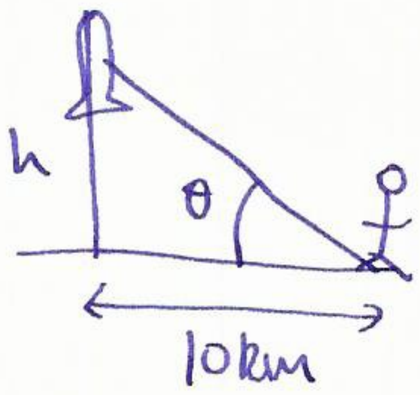
advice: ① give things names (height h, volume V etc...)
② write down relations between things and differentiate implicitly.
③ plug in numbers. (if necessary)

Example



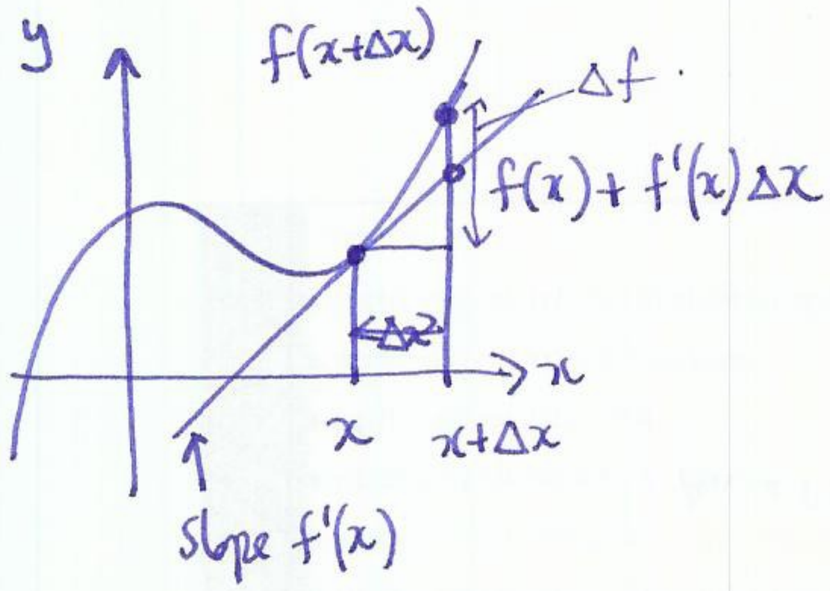
if angle is $\theta = \frac{\pi}{3}$ and rate of change $\frac{d\theta}{dt} = \frac{1}{2}$ rad/min

how fast is the rocket going?

$$\frac{h}{10} = \tan\theta \qquad \frac{1}{10}\frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt}$$

$$\frac{dh}{dt} = 10\sec^2\left(\frac{\pi}{3}\right)\cdot\frac{1}{2} \approx 20 \text{ km/min}.$$

§4.1 Linear approximations



If f is differentiable at x, and Δx is small, then

$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$$

so change in f is $\Delta f = f(x+\Delta x) - f(x)$,

$$\Delta f \approx f'(x)\Delta x$$

Example estimate $\sqrt{103}$

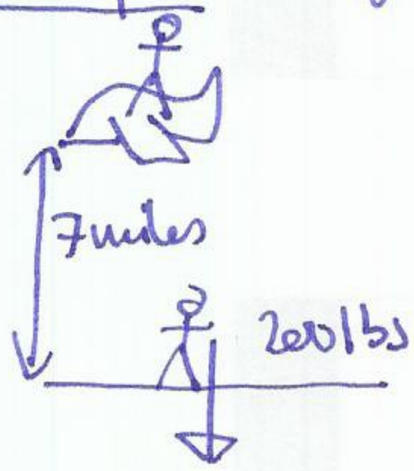
$f(x) = \sqrt{x} = x^{1/2}$ $\quad f(100) = 10$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(100) = \frac{1}{20}$

so $\Delta f \approx f'(x)\Delta x = \frac{1}{20} \cdot 3.$

so $\sqrt{103} \approx 10 + \frac{3}{20} = 10.15$

Example weight loss



$$w(x) = \frac{wR^2}{x^2}$$

$R = 3960$ miles (radius of earth)

x = distance from center of earth

w = weight at ground ($R = 3960$)

$$w'(x) = \frac{-2wR^2}{x^3}$$

$$w(3907) \approx w(3960) + w'(3960)\Delta x$$

$$\approx 200 + \frac{-2 \cdot 200\,(3960)^2}{(3960)^3} \cdot 7 \approx 200 - 0.7$$

Example pizza size: you make an 18" pizza, if your diameter is accurate to ± 0.4 in how much pizza do you gain or lose?

$A = \pi r^2 \qquad 2r = D \qquad A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$

$A'(D) = \frac{2\pi D}{4} = \frac{\pi D}{2}$

$\Delta A \simeq A'(18)\cdot \Delta D = \frac{1}{2}\pi 18 \cdot 0.4 \approx 11 \text{in}^2.$

Q. is this good or bad? absolute error = 11.

percentage error = $\left|\frac{\text{error}}{\text{actual value}}\right| \times 100.$ percentage error = $\frac{100 \times 11}{\pi \frac{18^2}{4}}$

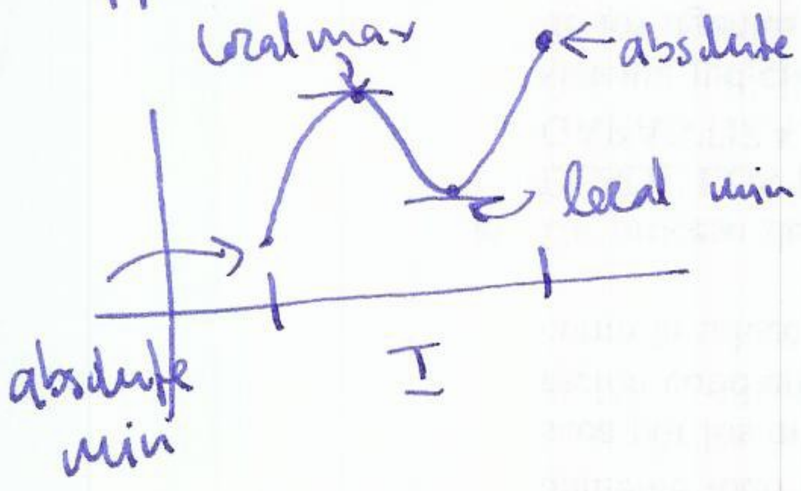
observation: when is the linear approximation a good approximation?

$f''(x)$ small

$f''(x)$ big

§4.2 Extreme values (max/min)

Suppose $f(x)$ is defined on an interval I



Defn $f(a)$ is the absolute max if $f(a) \geq f(x)$ for all $x \in I$

$f(a)$ is the absolute min if $f(a) \leq f(x)$ for all $x \in I$.

note: absolute max or min usually want value of function.

$\in$ can be many inputs which give local max or min