Math 231 Calculus I

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- math tutoring: 1S-214
- students with disabilities.

Text: Early transcendentals, by Rogawski.

§1.2 Linear and quadratic functions

recall:

$$\underset{\text{domain}}{\text{input set}} \xrightarrow{\text{function}} \underset{\text{range}}{\text{output set}}.$$

examples:
$$f: \mathbb{R} \longrightarrow \mathbb{R} \quad (\mathbb{R} = \text{real numbers}).$$
$$x \longmapsto x^2 \quad (\text{description of function})$$

e.g.
$$0 \longmapsto 0$$
$$1 \longmapsto 1$$
$$-1 \longmapsto 1$$
$$2 \longmapsto 4 \quad \text{etc.}$$

notation

$$f: \mathbb{R} \to \mathbb{R}$$

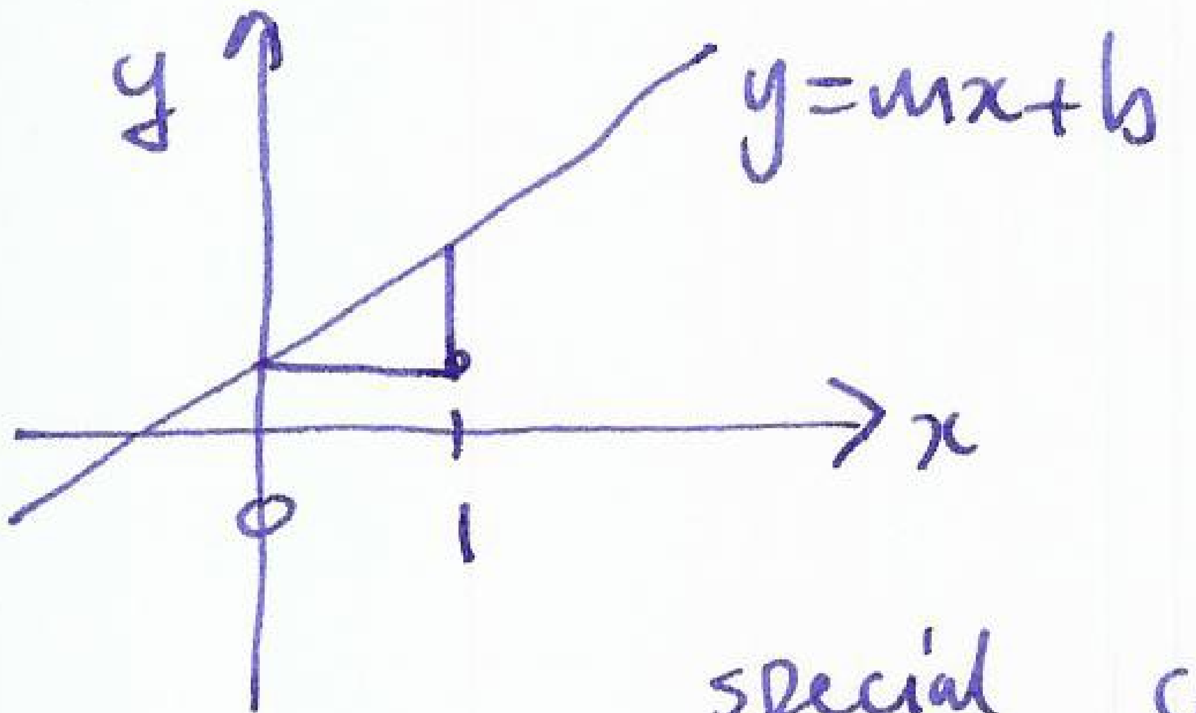
name, input, output

examples
$$+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}.$$
$$(a,b) \longmapsto a+b.$$

evaluation at 0:
$$\left\{\begin{matrix}\text{functions}\\ f:\mathbb{R}\to\mathbb{R}\end{matrix}\right\} \longrightarrow \mathbb{R}$$
$$f \longmapsto f(0)$$

