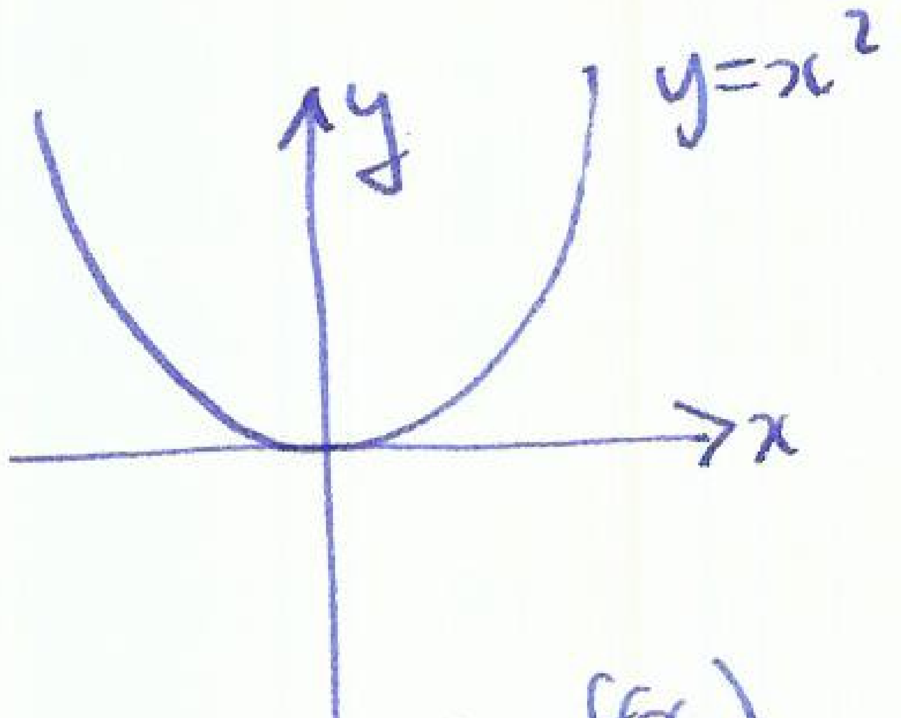
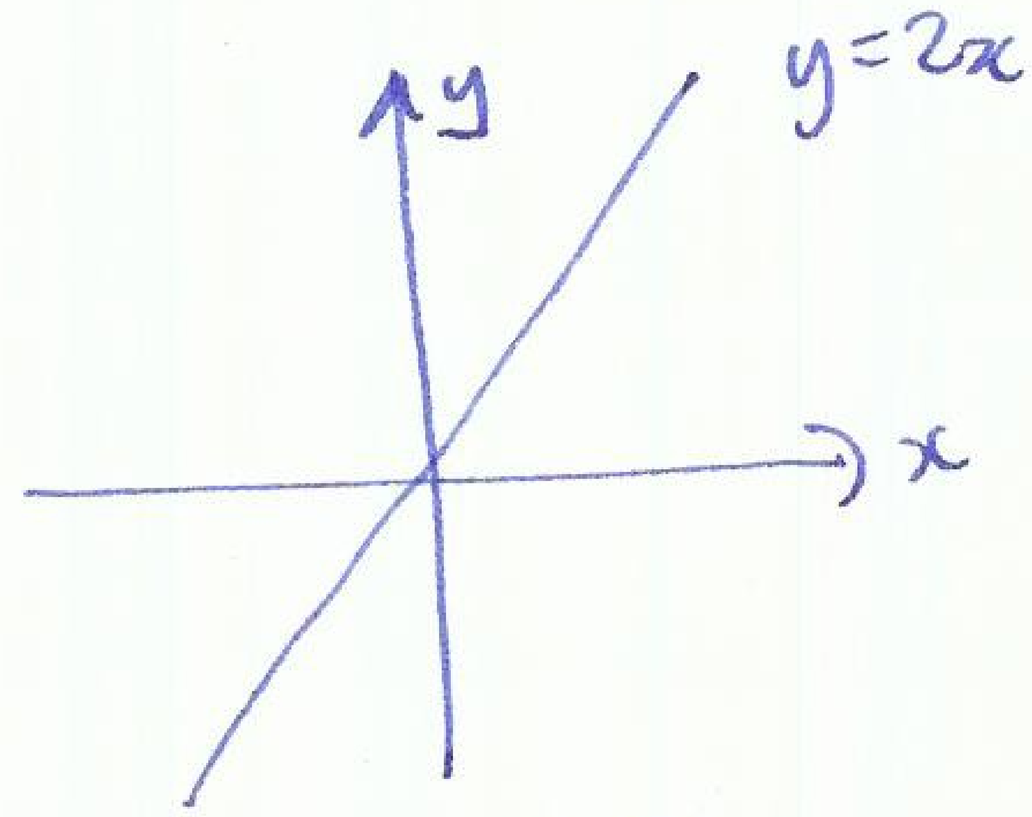
§3.5 Higher derivatives



