

MTTH 231 - FINAL EXAM December 19, 2007

NAME: _____

Please place your name on the top of the paper. Show your work in the space provided for each question. Partial credit can only be given for work shown (!). GOOD LUCK!

(1) (12 points) Evaluate the following limits. If the limit is not defined, state this. If the limit is $\pm\infty$, indicate it.

a. $\lim_{x \rightarrow 1} x^2 + 2x - 8$

b. $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$

c. $\lim_{x \rightarrow 0} \frac{\sin 4x}{x}$

d. $\lim_{x \rightarrow \infty} \frac{4x^3 - 3}{2x^3 + 8x^2 - 27}$

(2) (6 points) Write down the limit definition of the derivative. Use this definition to compute the derivative of the following function.

$$f(x) = 10x + 4$$

(3) (12 points) Compute the derivative of the following functions using the rules of differentiation.

a. $f(x) = \sqrt{x^2 - 1}$

b. $h(x) = x^4 \sin x$

c. $q(x) = \frac{\cos^2(x)}{\sin(x)}$

(4) (5 points) In the space provided, sketch an accurate graph of a differentiable function f that satisfies the following conditions.

$$f(0) = 4 \quad f(8) = 0 \quad f(-4) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = -4$$

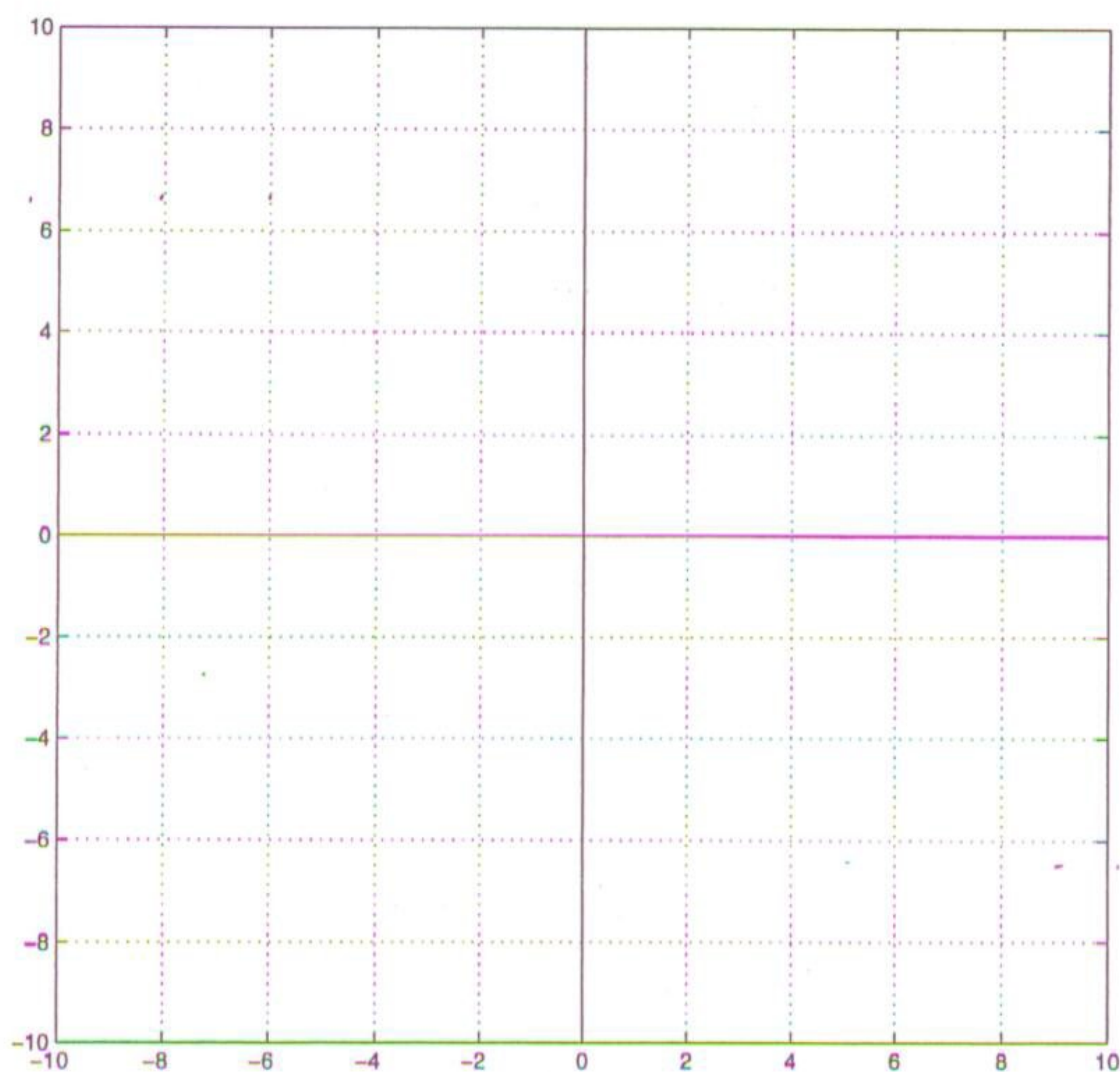
$$\lim_{x \rightarrow \infty} f(x) = 2$$