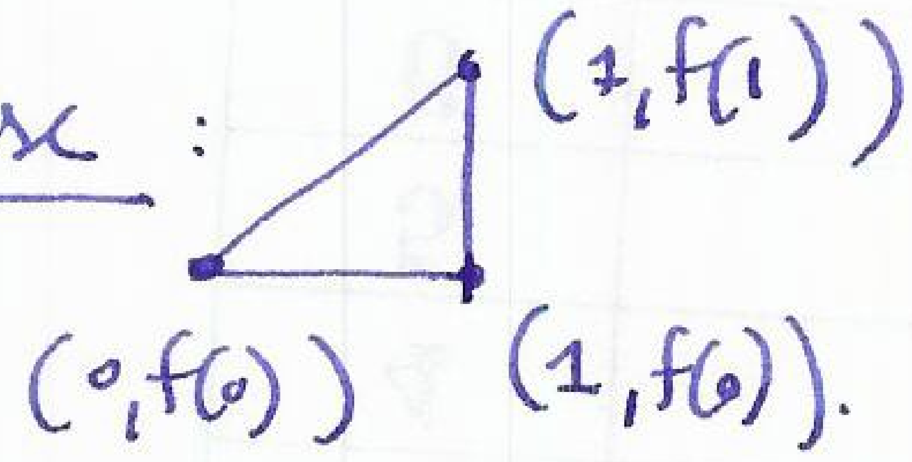
key property: every input gets sent to a single output, not a collection of values...

A linear function $f: \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx + b$ (m, b constants, i.e. don't depend on x) and the graph of a linear function is a straight line.



$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}.$$

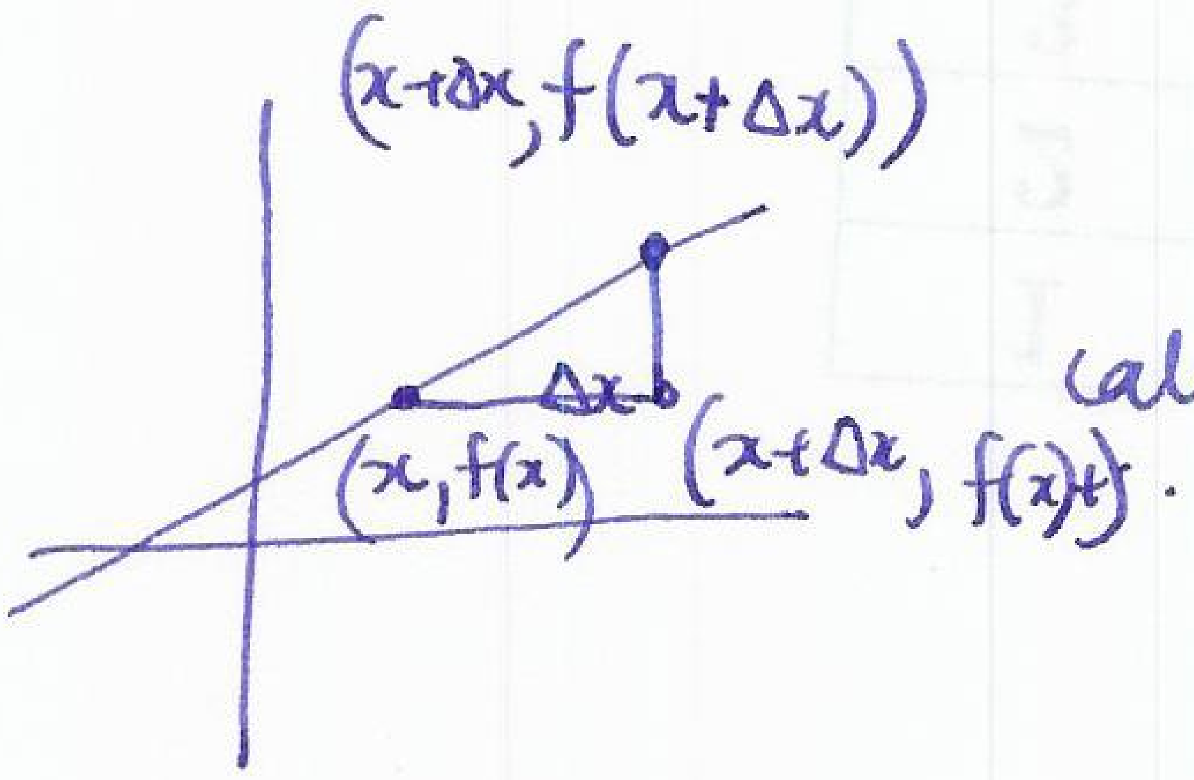
special case :



