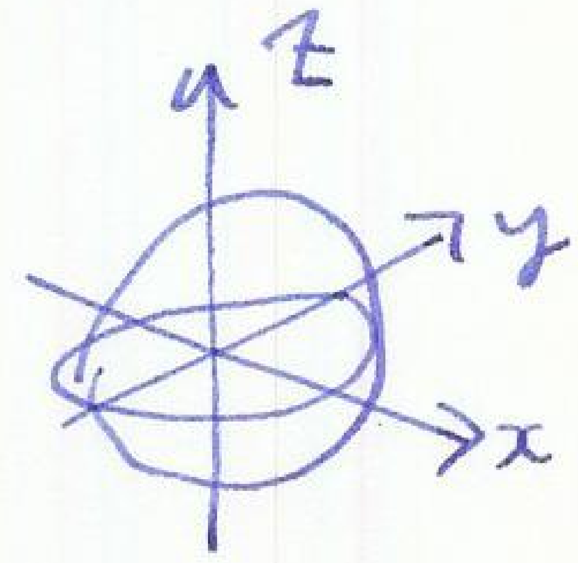
§12.6 Quadric surfaces

A quadric surface is defined by a quadratic equation in 3 variables.

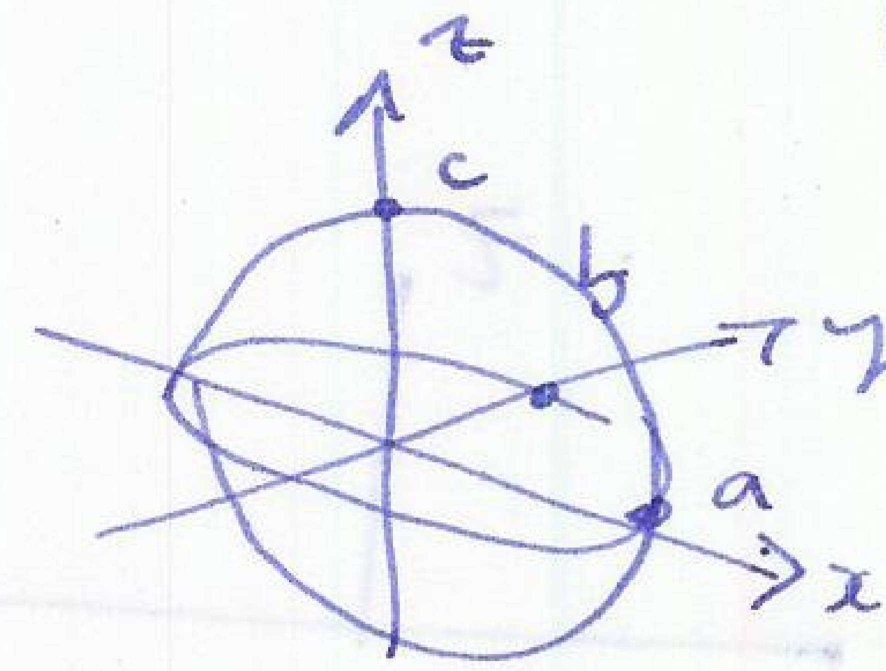
$$\left(Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + ax + by + cz + d = 0 \right).$$

↑ general quadratic in 3 vars.



Example sphere of radius r: $x^2+y^2+z^2=r^2$

ellipsoids: $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{d}\right)^2=1$



Traces: are intersections of the surface with planes parallel to the coordinate ~~axes~~ planes.

