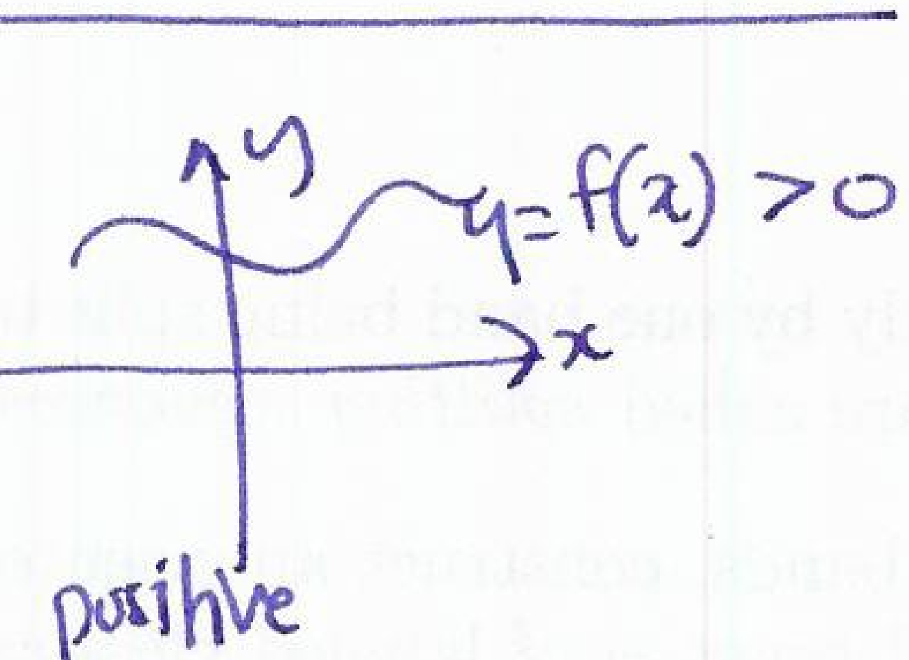
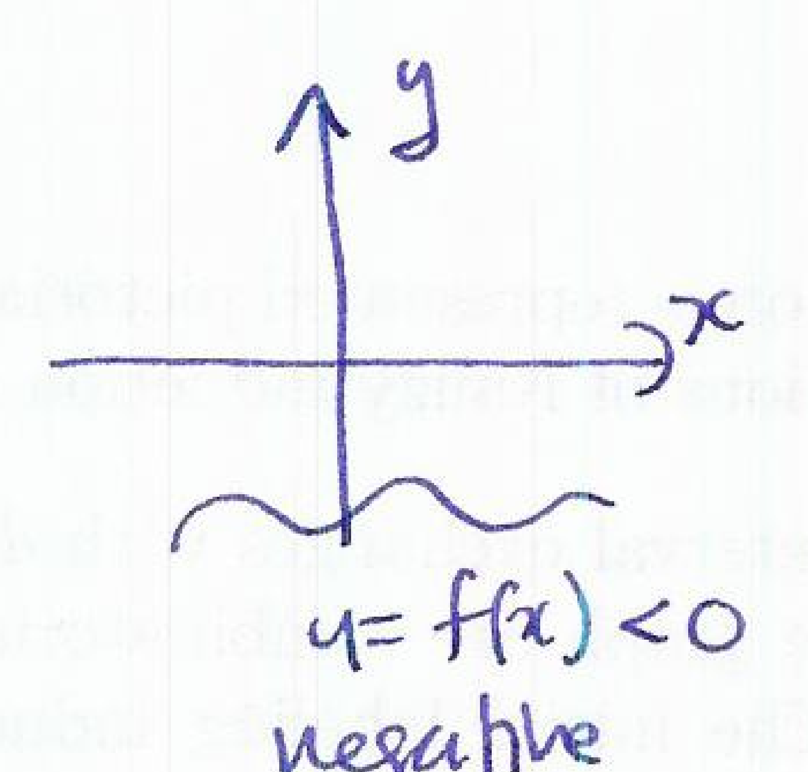