$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{f(1) - f(0)}{1}$$

$$= \frac{m + b - b}{1} = m.$$

general case :



call horizontal change Δx
vertical change Δy

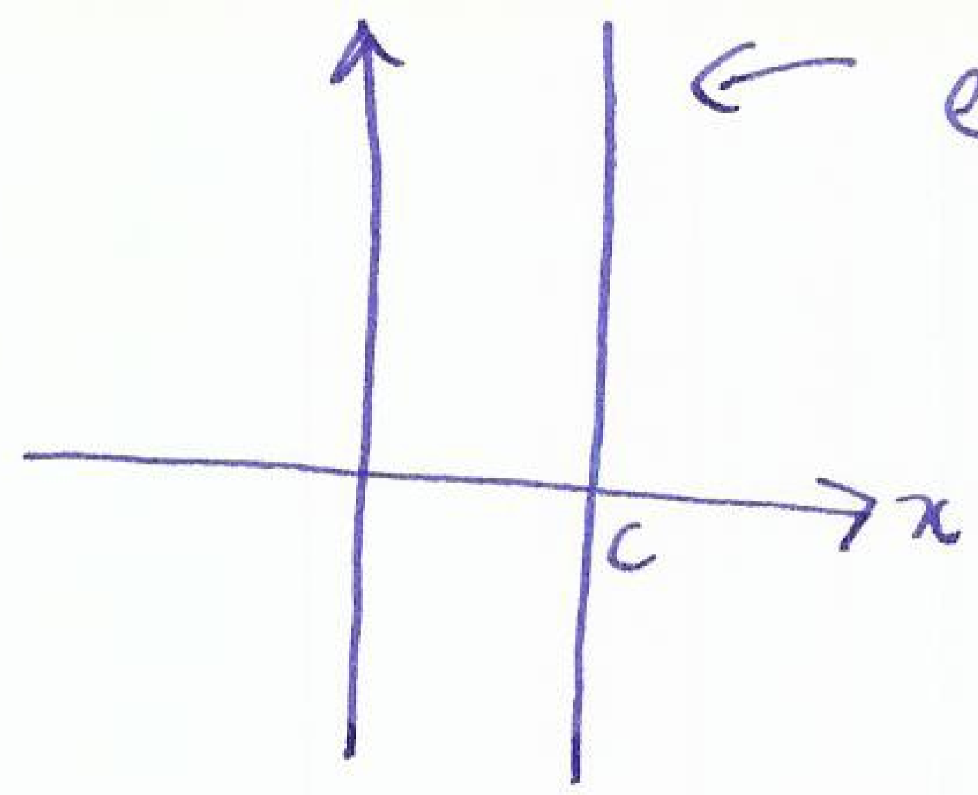
so

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{x + \Delta x - x} = \frac{m(x+\Delta x) + b - (mx+b)}{\Delta x}$$

$$= \frac{\cancel{mx} + m\Delta x + \cancel{b} - \cancel{mx} - \cancel{b}}{\Delta x} = \frac{m\Delta x}{\Delta x} = m.$$

useful fact : a straight line has constant slope m everywhere.

observations :
- if $|m|$ large, line is steep.
- $m = 0$ horizontal line.
- $m < 0$ slopes down from left to right.
- vertical lines are not graphs of functions!



