<u>observations</u>

① choice of anti-derivative doesn't matter: Let $F(x)$ and $F(x)+c$ be anti-derivatives for $f(x)$. Then

$$\int_a^b f(x)\,dx = F(b)-F(a)$$

$$= F(b)+c-(F(a)+c) = F(b)-F(a)$$

② $\int_a^t f(x)\,dx$ is a function of t! x is a <u>dummy variable</u>

i.e. $\int_a^t f(x)\,dx = \int_a^t f(y)\,dy$.

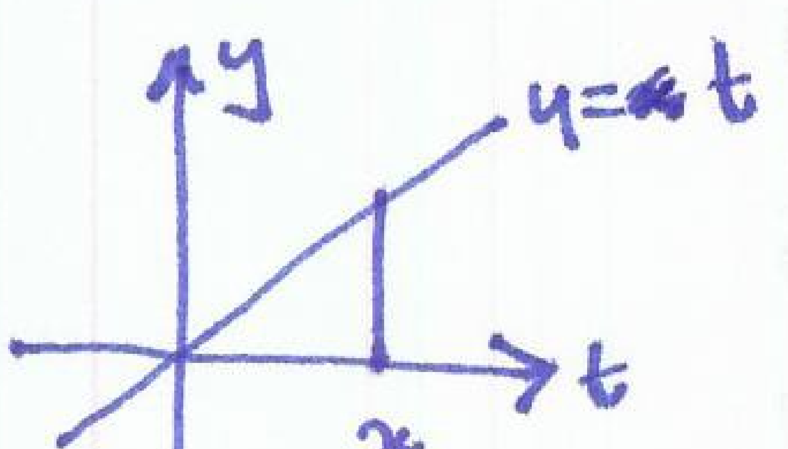
§5.4 Fundamental Thm of calculus II

<u>Thm</u> (FTC II) Let $f(x)$ be continuous on $[a,b]$, then

$A(x) = \int_a^x f(t)\,dt$ is anti-derivative of $f(x)$, i.e. $A'(x) = \frac{dA}{dx} = f(x)$.

i.e. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. Furthermore $A(a)=0$.

<u>Examples</u>



$$\int_0^x t\,dt = \left[\tfrac{1}{2}t^2\right]_0^x = \tfrac{1}{2}x^2 - 0 = \tfrac{1}{2}x^2$$

$\int_0^x e^{-t^2}\,dt \leftarrow$ a function with anti-derivative e^{-x^2}.

<u>Example</u> what about $\int_0^{x^2} \sin t\,dt$? $\leftarrow$ this is function of a function!

set $A(x) = \int_0^x \sin(t)\,dt$, then if $G(x) = \int_0^{x^2} \sin t\,dt$

then $G(x) = A(x^2) \leftarrow$ use chain rule to differentiate

$$\frac{d}{dx}(A(x^2)) = A'(x^2)\cdot 2x = \sin(x^2)\cdot 2x.$$