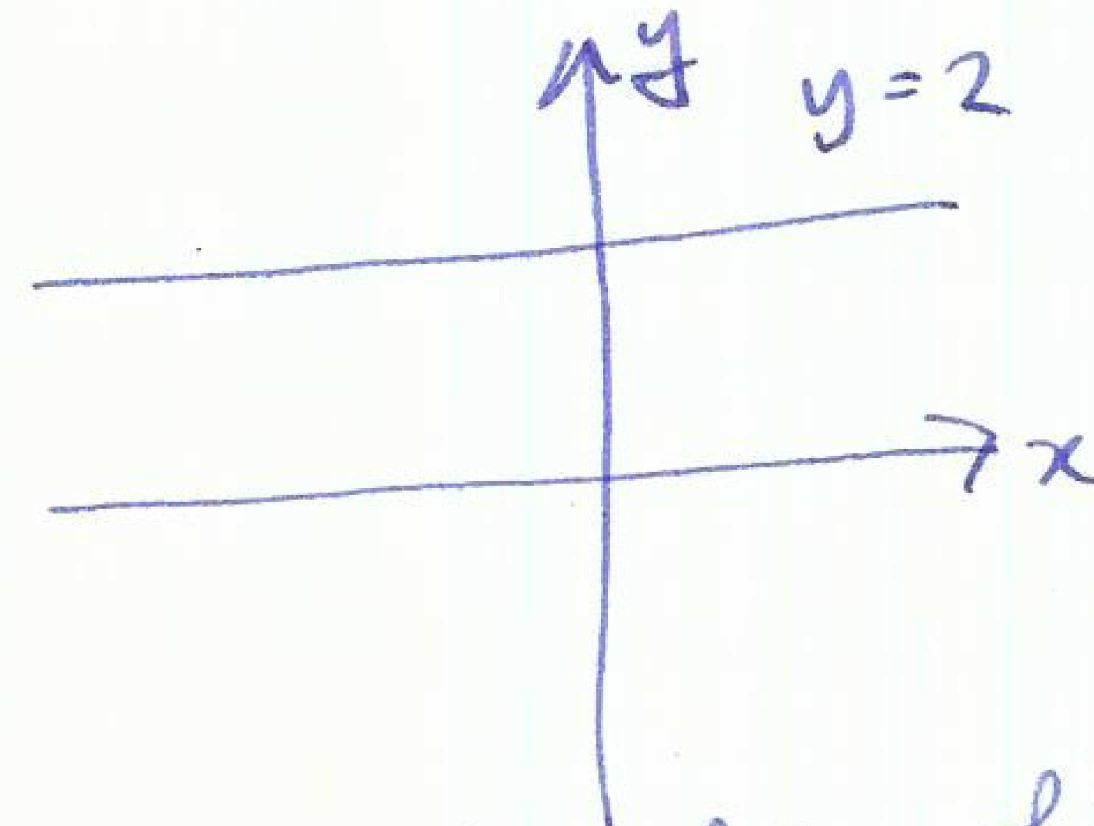
$y=f(x)$



slope of $f(x)$

$y=f'(x)$

"velocity"



slope of slope of f

$y=f''(x)$.

"acceleration"

Example $f(x) = xe^x$.

