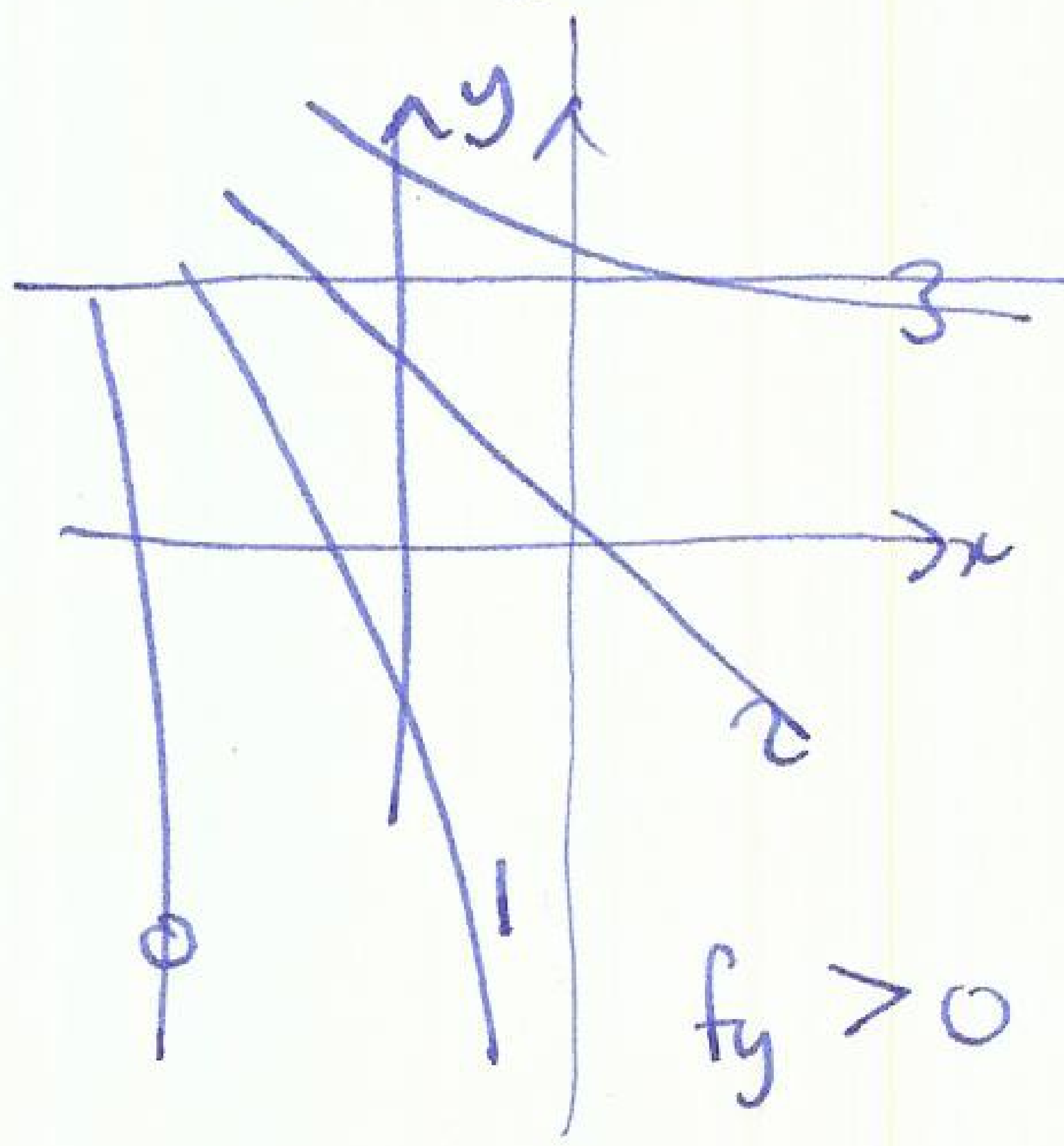
<u>Warning</u> f_x, f_y exist $\Rightarrow$ f is continuous!

Interpreting contour maps / level sets of $f(x,y)$



$f_x > 0$ (numbers on contours increasing).

$f_{xx} < 0$ (contour lines getting further apart)

$f_y > 0$ (numbers on contour lines increasing)

$f_{yy} > 0$ (contour lines getting closer together).

$f_{xy} > 0$ (contours get closer together in x-direction as we move up in y-direction).

<u>Examples</u> ① $f(x,y) = x\sin(x+y)$

$$f_x = \sin(x+y) + x\cos(x+y) \qquad f_y = x\cos(x+y).$$

② $$f(x,y) = \frac{ae^{xy}}{y} \qquad f_x = \frac{aye^{xy}}{x} \qquad f_y = \frac{yaye^{xy} - ae^{xy}}{y^2}$$

<u>Functions of 3 vars $f: \mathbb{R}^3 \to \mathbb{R}$</u>

$f(x,y,z)$. $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} =$ "diff wrt z keeping x,y fixed".

