§4.8 Newton's method

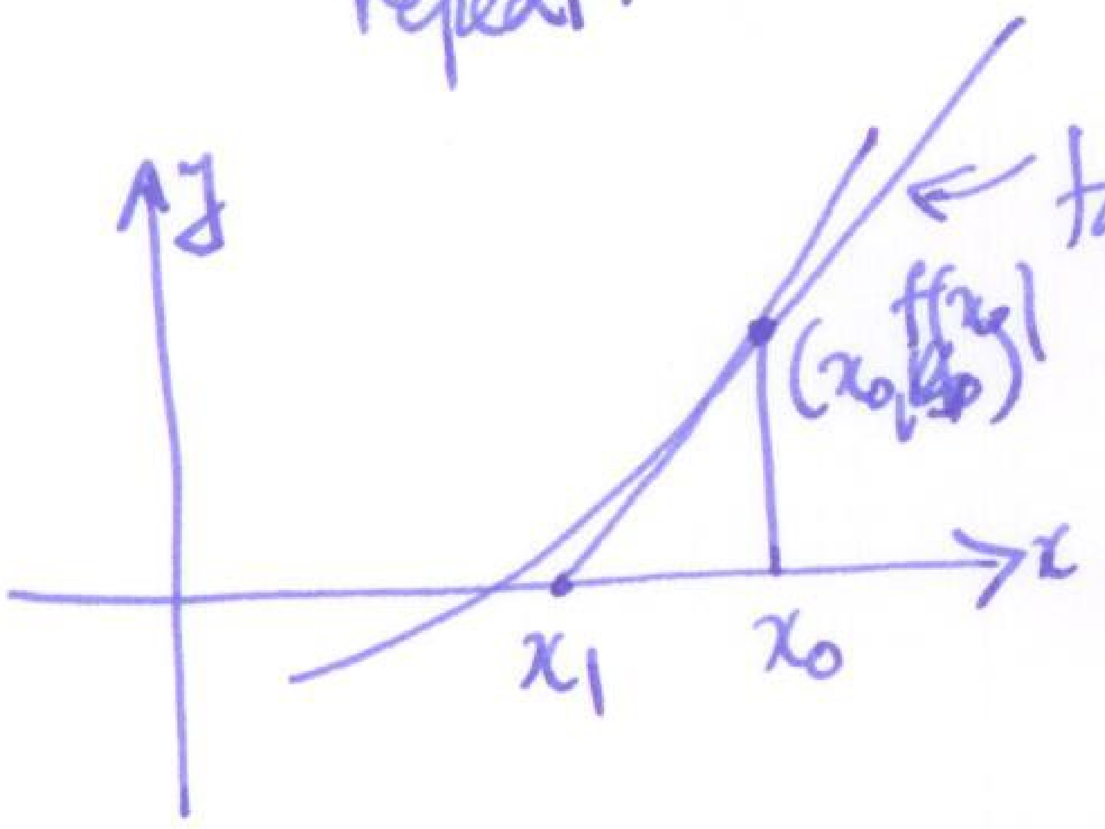
Q: find roots of x^5-x-1 (one real root) problem: no formula.

algorithm to find numerical approximation to solution.

idea: guess initial answer x_0.

use linear approximation to find better app. sol^{n}.

repeat.



← tangent line at x_0

$$y - f(x_0) = f'(x_0)(x - x_0)$$

where does tangent line hit x-axis?

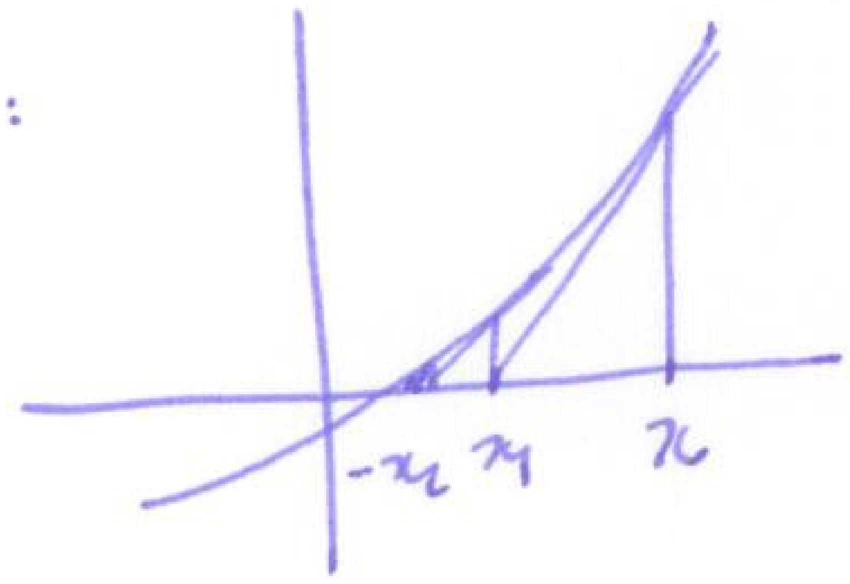
set $y=0$:
$$0 - f(x_0) = f'(x_0)(x_1 - x_0)$$
$$\frac{-f(x_0)}{f'(x_0)} = x_1 - x_0$$
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

summary start with x_0, set
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