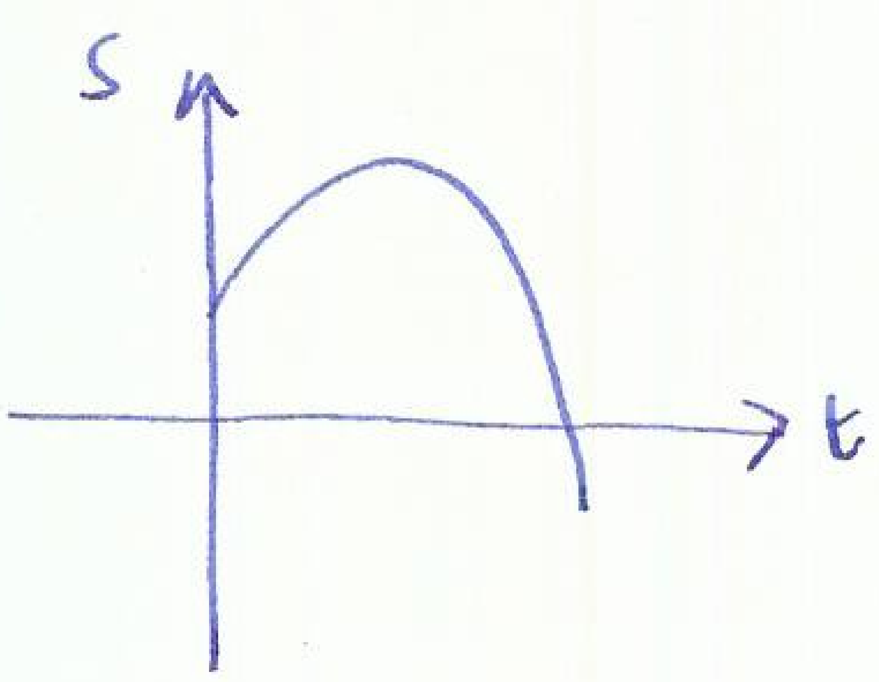
$f'(x) = xe^x + e^x$

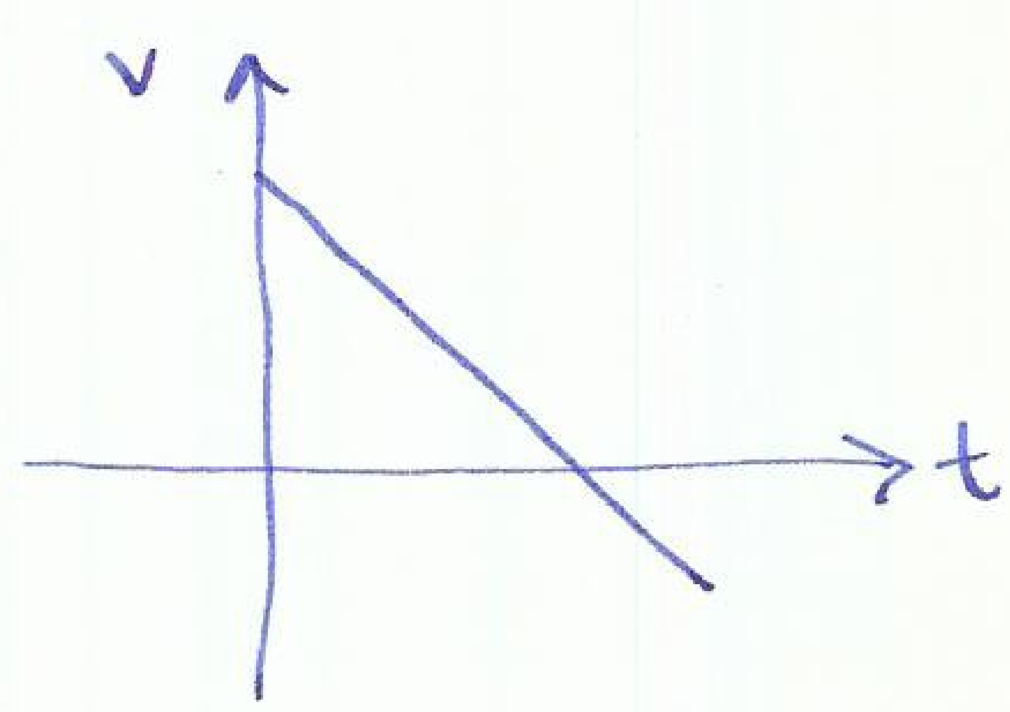
$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$