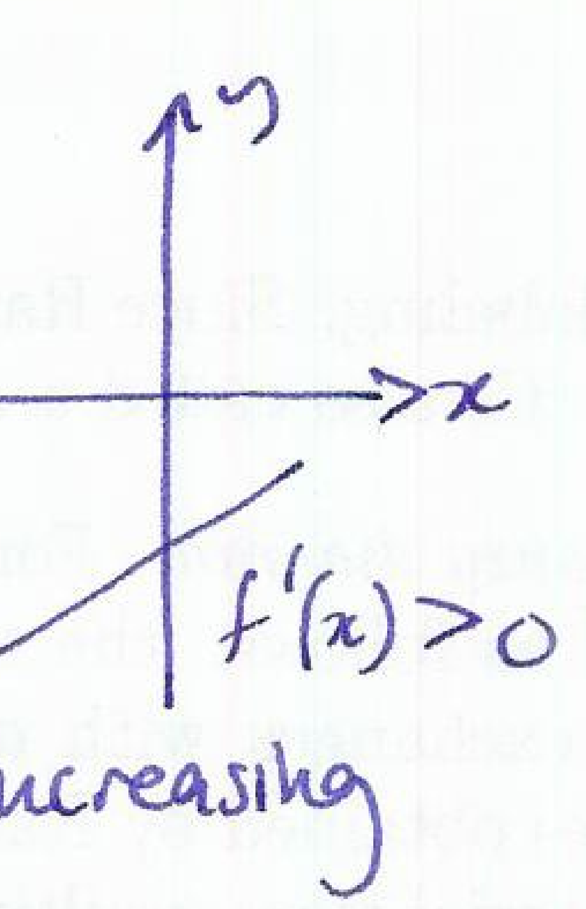
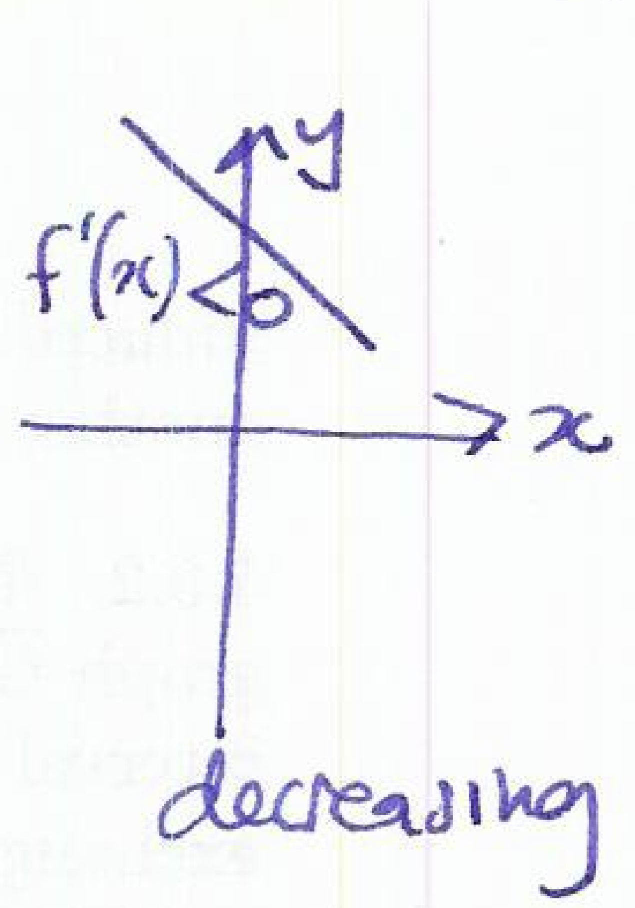
§4.4 Second derivative test