Example $\quad x^2+\frac{y^2}{4}+\frac{z^2}{9}=1.$

xy-trace (set $z=0$): $\quad x^2+\frac{y^2}{4}=1 \quad$ (ellipse in xy-plane)

yz-trace (set $x=0$): $\quad \frac{y^2}{4}+\frac{z^2}{9}=1 \quad$ (ellipse in yz-plane)

trace at height $z_0=1$: $\quad x^2+\frac{y^2}{4}+\frac{1}{9}=1.$

$$x^2+\frac{y^2}{4}=\frac{8}{9}. \quad \text{(ellipse).}$$

$z_0=3$: $\quad x^2+\frac{y^2}{4}+\frac{9}{9}=1$

$$x^2+\frac{y^2}{4}=0 \quad \text{(single point)}$$

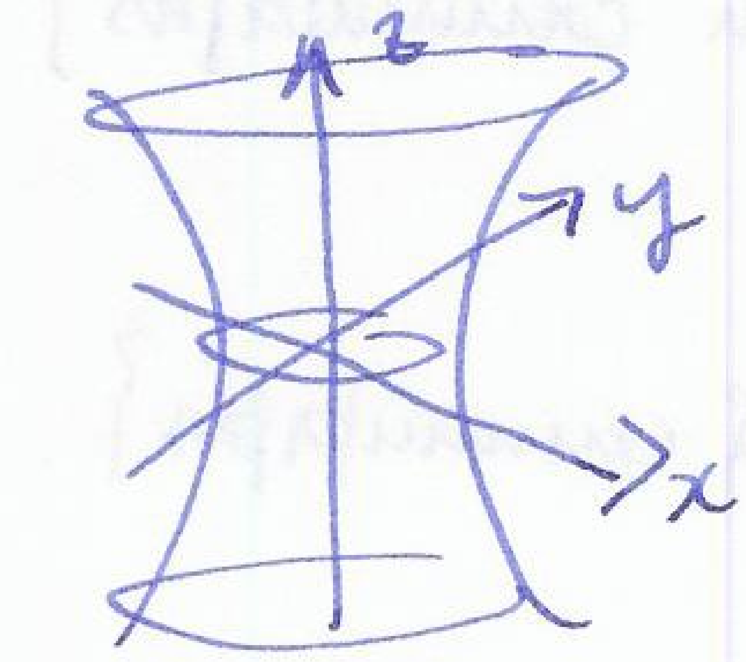
$z_0=4$: $\quad x^2+\frac{y^2}{4}\ \frac{16}{9}=1$

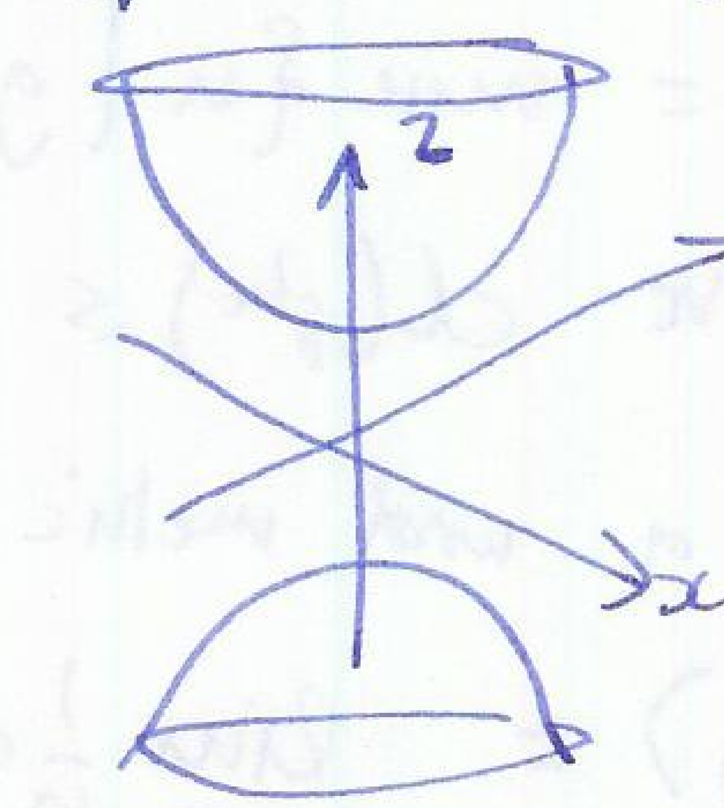
$$x^2+\frac{y^2}{4}=-\frac{7}{9} \quad \text{(empty: no solutions)}$$

<u>hyperboloids</u>

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=1 \qquad\qquad \left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=-1$$

hyperboloid of 1-sheet   hyperboloid of two sheets.