$$\lim_{x \rightarrow 6^-} f(x) = \infty$$

$$\lim_{x \rightarrow 6^+} f(x) = -\infty$$

$$f'(x) \geq 0 \quad \text{for} \quad -\infty < x < \infty$$

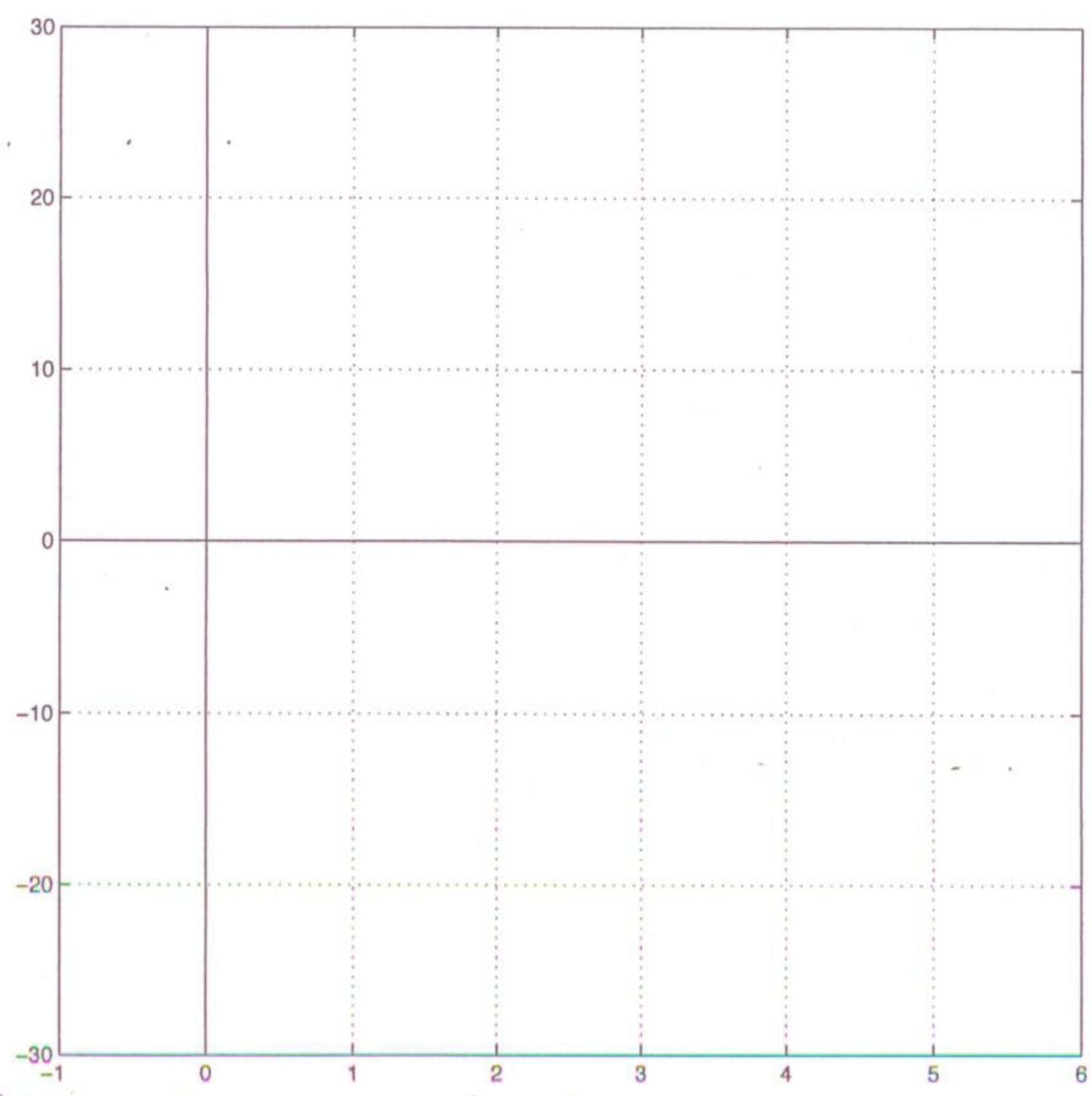
$$f'(0) = 0$$



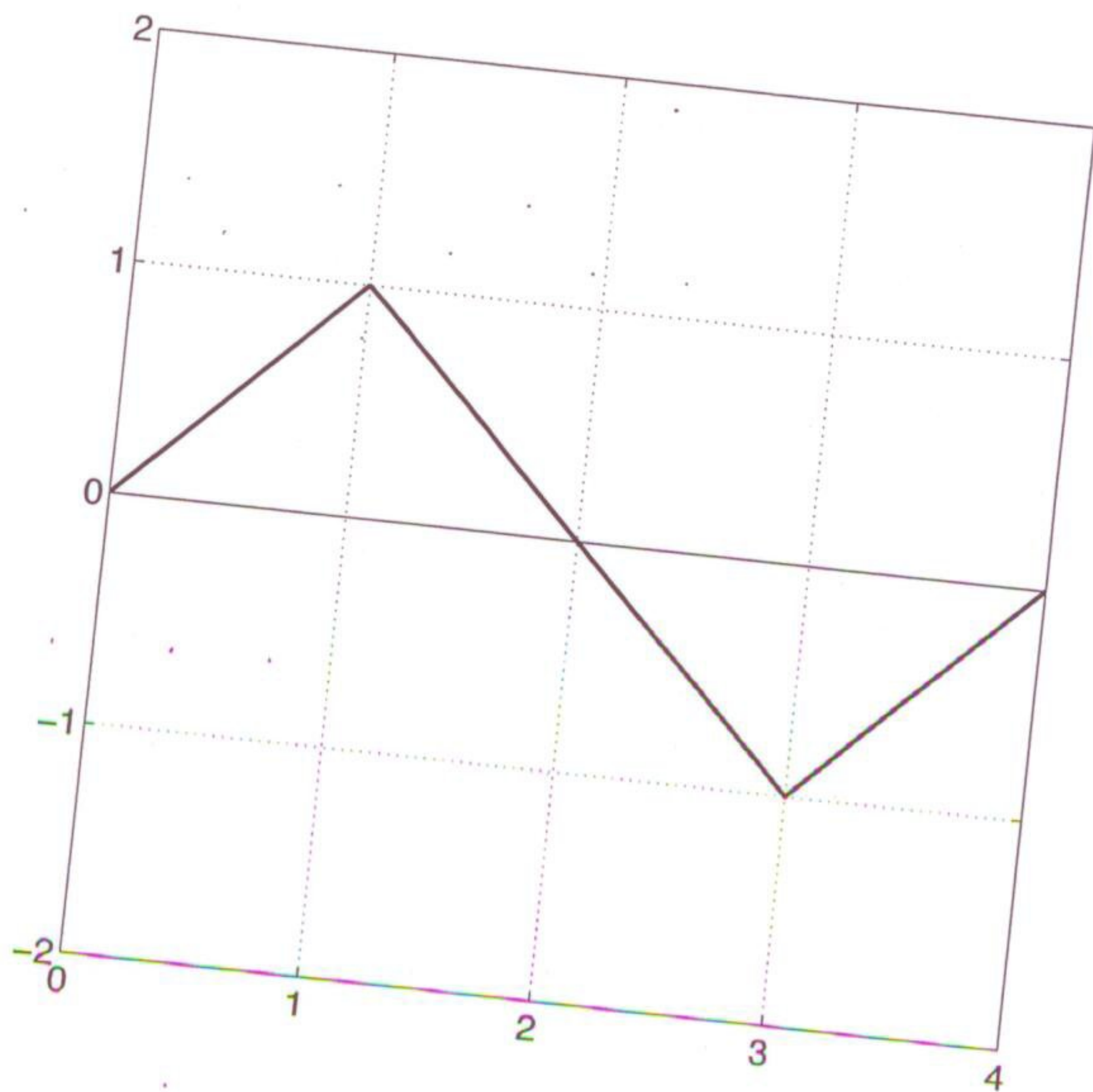
(5) (12 points) Consider the following function:

