§14.1 Functions of many variables

Examples height above sea level $h(x,y)$

temperature $t(x,y,z)$

Defn a function of many variables is a function $f: \mathbb{R}^n \to \mathbb{R}$ (domain, range).

or if $D \subset \mathbb{R}^n$ $f: D \to \mathbb{R}$

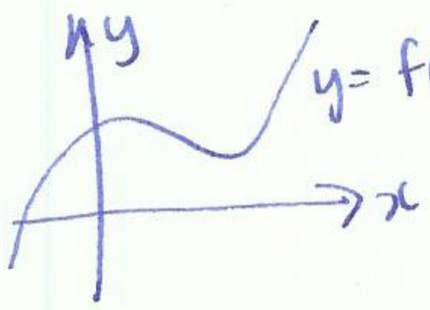
Examples $f: \mathbb{R}^2 \to \mathbb{R}$

$(x,y) \mapsto \sqrt{9-x^2-y^2}$ Q: what is D?

$f: \mathbb{R}^3 \to \mathbb{R}$

$(x,y,z) \mapsto x\sqrt{y} + \ln(z-1)$ Q: what is D?

Recall function of one variable: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto f(x)$ has a graph.

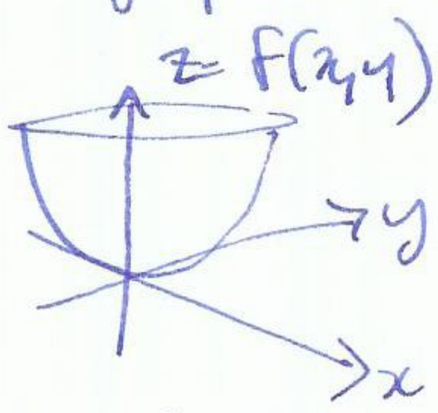


graph is a curve in $\mathbb{R}^2$

similarly we can draw the graph of a function of two variables.

$f: \mathbb{R}^2 \to \mathbb{R}$

$(x,y) \mapsto f(x,y)$



graph is now a surface in $\mathbb{R}^3$

Examples ① draw graph of $f(x,y) = x^2+y^2$.

traces: horizontal: solve $f(x,y) = c$ $x^2+y^2 = c$ (circles of radius $\sqrt{c}$)

vertical: fix $x=c$ or $y=c$ $f(x,c) = x^2+c^2$ (parabola)

$f(c,y) = c^2+y^2$ (")

② draw graph of $f(x,y) = x^2-y^2$

vertical traces are: $f(c,y) = c^2-y^2$ ∩

$f(x,c) = x^2-c^2$ ∪

③ linear functions: $f(x,y) = ax+by+c$.

the graph of a linear function is a plane ($z = ax+by+c$)

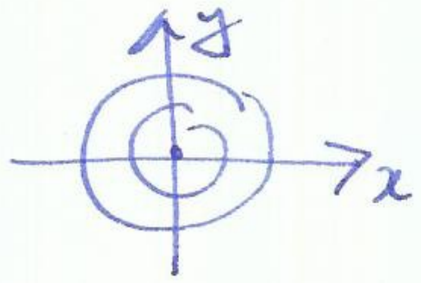
the traces are straight lines.

Contour maps

The horizontal traces are also known as contour lines or level sets.

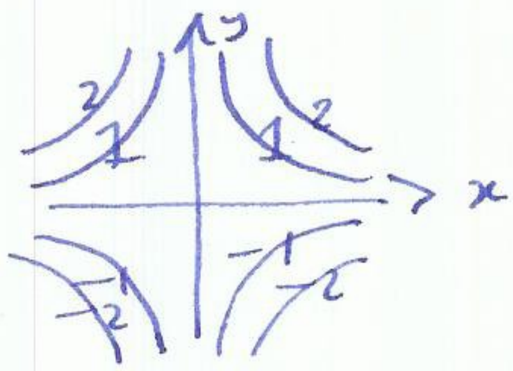
($\leftrightarrow$ solutions to $f(x,y)=c$). contour lines live in the domain!

Example ① $f(x,y) = \sqrt{4-x^2-y^2}$.

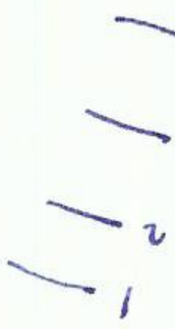


② $f(x,y) = x^2y$

$f(x,y)=c \leftrightarrow x^2y = c \leftrightarrow y = \frac{c}{x^2}$



On the interpretation of contours.



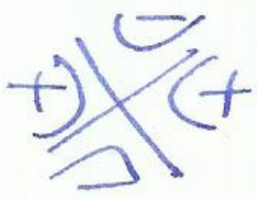
far apart $\leftrightarrow$ slow change/ shallow

close together $\leftrightarrow$ fast change/ steep

direction of fastest change $\perp$ to contour.

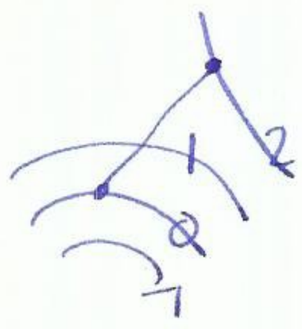
$\parallel$ contour stays same.

could be local max or min depending on labels.



saddle.

average rate of change: $\frac{\Delta f}{\Delta r}$



distance.

Functions of 3 vars.

graphs: live in $\mathbb{R}^4$, hard to draw.

however level sets $f(x,y,z)=c$ are surfaces in $\mathbb{R}^3$, so we can draw these.

Example ① $f(x,y,z) = x^2+y^2+z^2$