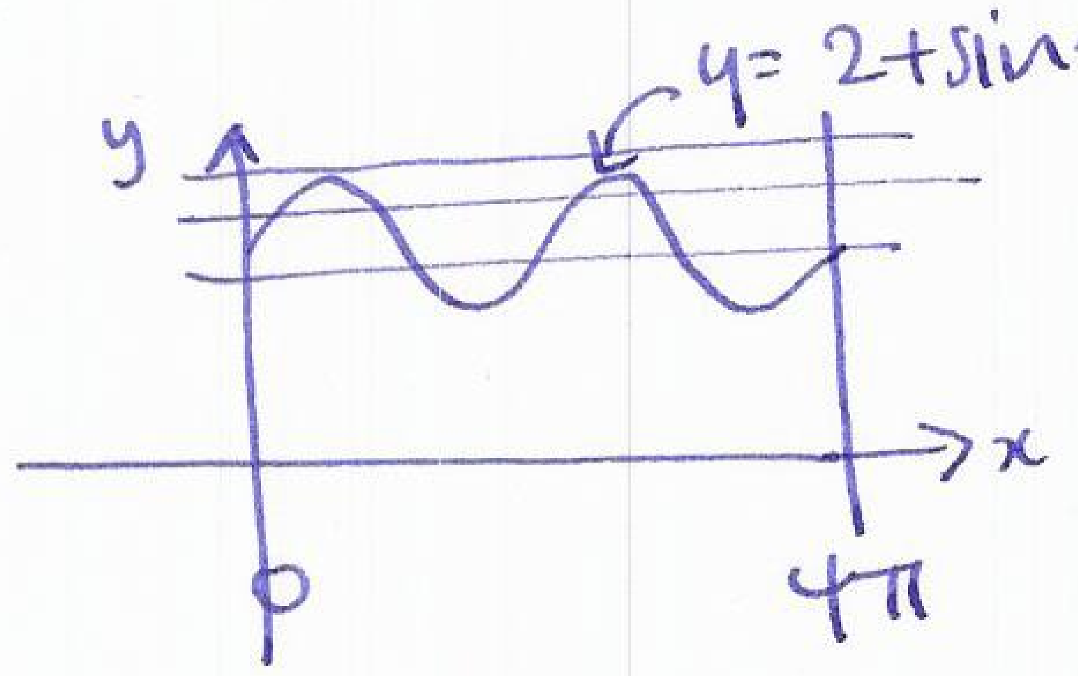
$x=\pm\sqrt{1-y^2}$



$$\int_0^1 \int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} f(x,y)\, dx\, dy$$

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x,y)\, dy\, dx$$

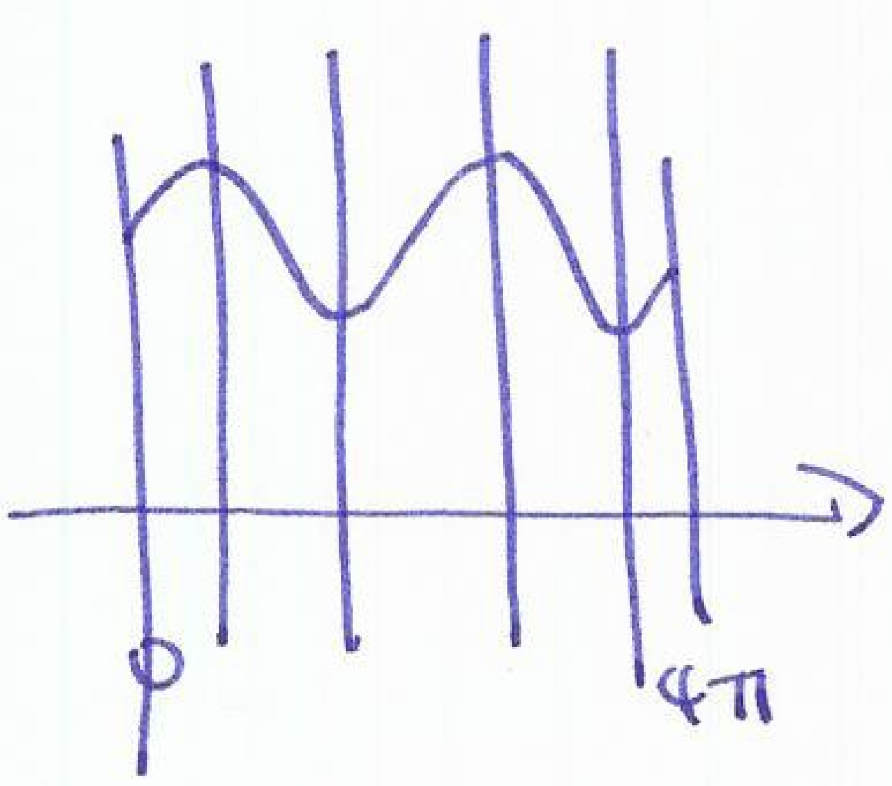
__Example__ D =



$$\int_0^{4\pi} \int_0^{2+\sin x} f(x,y)\, dy\, dx$$

$$\int_0^3 \int_{\star}^{\star} f(x,y)\, dx\, dy$$

↑ __problem__! boundary not a function of y!