$$g(x) = x(x - 4)^3$$

- a) Calculate the derivative and find all critical points of the function.
- b) Determine the interval(s) where g is increasing and those where g is decreasing.
- c) Use the first derivative test to find the coordinates of all relative maxima and minima.
- d) Find, if they exist, the coordinates of all points of inflection and determine the intervals where g is concave up and those where g is concave down.
- e) Sketch the curve as accurately as possible in the space provided on the next page. Label the points and regions found in parts a-d.



(6) (12 points) Consider the function $f(x)$ defined by the graph below.



- Sketch a graph of $f'(x)$ on the figure.
- Calculate $\int_0^1 f(x) dx$.
- What is the average value of $f(x)$ on the interval $[0, 2]$?
- Sketch a graph of $F(x) = \int_0^x f(t) dt$ on the figure.

(7) (12 points) Short Answers - Provide numerical answers to each of the following. ASSUME $f(x)$ is a DIFFERENTIABLE FUNCTION defined everywhere in each.

a. If $f(10) = 20$ and $f(0) = -10$ then $f(x) = 0$ on what interval?

b. If $f(10) = 20$ and $f(0) = -10$ then $f'(x)$ MUST have what value on the interval $0 < x < 10$?

c. If $f(a) = 8$ and $f(b) = 16$ and $b > a$, then there must exist some c , $a < c < b$, where $f(c) =$

d. If $\int_{-5}^5 f(x) dx = 50$, then there must exist some c , $-5 < c < 5$, where $f(c) = ?$

(8) (20 points) Calculate the following integrals:

a) $\int_0^3 x^2 - 1 dx$

b) $\int 10e^{5x} dx$

c) $\int \frac{x}{(x-2)^2} dx$

d) $\int \sin^3 x \cos x dx$

e) $\int \frac{\ln x^2}{x} dx$

(9) (9 points) Calculate the derivatives ($df/dx = f'$) of the functions given below

a) $f(x) = \ln(x^2 + 4)$

b) $f(x) = y = x^{e^{3x}}$ (Hint: Use natural log?)

c) $f(x) = e^{e^{e^x}}$