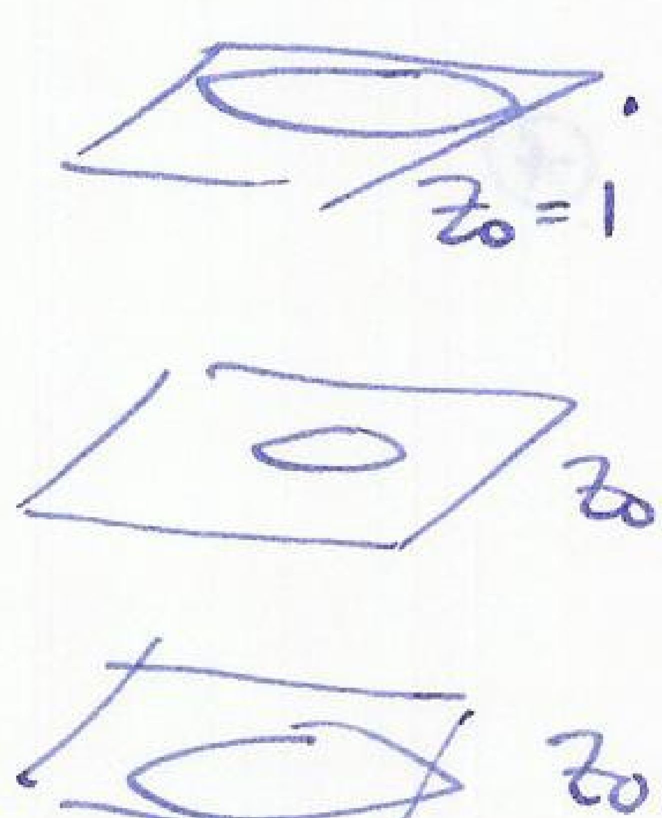
traces parallel to xy-plane: $z=z_0$.

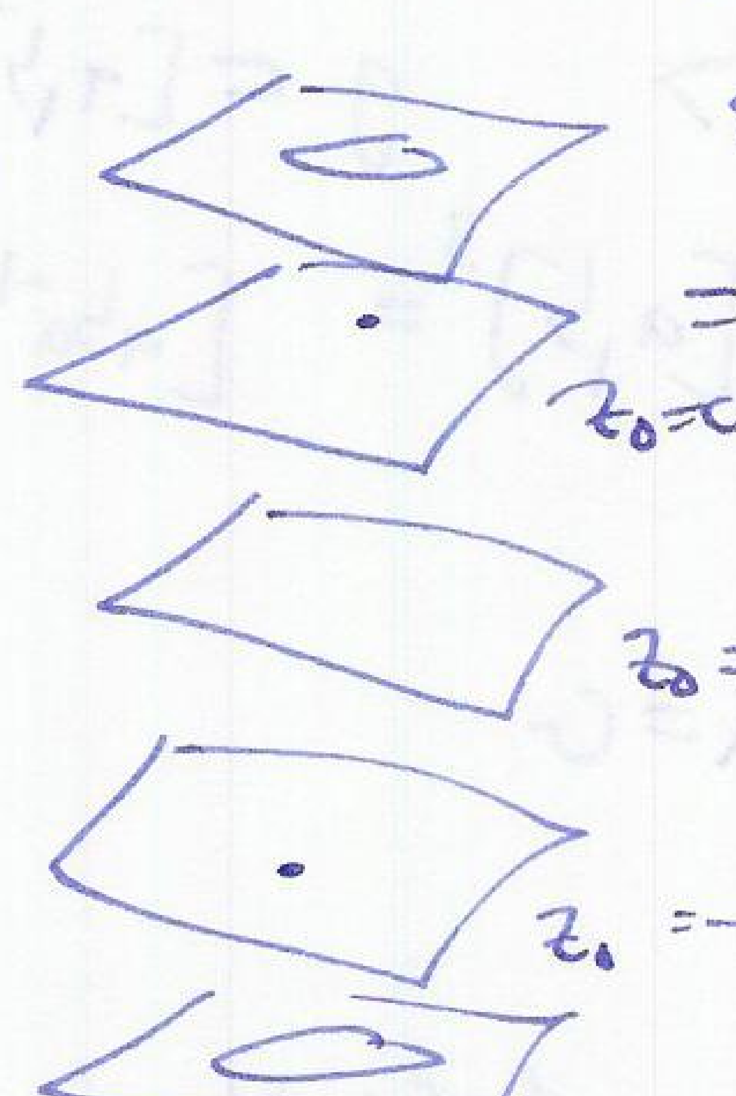
$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\underbrace{1+\left(\frac{z_0}{c}\right)^2} \qquad\qquad \left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\underbrace{\left(\frac{z}{c}\right)^2-1}.$$

