Math 233 Calculus 3 Spring 12 Review questions

(1) Let $f(x, y, z) = z^2 + xy$. Evaluate

$$\int_C f \, ds$$

where C is the straight line path from $(1, 1, 1)$ to $(4, 3, 2)$.

(2) Show that the vector field $\mathbf{F} = \langle y, -x, z \rangle$ is not conservative. Evaluate

$$\int_C \mathbf{F}.d\mathbf{s}$$

where C is the circle of radius 3 in the plane $z = 1$ centered on the z-axis.

(3) Show that the vector field $\mathbf{F} = \langle ye^{xy}, xe^{xy} + ze^{yz}, ye^{yz} \rangle$ is conservative, and find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$. Evaluate

$$\int_C \mathbf{F}.d\mathbf{s}$$

where C is the geodesic on the sphere of radius $\sqrt{6}$ centered at the origin between the points $(1, -1, 2)$ and $(-2, 1, 1)$.

(4) Write down limits for an integral over the region in the plane which lies in the intersection of two circle of radius 1, one centered at $(0, 0)$, and the other at $(-1, 0)$, in both cartesian and polar coordinates.

(5) Write down the limits for a triple integral over the tetrahedron with vertices $(1, 1, 0)$, $(1, 1, 1)$, $(1, 0, 1)$ and $(0, 1, 1)$.

(6) Write down limits for an integral over the region in the octant $x \geq 0, y \leq 0, z \leq 0$ inside the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 4$.

(7) Write down limits for the integral over region with $y \leq 0$, which lies below the negative cone $z^2 = -3x^2 - 3y^2$, and inside the sphere of radius 5.

(8) A rectangular/cuboid building loses four times as much heat per unit area from the roof than from the floor or the walls. What shape should the building be to minimize heat loss if the volume is 5000m^3?