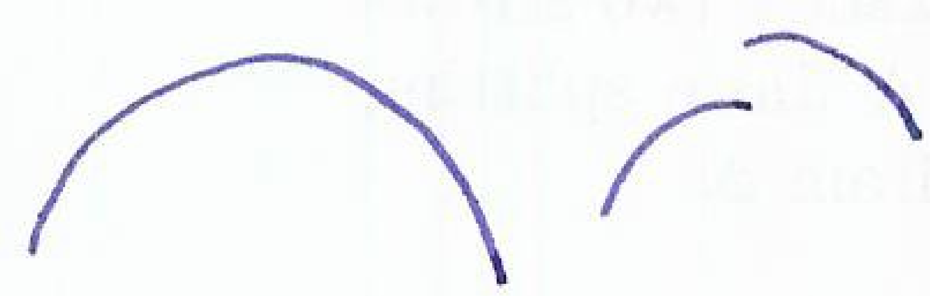
recall



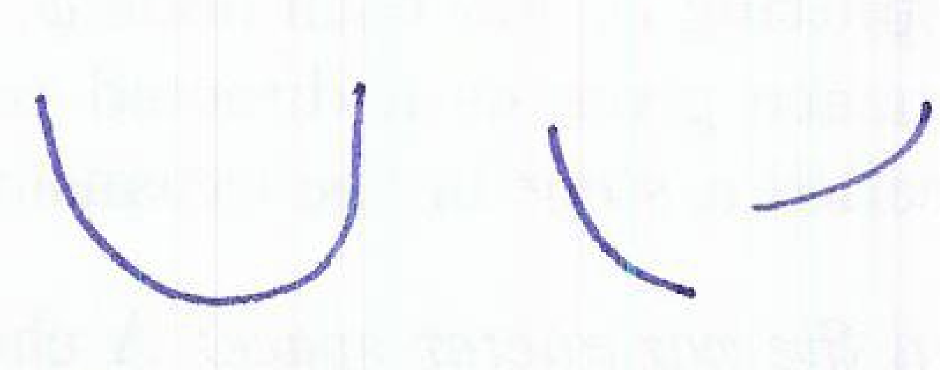








concave down

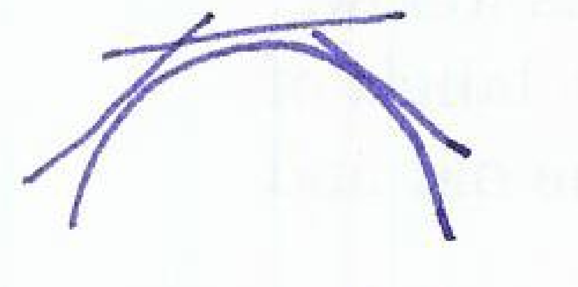


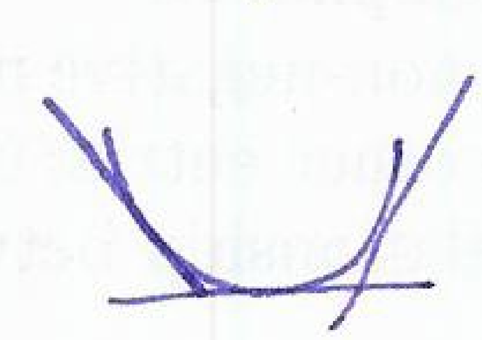
concave up

mnemonic

down up

Q: how do the slopes change?





concave down

$\Leftrightarrow$ slopes decreasing

$\Leftrightarrow$ $f''(x) < 0$

concave up

$\Leftrightarrow$ slopes increasing

$\Leftrightarrow$ $f''(x) > 0$

Def Let $f(x)$ be differentiable on the interval (a,b) then

$f(x)$ is concave up $\Leftrightarrow$ $f''(x) > 0$

$f(x)$ is concave down $\Leftrightarrow$ $f''(x) < 0$

Def A point of inflection is where the graph changes from concave up to concave down (or vice versa)

Note x point of inflection $\Rightarrow$ $f''(x) = 0$ ($\not\Leftarrow$)