- always positive, so always solutions
- smallest value $z_0=0$

<0 if $-c<z<c$ so no solutions here

$=0$ if $z=\pm c$ single point





These hyperboloids are symmetric about z-axis.

For symmetry about x-axis:

$$\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2=1+\left(\frac{x}{a}\right)^2 \qquad\qquad \left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2=\left(\frac{x}{a}\right)^2-1.$$

one-sheet.

(3) (15 points) The value of $\tan x$ at $\pi/4$ is 1. Use a linear approximation to estimate $\tan(0.8)$. Do you consider this to be a good approximation?

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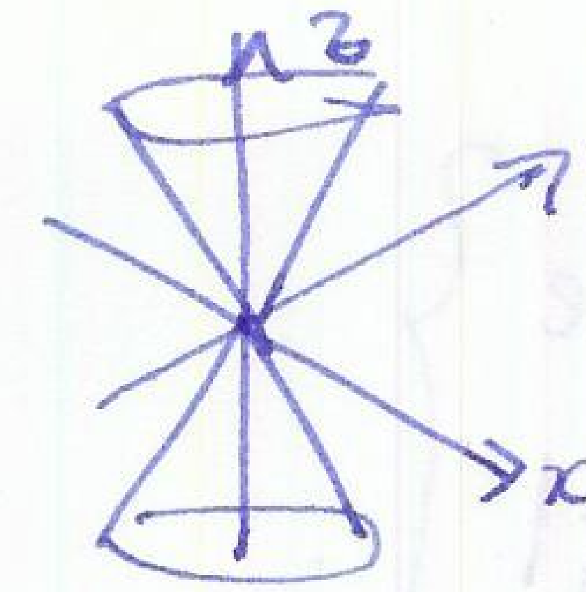
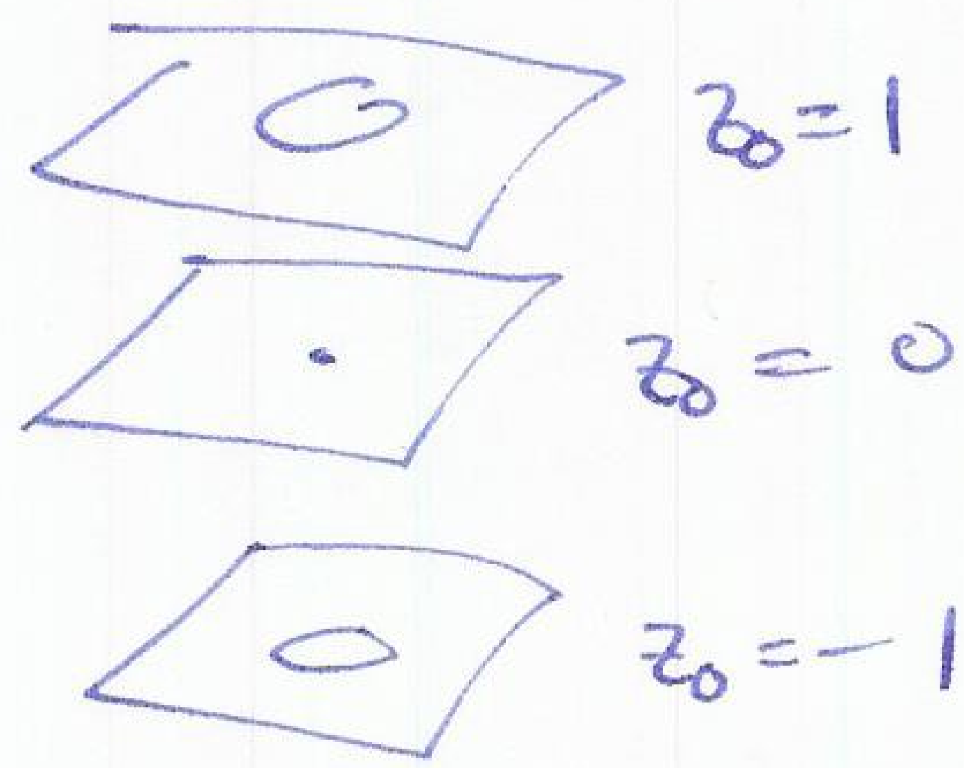
(elliptic) cones