← equation of vertical line is $x = c$

equation vs function

↑ relation between variables, eg $x = c$, $x^2 + y^2 = 1$.

↑ map from one set to another. $f: \mathbb{R} \to \mathbb{R}$. $x \mapsto f(x)$.

the graph of $f: \mathbb{R} \to \mathbb{R}$ is $y = f(x)$ (an equation!).
but not every equation comes from a function...

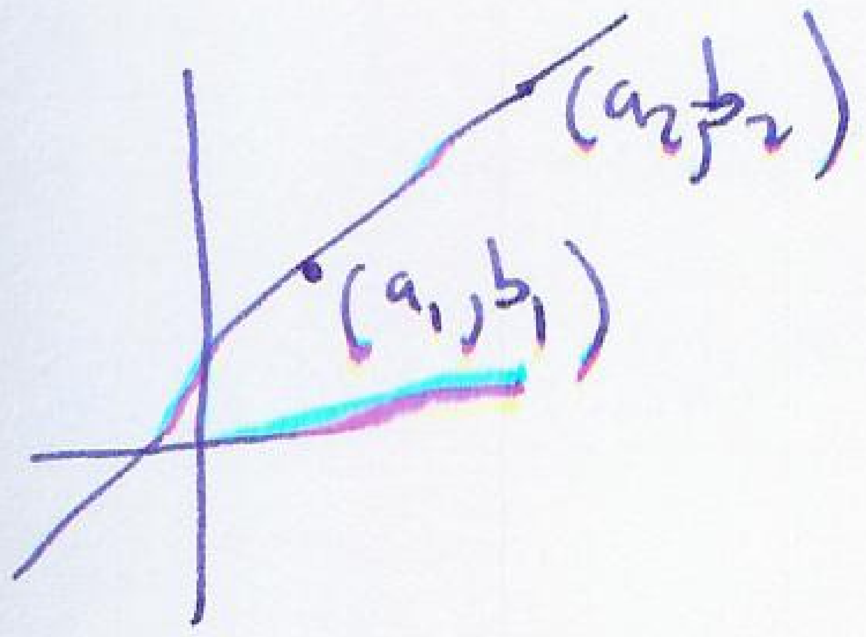
how to deal with <u>any</u> straight line: use the <u>general linear equation</u>

$ax + by = c$ (at least one of a, b not zero)

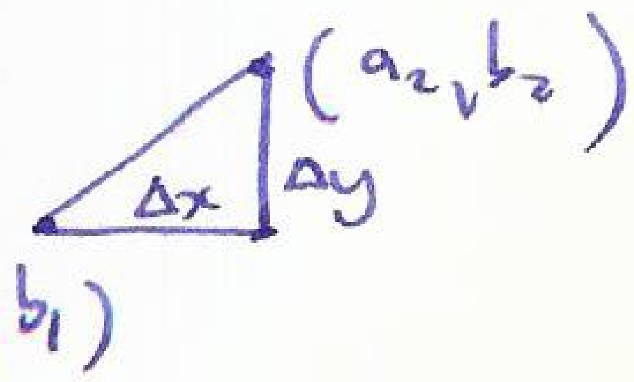
can write <u>any</u> line in this form.

$y = mx + b$ | $b = 1$, $a = m$, $c = b$ in general linear equation.

<u>useful technique</u>: find equation of line through two points.