Example $f(x) = x^3 - x = x(x^2-1) = x(x-1)(x+1)$

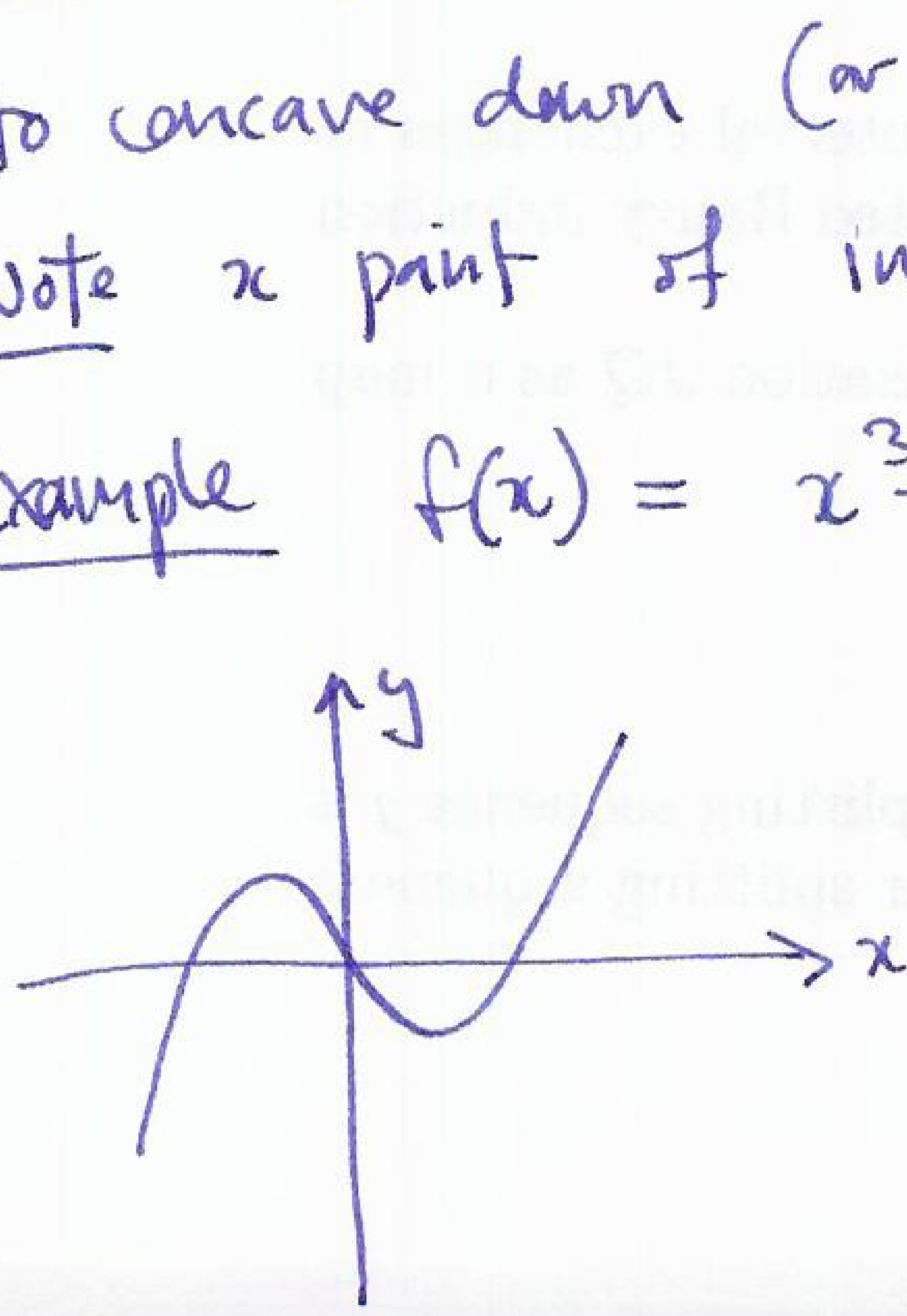
~~f(x) positive~~

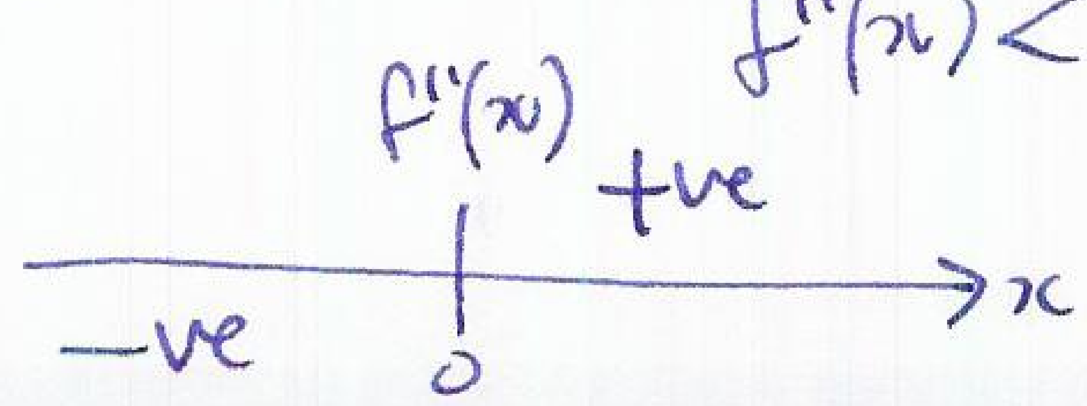
$f'(x) = 3x^2 - 1$

$f''(x) = 6x$

$f''(x) > 0$ for $x > 0$