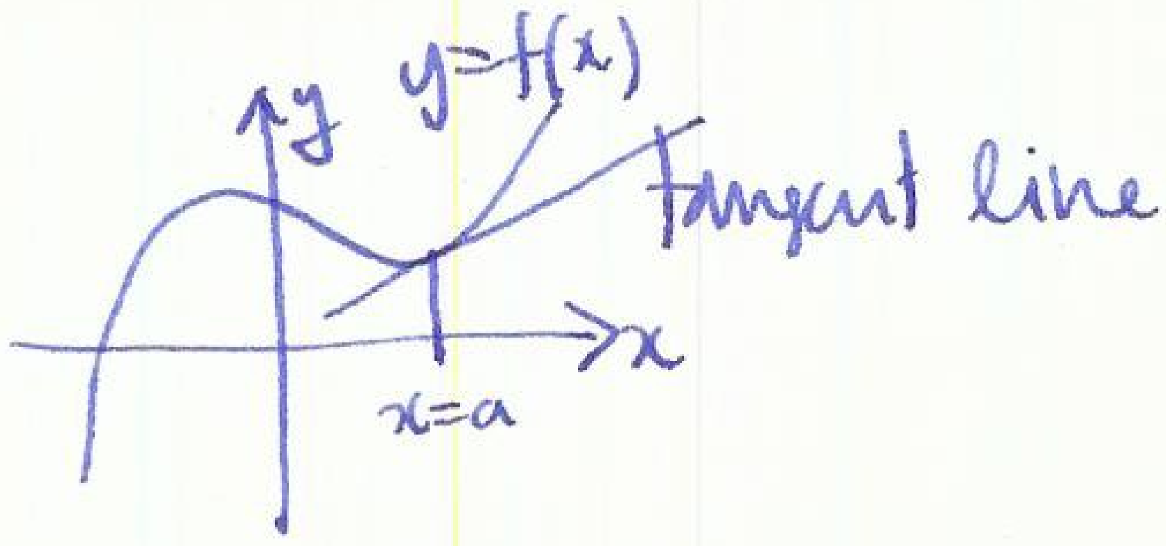
<u>Example</u> $f(x,y) = xy + yz + xyz$

$$f_x = y + yz \qquad f_y = x + z + xz \qquad f_z = y + xy.$$

<u>Th^m</u> If 2nd order derivatives are cts then mixed partials are equal, i.e. $f_{xy} = f_{yx}$ $\quad f_{xz} = f_{zx}$ etc.

§14.4 Differentiability, linear approximations and tangent planes

$f : \mathbb{R} \to \mathbb{R}$



$f : \mathbb{R}^2 \to \mathbb{R}$

linear approximation at $x=a$ is

$$L(x) = f(a) + f'(a)(x-a)$$

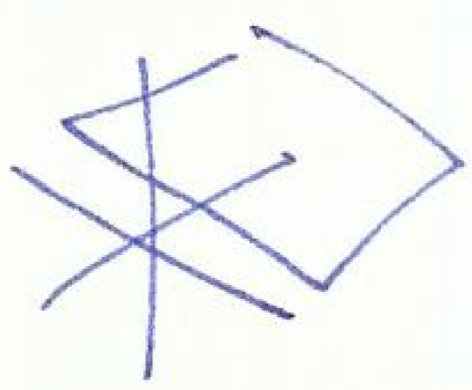
tangent line is $y = L(x)$

linear approximation is

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

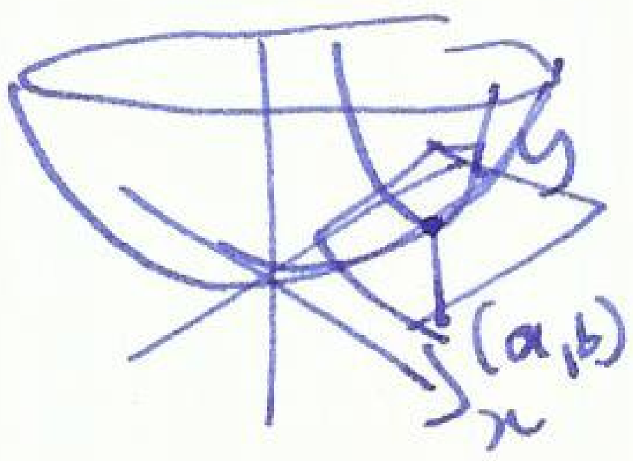
tangent plane is $z = L(x,y)$.

explanation consider plane $z = cx + dy$



slope in x-direction is $\frac{\partial z}{\partial x} = c$

y-direction is $\frac{\partial z}{\partial y} = d$.



at (a,b) slope in x-direction is $\frac{\partial f}{\partial x}(a,b) = f_x(a,b)$

y-direction $\frac{\partial f}{\partial y}(a,b) = f_y(a,b)$.

Example find tangent plane to $f(x,y) = x^2 + y^2$ at $(1,1)$

$f(1,1) = 2 \qquad \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = 2y$

so $L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$\qquad\qquad = 2 + 2(x-1) + 2(y-1)$.

Def^n^ Local linearity $f(x,y)$ is locally linear at (a,b) if $L(x,y)$ approximates $f(x,y)$ to first order, i.e.

$$f(x,y) = L(x,y) + \underbrace{\epsilon(x,y)}\sqrt{(x-a)^2 + (y-b)^2} \quad \text{for } (x,y) \text{ close to } (a,b)$$

any function s.t. $\epsilon(x,y) \to 0$ as $(x,y) \to (0,0)$.

Problem f_x, f_y exist $\not\Rightarrow$ locally linear.