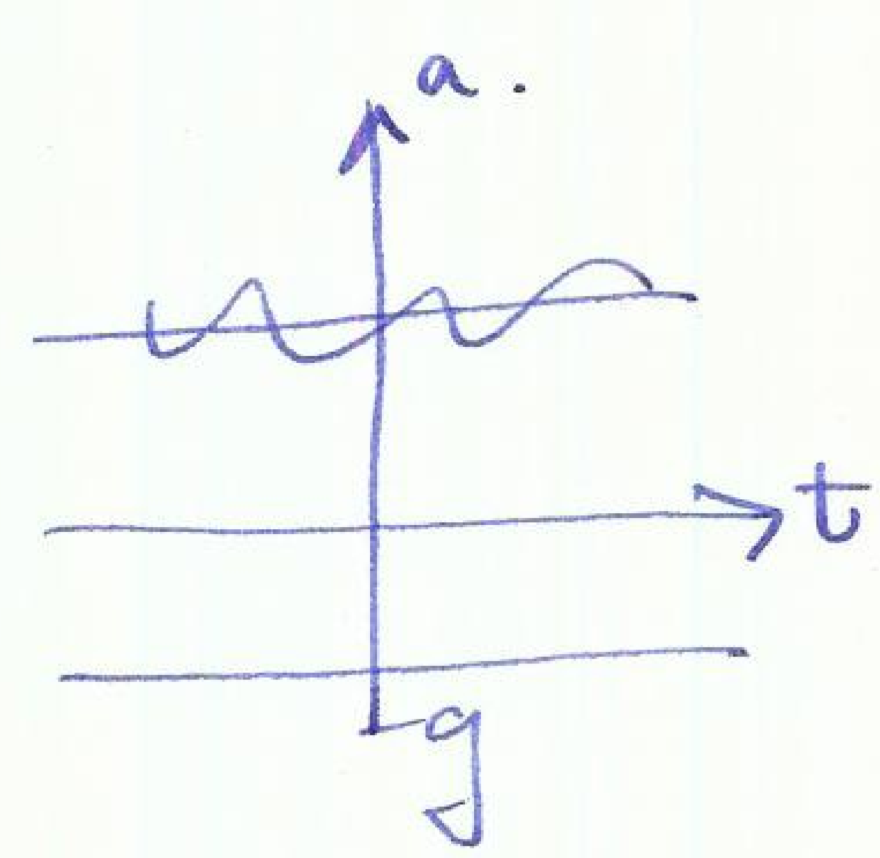
$f^{(3)}(x) = f'''(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$ etc.

Example acceleration due to gravity constant $-g$ m/s

$s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$ $\quad v(t) = s'(t) = -gt + v_0$ $\quad a(t) = s''(t) = -g$







§3.6 Trigonometric functions

Thm $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

Proof (for $\sin(x)$)

$$\frac{d}{dx}(\sin(x)) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

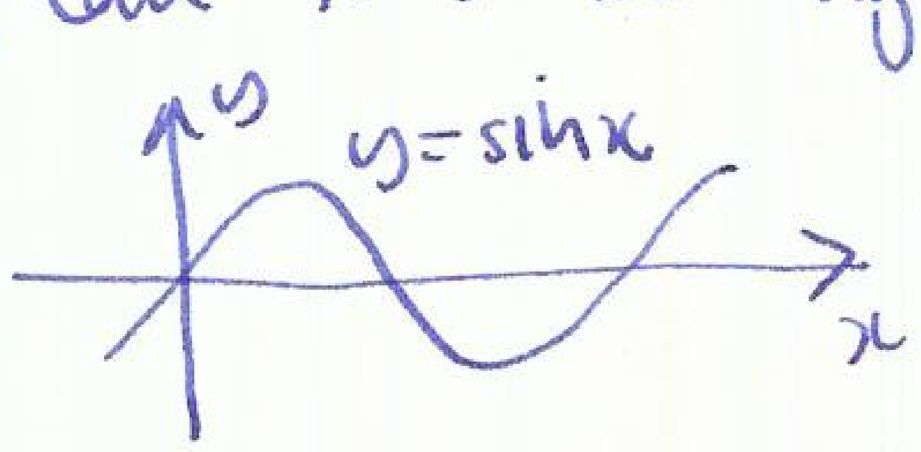
recall: $\sin(x+h) = \sin x \cos h + \cos x \sin h$