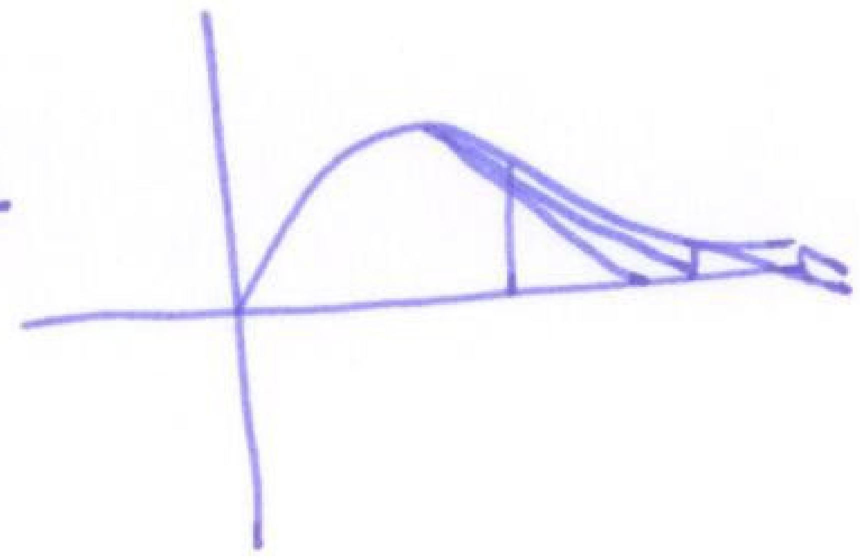
Example $f(x) = x^5 - x - 1$
$f'(x) = 5x^4 - 1$

$x_0 = 1$
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-1}{4} = 1.25$$
etc...

Picture:



Problems



may not converge

may be many roots

§4.9 Antiderivatives

Defn A function $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

Example $f(x) = x^2$, then $F(x) = \frac{1}{3}x^3$ is an anti-derivative.

check: $\frac{d}{dx}\left(\frac{1}{3}x^3\right) = x^2$. **Note** $\frac{1}{3}x^3 + 4$ is also an antiderivative.

General anti-derivative.

Thm Let $F(x)$ be an anti-derivative for $f(x)$. Then any other antiderivative for $f(x)$ is of the form $F(x) + c$ for some constant c.

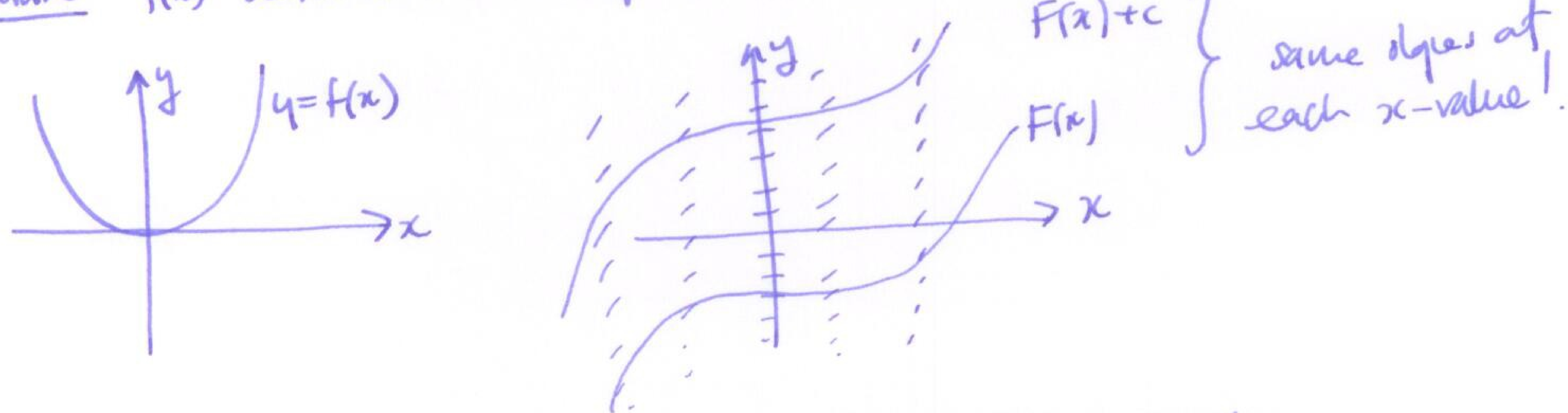
Proof Suppose $F(x)$ and $G(x)$ are antiderivatives of $f(x)$,

i.e. $F'(x) = f(x)$ and $G'(x) = f(x)$.

Consider $G(x) - F(x)$, which has derivative $\left(G(x) - F(x)\right)' = G'(x) - F'(x) = f(x) - f(x) = 0$

so $G(x) - F(x) = \text{constant}$ $\square$.

Picture $f(x)$ determines the slope function of $F(x)$.



Example: find the general anti-derivative to $f(x) = \sin 4x$

guess: $\frac{d}{dx}(\cos 4x) = -4\sin 4x$

so try $\frac{d}{dx}\left(-\frac{1}{4}\cos 4x\right) = -\frac{1}{4}\cdot -4\sin 4x = \sin 4x$

so $F(x) = -\frac{1}{4}\cos(4x) + c$