Sample problems for Final for Math 233

- The final exam will be comprehensive, covering all sections covered in class. Changes to the syllabus: added Sections 13.10, 14.5, 14.8; and removed Sections 14.4, 15.1. The untested material will weigh more on the final exam.
- For problems on the material for Exams 1, 2, 3 please refer to the sample problems for those exams.

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1. Evaluate the following double integrals.
   (a) $\iint_D xy + 2x + 3y \, dA$ where D is the region in the first quadrant bounded by $x = 1 - y^2,\ x = 0,\ y = 0$.
   (b) $\iint_D xe^y \, dA$ where D is bounded by $x = 1,\ y = 0,\ y = x^2$.
   (c) $\iint_D xy \, dA$ where D is bounded by $y = 5 - x^2,\ y = x^2 - 3$.
2. Evaluate the following double integrals using polar coordinates.
   (a) $\iint_D (x^2 + y^2)^{3/2} \, dA$ where D is bounded by $y = 0,\ y = \sqrt{3}x,\ x^2 + y^2 = 9$.
   (b) $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x^2 + y^2 \, dy \, dx$
3. Evaluate the following integral. (You must switch the order of integration.)
   $\int_0^1 \int_{y/2}^{1/2} e^{-x^2} dx \, dy$
4. Find the surface area of the paraboloid $z = 16 - x^2 - y^2$ in the first octant.
5. Evaluate the following Triple integrals
   (a) $\iiint_E x^2 \, dV$ where $E = \{(x, y, z) | 0 \le x \le 2, 0 \le y \le 2x, 0 \le z \le x\}$.
   (b) $\iiint_T y \, dV$ where T is the tetrahedron bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $2x + y + z = 2$.
6. Evaluate the following integrals using Cylindrical or Spherical Coordinates.
   (a) $\iiint_E x^2 + y^2 \, dV$ where E is the region bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.
   (b) $\iiint_H z^5 \sqrt{x^2 + y^2 + z^2} \, dV$ where H is the solid hemisphere with the center at the origin and radius 1 which lies above the xy-plane.

7. Set up the integrals for volumes of the given solids and indicate with coordinates you will use.

   (a) The volume inside the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$. the planes $z = 0,\ y = 0,\ y = x,\ z = 1$ in the first octant.

   (b) The volume of a wedge of cheese bounded by the cylinder $x^2 + y^2 = 1$, and the planes $z = 0$, $z = 1$, $y = 0$, $y = x$.

   (c) The volume of the chocolate ice cream bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

   (d) The volume of the region between the paraboloids $z = x^2+y^2-1$ and $z = 1-x^2-y^2$.

   (e) The volume of the tetrahedron bounded by the plane $2x + 2y + z = 4$ in the first octant.

   (f) The volume bounded by the sphere of radius a.

8. Evaluate the integral by changing to an appropriate coordinate system.

   (a) $\displaystyle\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx$

   (b) $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{1+\sqrt{1-x^2-y^2}}^{1-\sqrt{1-x^2-y^2}} 2\, dz\, dy\, dx$

9. Use change of variables to evaluate the following integrals.

   (a) $\iint_R (x + y)\, dA$ where R is the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

   (b) $\iint_R (y - x)^2\, dA$ where R is the region bounded by lines $y = x,\ y = 2x,\ y = x + 2$ and $y = 2x - 1$. Hint: Use the transformation $T(u, v) = (u - v,\ 2u - v)$.