- work out slope



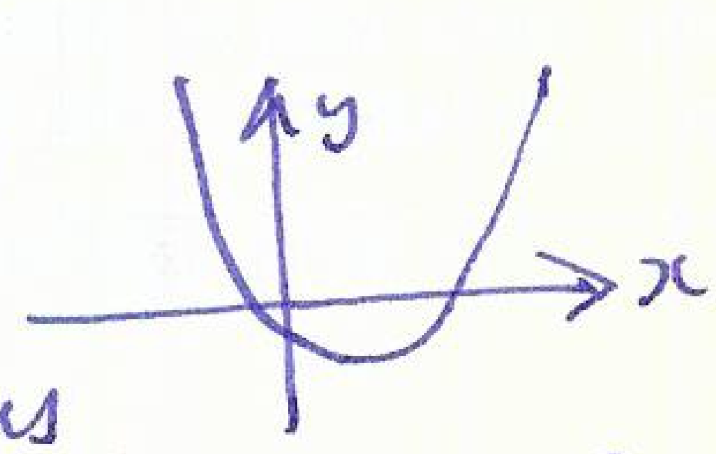
$$m = \frac{\Delta y}{\Delta x} = \frac{b_2 - b_1}{a_2 - a_1}$$

line of slope m through (a_1, b_1) given by $y - b_1 = m(x - a_1)$

<u>Quadratic functions</u>

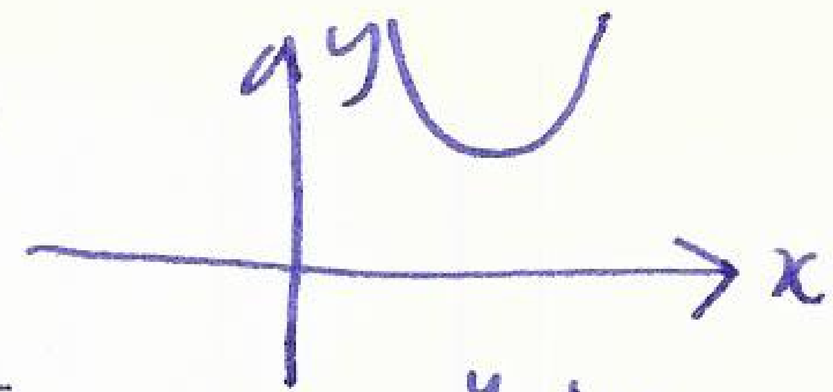
given by $f(x) = ax^2 + bx + c$ (a, b, c constants, i.e. do not depend on x)

<u>useful facts</u>: · graphs are parabolas:

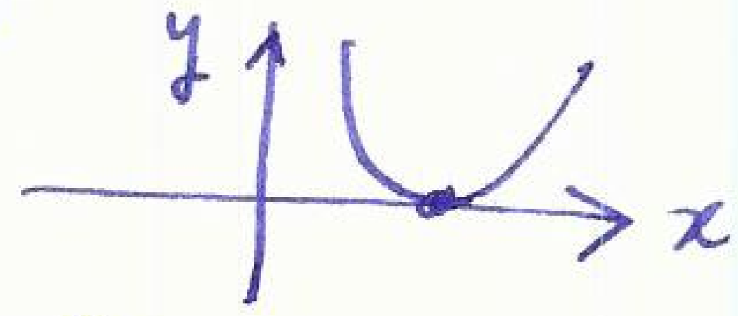


- at most two distinct real solutions to equation $f(x) = 0$ (corresponds to values of x where parabola hits x-axis) given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

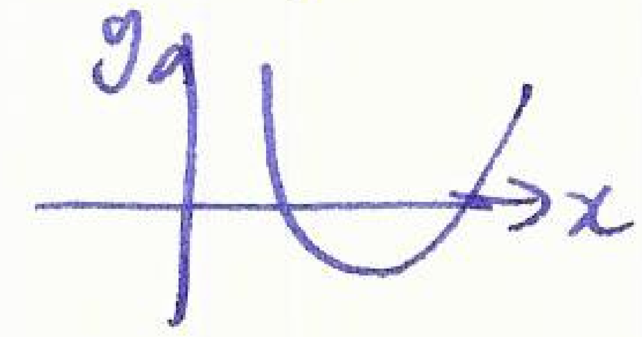
$b^2 - 4ac < 0 \Leftrightarrow$ no real solutions



$b^2 - 4ac = 0 \Leftrightarrow$ exactly one distinct solution.