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z}{c}\right)^2$$



traces parallel to xy-plane:

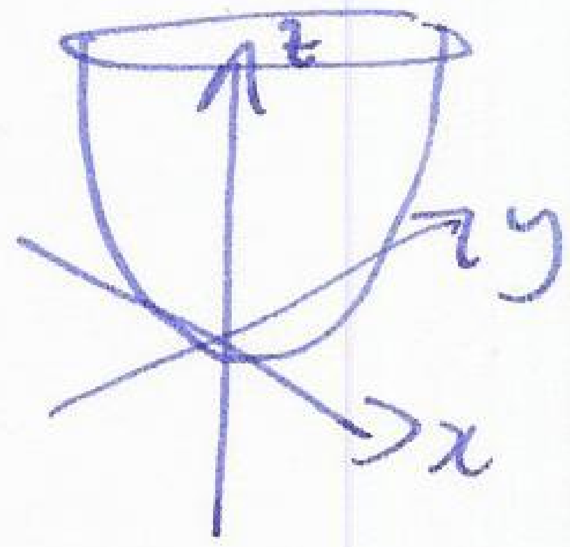
$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z_0}{c}\right)^2$$

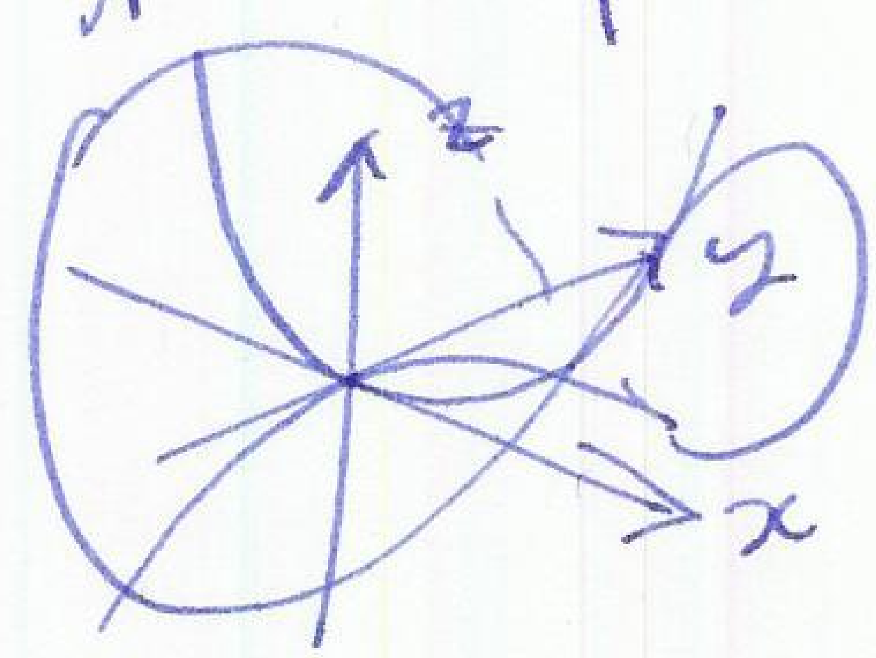


Paraboloids

$$z=\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2 \qquad z=\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2$$

elliptic paraboloid — hyperbolic paraboloid.