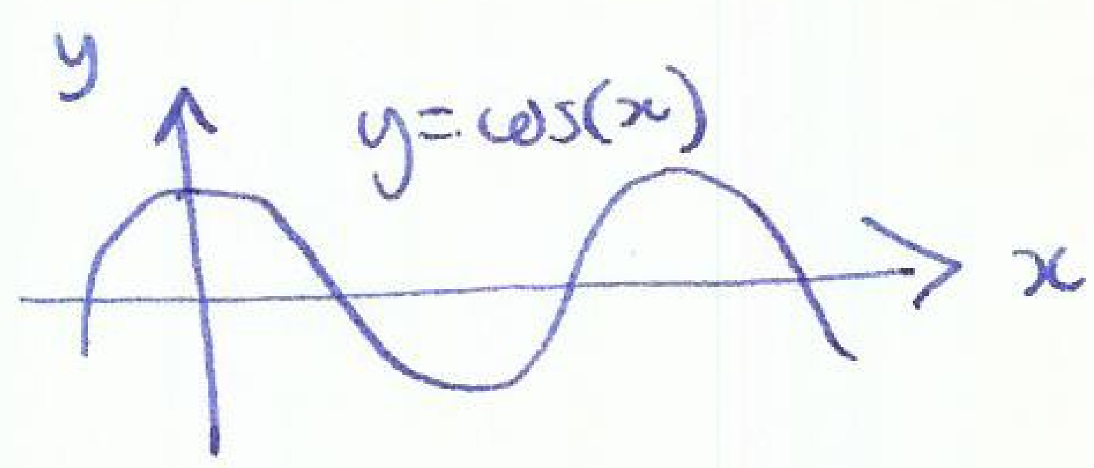
$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \frac{\sin h}{h} = \cancel{\cos}$$

$$= \sin x \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{1} = \cos x \qquad \square$$

Q can this be right?





Example $f(x) = x \sin x$

$f'(x) = x \cos x + \sin x$

Thm $\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \qquad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Proof (of $\tan x$).

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x . \cos x - \sin x . (-\cos x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x . \quad \square$$

Example $\frac{d}{dx}\left(e^x \cos x\right) = e^x(-\sin x) + e^x \cos x$.

§3.7 The chain rule

new functions from old: $f \pm g$ $\quad f/g$ $\quad f.g = f(x)g(x)$.

what about composition: $f(g(x)) = (f \circ g)(x)$

Examples e^{4x}, $\sin^2(x)$, etc...

Thm Chain rule If f and g are differentiable, then $f \circ g$ is differentiable,

and $\boxed{[f(g(x))]' = f'(g(x)).g'(x)}$

mnemonic: $[f(g(x))]' =$ outside'(inside) . inside'

Examples

① $e^{4x} = f(g(x))$ where $f(x) = e^x \quad f'(x) = e^x$

$g(x) = 4x \quad g'(x) = 4$

so $(e^{4x})' = f'(g(x)).g'(x) = 4e^{4x}$

② $\sin^2(x) = f(g(x))$ where $f(x) = x^2 \qquad f'(x) = 2x$

$g(x) = \sin(x) \quad g'(x) = \cos(x)$

so $(\sin^2(x))' = f'(g(x)).f'(x) = 2\sin(x).\cos(x)$

③ $\sqrt{x^3+1}$ etc...

Alternate notation $\quad f(g(x)) \leftrightarrow f(u),\ u = g(x)$

$$\frac{df}{dx} = f'(u)\frac{du}{dx} = \frac{df}{du} \neq \frac{du}{dx}$$

$$\boxed{\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}}$$

mnemonic "cancelling fractions"

Examples $\cos(x^2)$, $e^{\sqrt{x}}$, $\sin\left(\frac{\pi x}{180}\right)$, $\sqrt{x+\sqrt{x^2+1}}$.