← now each subregion has boundary which works.

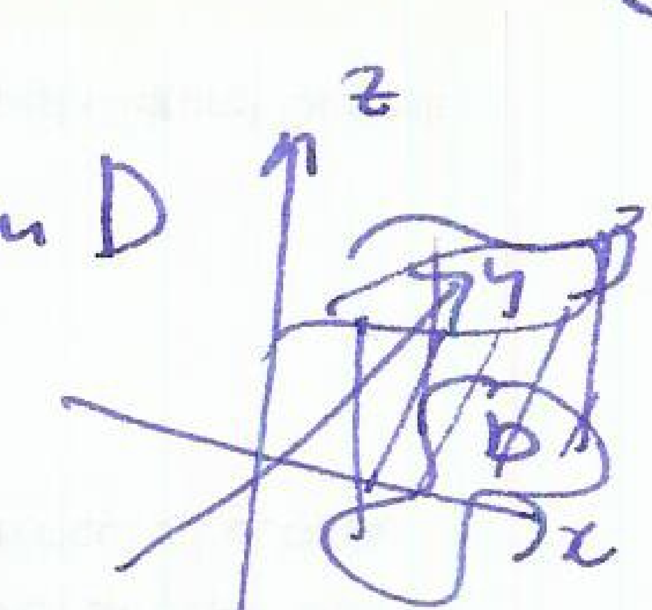
__solution__: chop up region.

__Example.__

can chop up __or__ do $\int_{\text{(shaded disc)}} - \int_{\text{(small disc)}}$

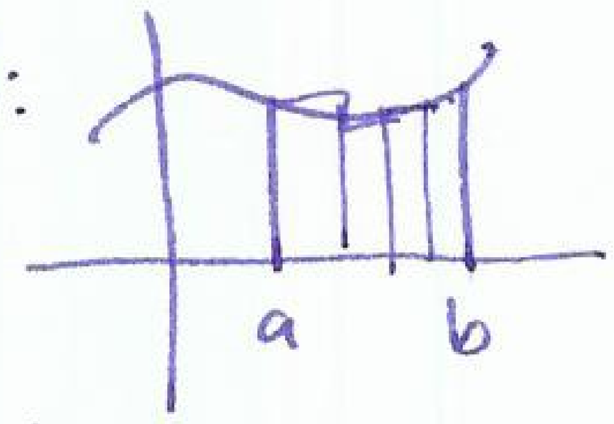
§15.2 Double integrals over general regions

<u>Notation</u>: $\iint_D f(x,y)\,dA$ means volume under surface over region D



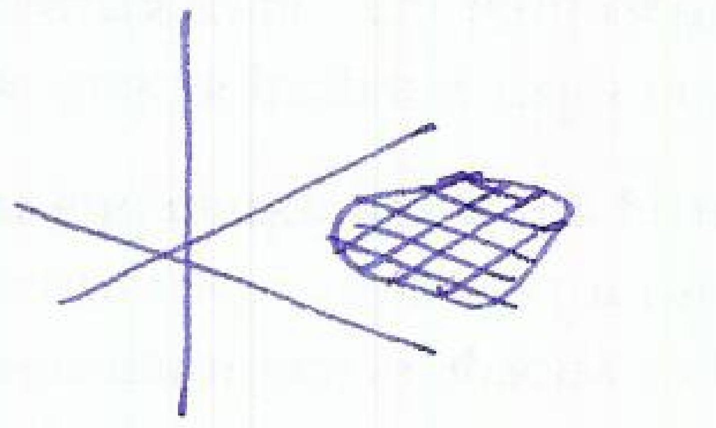
<u>Observation</u>: we can approximate integrals.

1d:



$\int_a^b f(x)\,dx \approx$ rectangle sum $\sum \underbrace{f(x_i)}_{\text{height}} \underbrace{\Delta x}_{\text{width}}$

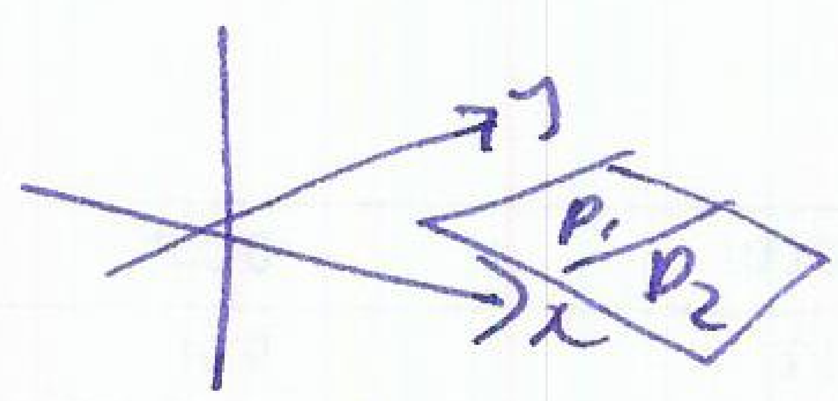
2d:



