Math 233 Calculus 3 Spring 12 Sample midterm 3

(1) Find the partial derivatives of $f \circ g$ using the chain rule, where

$$f(x, y) = (x^2 + y^2, xy)$$

and

$$g(x, y) = (x + y, x - y).$$

(2) Find the critical points of the following functions, and use the second derivative test to classify them, if possible.

(a)

$$f(x, y) = x^3 - xy + y^3$$

(b)

$$f(x, y) = e^x - xe^y$$

(c)

$$f(x, y) = x \ln(x + y)$$

(3) Use Lagrange multipliers to find the minimum and maximum values of $x^2y + x + y$ subject to $xy = 4$.

(4) Use Lagrange multipliers to find the dimensions of the cylindrical tin can of volume V with least surface area.

(5) Integrate the function $f(x, y) = xy$ over the triangle in the xy-plane with vertices $(0, 2), (4, 0)$ and $(4, 2)$.

(6) Consider the following integral:

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{y}{(1 + x^2 + y^2)^2} dxdy$$

(a) Evaluate the integral by changing the order of integration.
(b) Evaluate the integral by changing to polar coordinates.

(7) Write down limits for an integral over the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$ and $(1, 1, 1)$.

(8) Find the volume of the solid contained in the cylinder $x^2 + y^2 = 1$, below the surface $z = (x + y)^2$ and above the surface $z = -(x - y)^2$.

(9) Use spherical coordinates to evaluate the following integral.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}}dzdydx$$

(10) Use the map

$$(u,v) \mapsto (\frac{u+v}{2}, \frac{u-v}{2})$$

to evaluate

$$\int\int_R ((x-y)\sin(x+y))^2 dxdy$$

where R is the square with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi)$, and $(0, \pi)$.