vertical traces are parabolas:

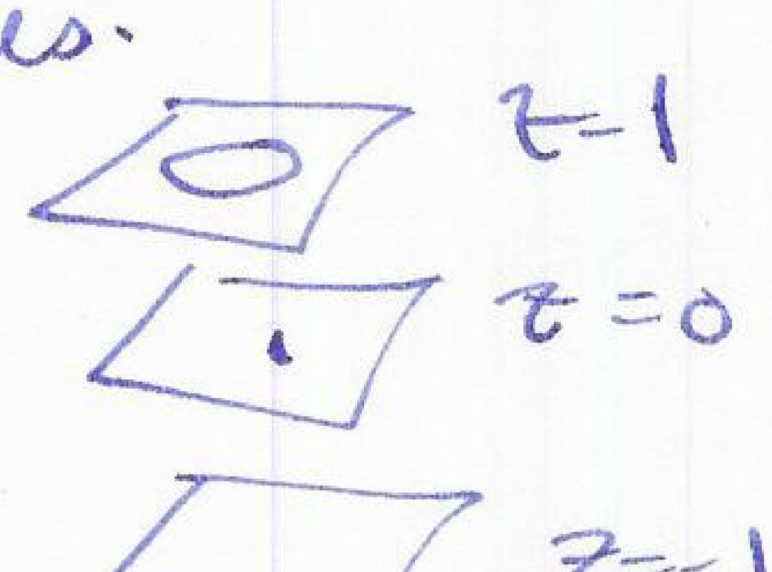
$x=x_0$:

$$z=\left(\frac{x_0}{a}\right)^2+\left(\frac{y}{b}\right)^2 \text{ etc.}$$

horizontal traces:

$$z_0=\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2 \qquad z_0=\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2.$$

ellipses. — hyperbola.

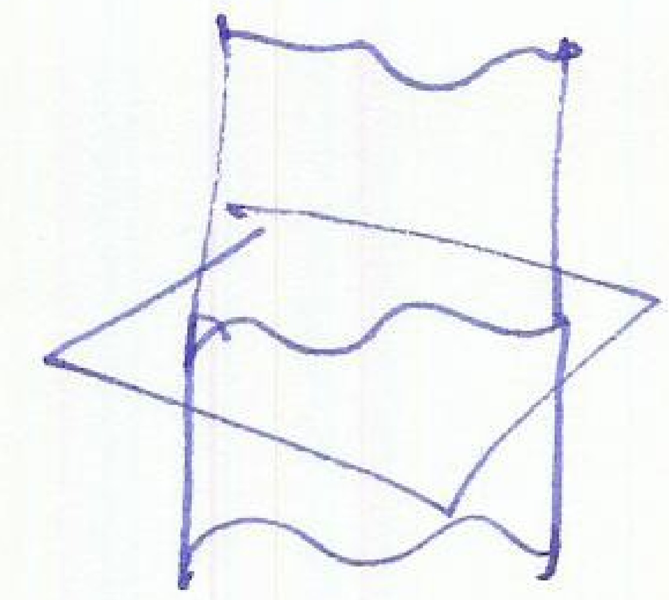


alternate form

$z=xy$.