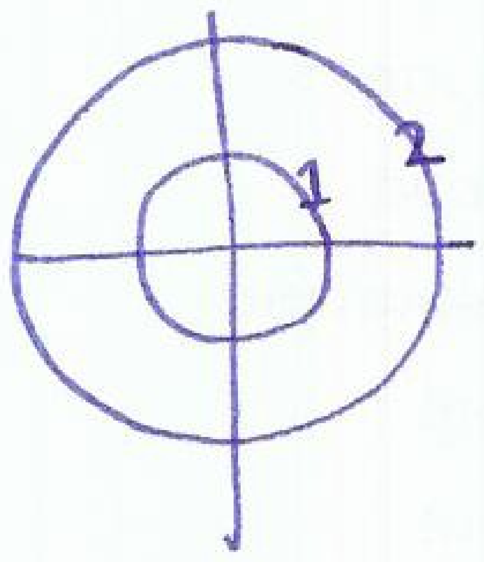
$$\iint_D f(x,y)\,dA \approx \sum \underbrace{f(x_i,y_i)}_{\text{hight}} \underbrace{\Delta x \Delta y}_{\text{base}}.$$

<u>Observation</u>: if a region D is the union of two subregions D_1, D_2 then

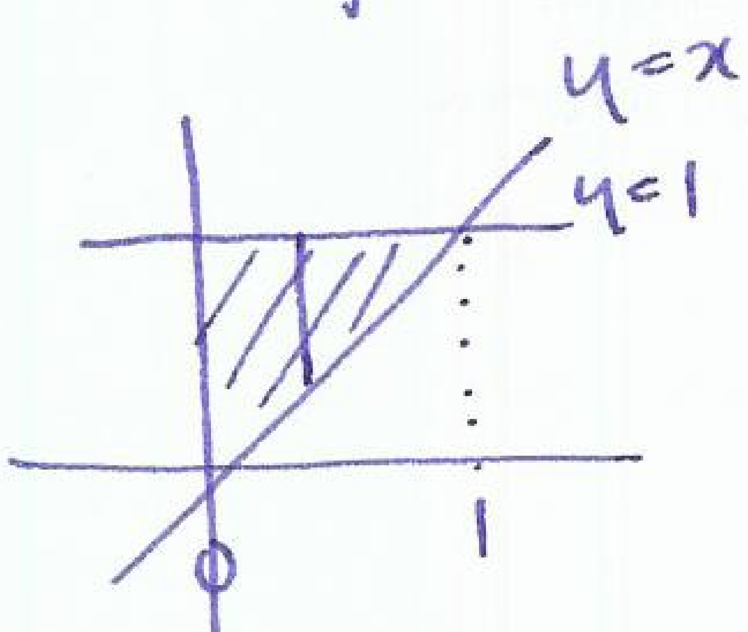
$$\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$$



<u>Example</u> find limits for:



$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{+\sqrt{4-x^2}} f(x,y)\,dy\,dx - \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} f(x,y)\,dy\,dx$$

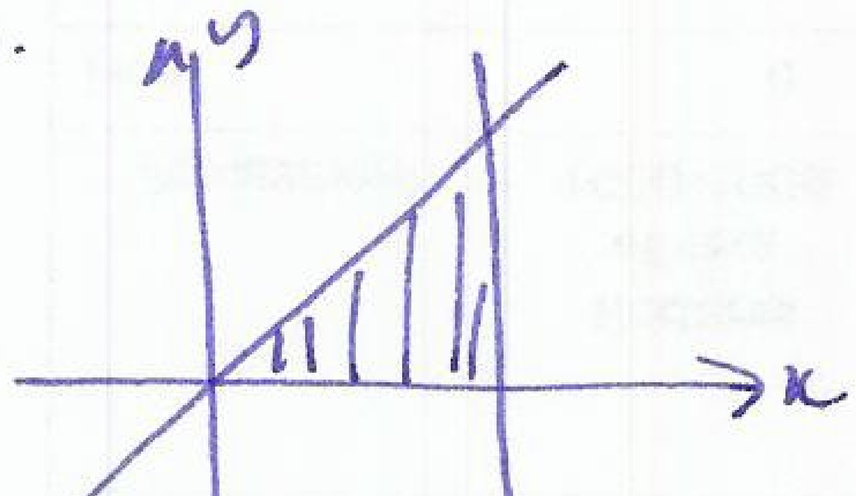


$$\int_0^1\int_0^y f(x,y)\,dx\,dy \qquad \int_0^1\int_x^1 f(x,y)\,dy\,dx$$

can now change limits.

translate limits into regions.

$$\int_0^1\int_y^1 \frac{\sin x}{x}\,dx\,dy$$



$$\int_0^1\int_0^x \frac{\sin x}{x}\,dy\,dx$$

can now do integral