$b^2 - 4ac = 0 \Leftrightarrow$ two different solutions



useful techniques:

- factorization $ax^2 + bx + c = a(x - r_1)(x - r_2)$

  r_1, r_2 are called solutions or roots of quadratic function.

- completing the square. any quadratic function can be written as $(x+a)^2 + b$.

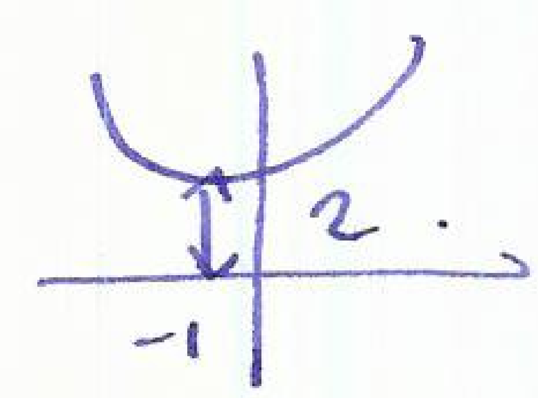
  $x^2 + 2ax + a^2$.

example • $x^2 + 2x + 3$

$(x+1)^2 + 2$

$x^2 + 2x + 1 + 2$

$f(x) = (x+1)^2 + 2$ has no solutions to $f(x) = 0$



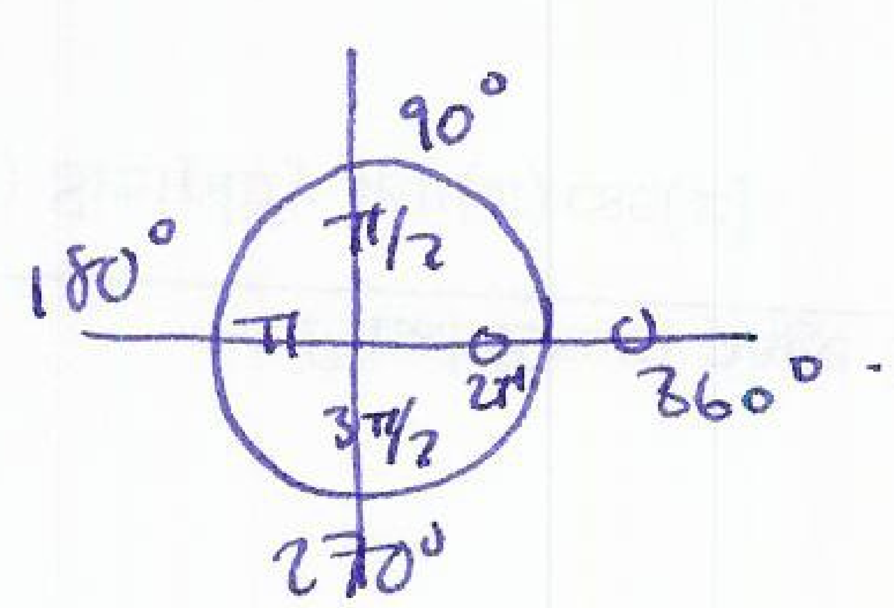
example • $2x + x + 1 = 2(x + \frac{1}{2}x + \frac{1}{2})$.

$= 2\left((x + \frac{1}{4})^2 + \frac{7}{16} \right)$.

$x^2 + \frac{1}{2}x + \frac{1}{16} + \frac{7}{16}$.

§1.4 Trigonometric functions

angles vs radians



degrees $= \frac{360}{2\pi}$ radians

radians $= \frac{2\pi}{360}$ degrees.

• angle in radians = distance travelled around unit circle.