MTH 231 College of Staten Island, CUNY Spring 2007

FINAL EXAM

NAME: Last 4 digits of your SS # :

ANSWER ALL QUESTIONS IN THE SPACE PROVIDED.
Please present clear solutions and fully explain your reasoning in complete sentences. Answers submitted without justification will not receive full credit. No books or notes are permitted. Calculator is permitted. ANSWERS SHOULD BE GIVEN BY CALCULUS NOT BY USING THE CALCULATOR

1. Find the following limits (5 points each)

(a) $\lim_{x\to 3} \dfrac{x+3}{x^2-9} =$

(b) $\lim_{x\to \frac{\pi}{2}} (\sin x + \cos x + \sin x \cos x + \dfrac{1}{2x}) =$

(c) $\lim_{x\to 0} \dfrac{x}{\sqrt{9+x}-3} =$

(d) $\lim_{x\to +\infty} \dfrac{-3x^4+2x^2-10}{5x^4-100x+3} =$

2./ (5 points) Consider the following function

$$g(x) = \begin{cases} 5 - 3\cos x & x \leq 0 \\ 7x + b & x > 0 \end{cases}$$

Select b such that $g(x)$ should be continuous on the whole real line.

3.(5 points each) Find the first derivative for each of the following functions:

(a) $f(x) = x^3 + xe^x + 3^3$

Be careful!

$f'(x) =$

(b) $f(x) = \dfrac{\sin x}{x^2 + 1}$

$f'(x) =$

(c) $f(x) = \sqrt{x^3 + \cos^2 x}$

$f'(x) =$

4. (8 points) Consider the curve described by the following equation $x^3 + y^2 + y^4 = 10$ The point $(2, 1)$ is on this curve. Use implicit differentiation to find the tangent line to the curve at this point.

5.(8 points) Find the **absolute** maximum and minimum of the following function

$$f(x) = \frac{x^2 + 8}{x + 1}$$

in the interval $[1, 3]$. Make a sketch of the function indication your finding.

6. (12 points) Investigate the following function

$f(x) = (x^2 - 1)^2$

(a) List the intervals on which $f(x)$ is increasing: _____

(b) List the intervals on which $f(x)$ is concave up:_____

(c) Find the relative maxima and relative minima, and clearly state which test you used.:_____

(d) Graph the function.

7.(5 points each) Find the following integrals:

(a) $\int_1^3 \left(x^2 + \frac{1}{x} + \sqrt{x}\right) dx =$

(b) $\int \left(\frac{e^{2x} + 3e^x + 1}{e^x}\right) dx =$

(c) $\int_0^1 \left(e^{x^3} x^2\right) dx =$

(d) $\int_0^{\pi/4} (\sec^2 x)\, dx =$

DO ANY TWO OF THE LAST THREE PROBLEMS. INDICATE CLEARLY WHICH TWO YOU ARE DOING

8. (6 points) Find all the values c in the interval $[a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

where $f(x) = x + \sin x$. and $[a, b] = [0, \pi]$ In other words apply the mean value theorem for the above function in the interval $[0, \pi]$. Explain how do you know that theorem can be applied.

9. (6 points) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at a constant rate 1.5 foot per second. When the radius is 3 feet, **what rate is the total area** A of the disturbed water **changing.**

10. (6 points) Even though you know very well how to calculate the integral

$$\int_0^1 (x+1)\,dx$$

make the calculation using an approximation sum with n **subintervals and taking the limit when** $n \to +\infty$. (It is up to you whether you make an upper or lower sum, but the upper sum is easier.)When you are done, do the regular integration as well to check your result. You might need some of the following formulas:

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} 1^3 = \frac{n^2(n+1)^2}{4}$$