$f''(x) < 0$ for $x < 0$

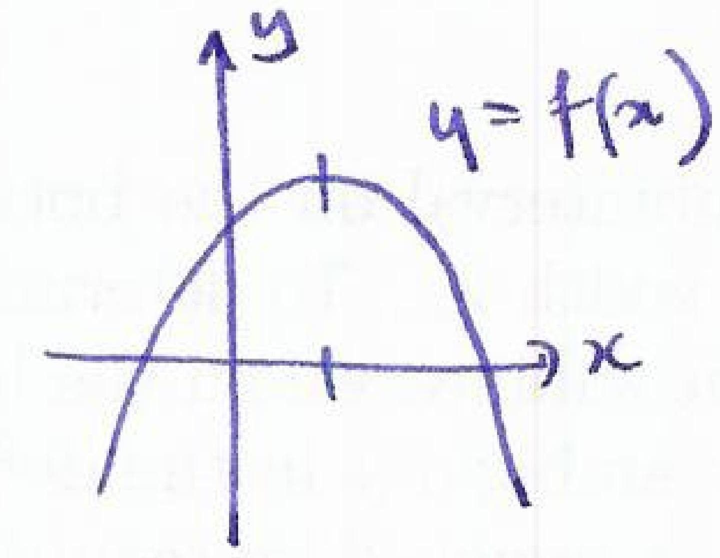


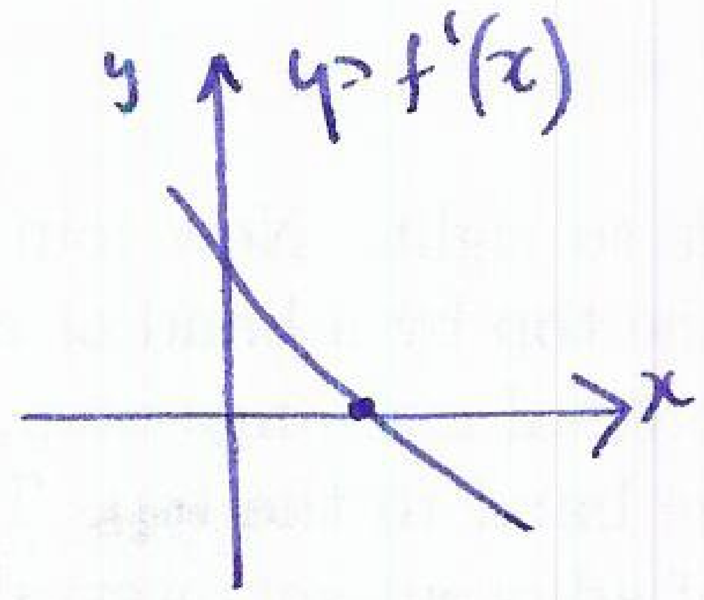


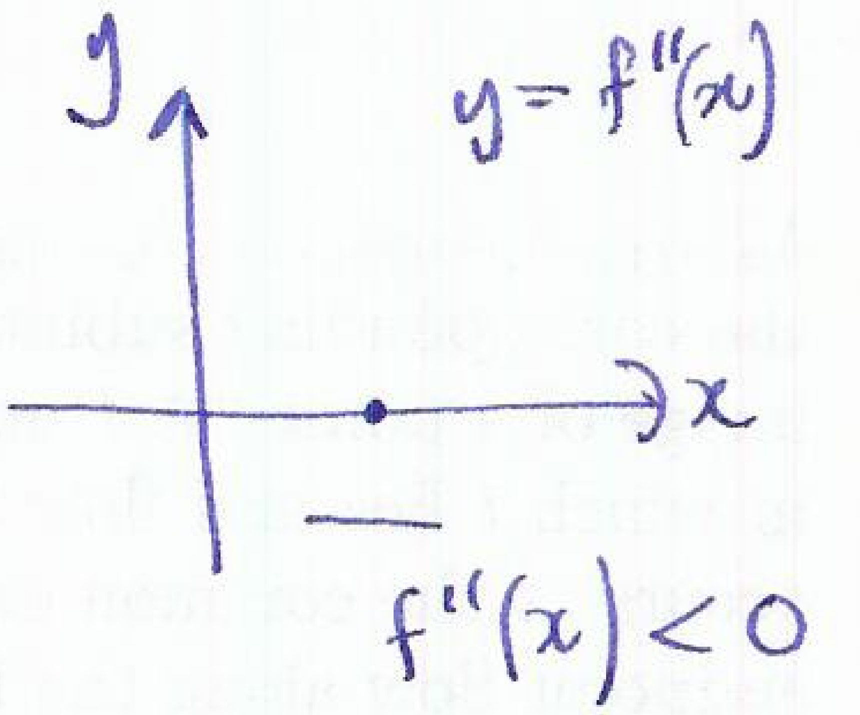
so there is a point of inflection at $x=0$