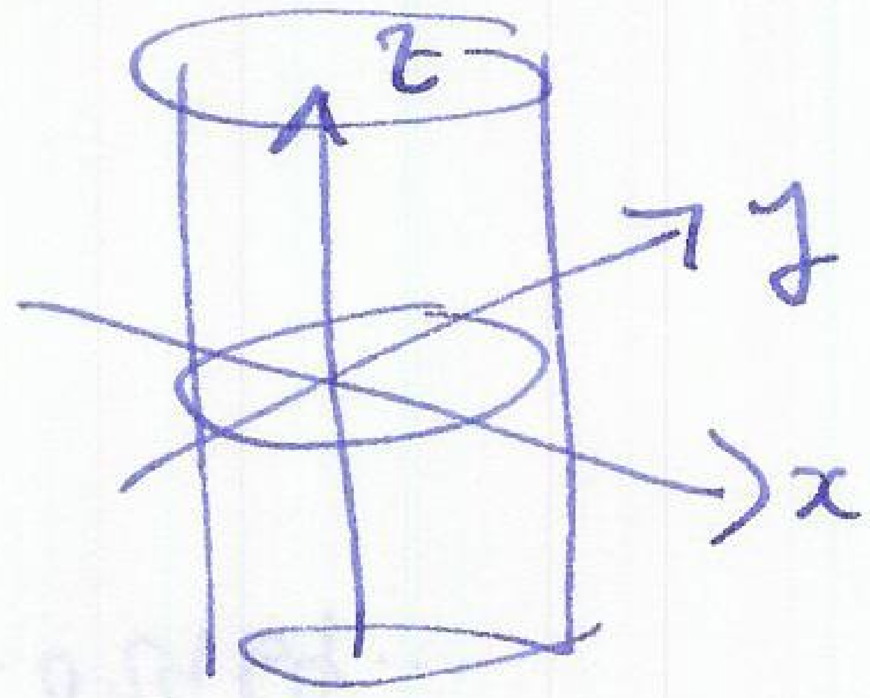
Cylinders

general cylinders: $\mathcal{C}$ some curve in xy-plane:

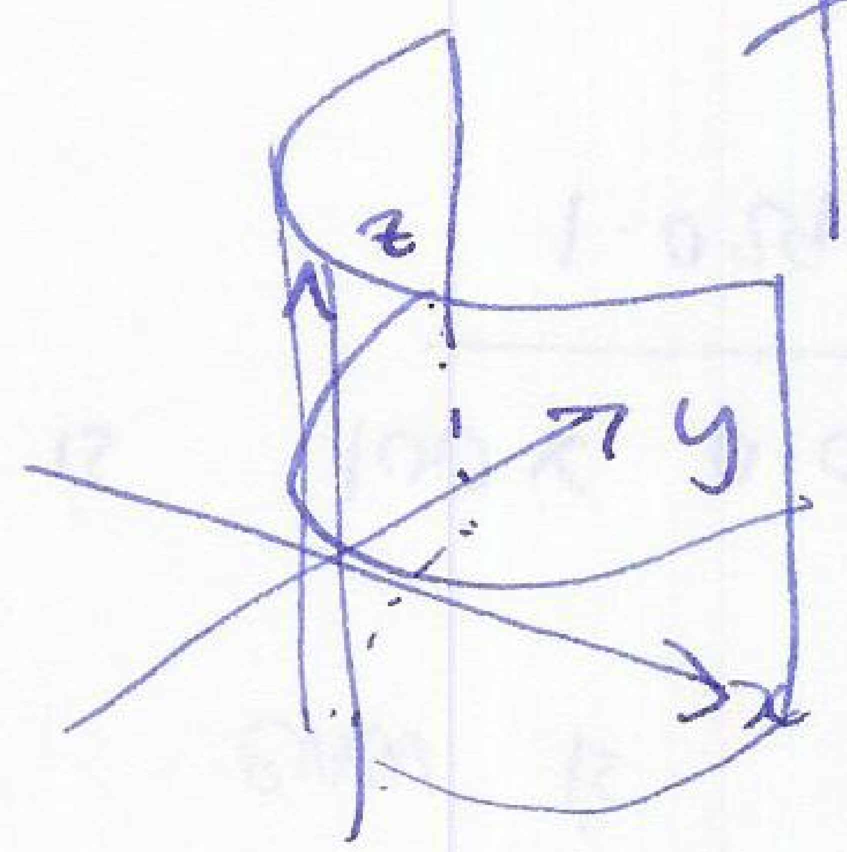


the cylinder over $\mathcal{C}$ is all points directly above or below $\mathcal{C}$.

Example $x^2+y^2=r^2$



$y=x^2$



$\left(\frac{x}{9}\right)^2+\left(\frac{y}{4}\right)^2=1$