Second derivative test

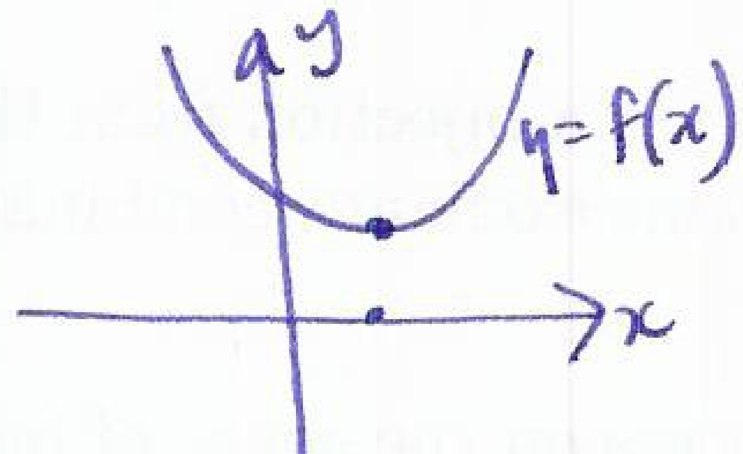
local max

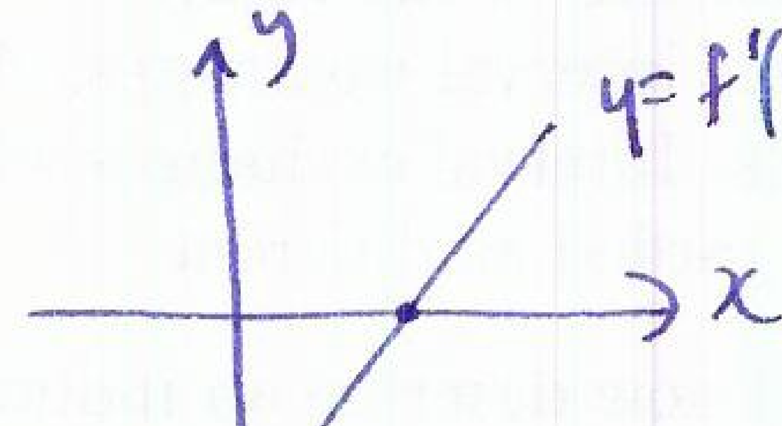


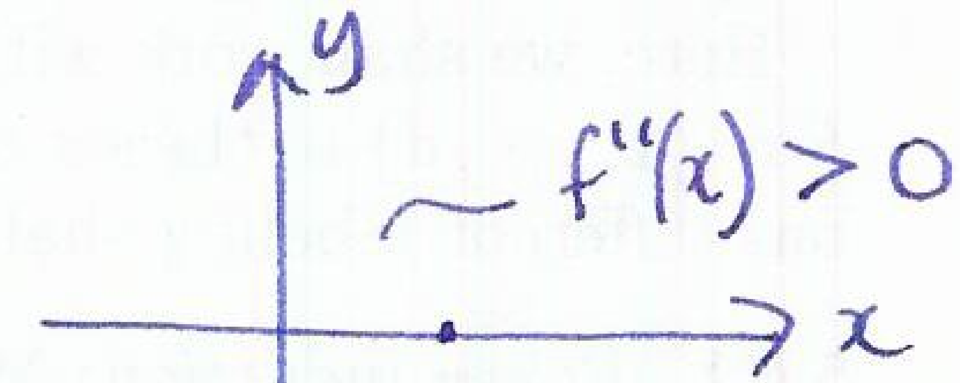




local ~~max~~ min







Thm Suppose $f(x)$ differentiable and c is a critical point, then

if $f''(c) > 0 \Rightarrow f(c)$ is a local max

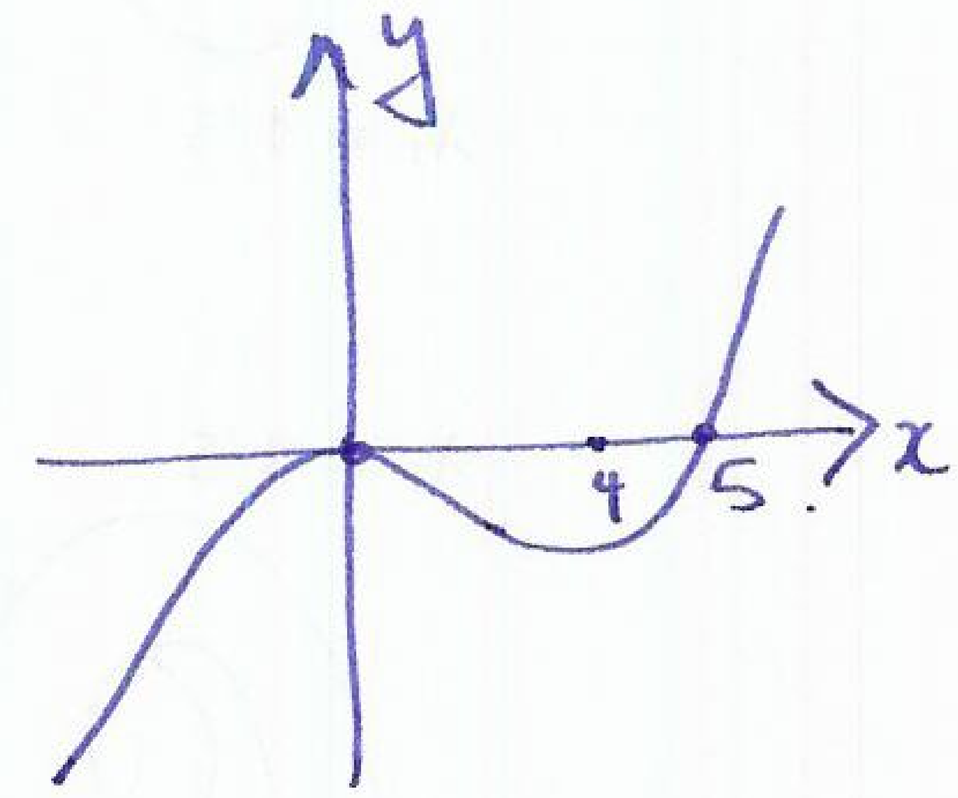
$f''(c) < 0 \Rightarrow f(c)$ is a local min

$f''(c) = 0$ NO INFORMATION : may be local max or local min or neither.

Example $f(x) = x^5 - 5x^4 = x^4(x-5)$

$f'(x) = 5x^4 - 20x^3 = 5x^3(x-4)$

$f''(x) = 20x^3 - 60x^2$



critical points $f'(x) = 0 \Rightarrow x = 0, 4.$

at $x = 0$ $f''(0) = 0$ no information

$x = 4$ $f''(4) = 320 \Rightarrow$ local min

at $x = 0$ use first derivative test: $f'(x)$: +ve | −ve, at 0, $\rightarrow x \Rightarrow$ local max