

### Math 231 Calculus 1 Spring 12 Sample midterm 3

(1) Consider the function  $f(x)$  defined by the following graph.

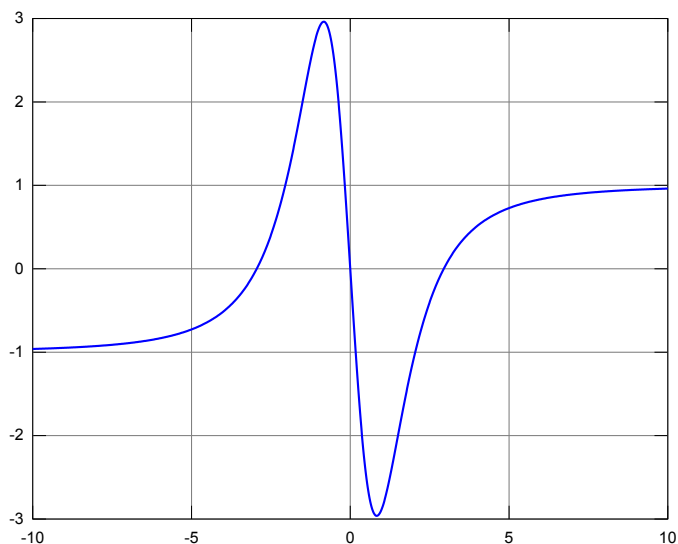


FIGURE 1

- (a) Label all regions where  $f'(x) < 0$ .
  - (b) Label all regions where  $f'(x) > 0$ .
  - (c) What is  $\lim_{x \rightarrow \infty} f'(x)$ ?
  - (d) What is  $\lim_{x \rightarrow -\infty} f''(x)$ ?
  - (e) Sketch a graph of  $f'(x)$  on the figure.
  - (f) Label the approximate locations of all points of inflection.
- (2) Sketch a graph of a differentiable function  $f$  that satisfies the following conditions and has  $x = -1$  as its only critical point.

$$\begin{aligned}f(-1) &= 8 \\f'(-1) &= 0 \\f'(x) &> 0 \text{ for } x < -1 \\f'(x) &< 0 \text{ for } x > -1 \\ \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow -\infty} f(x) = -1\end{aligned}$$

(3) Consider the function

$$f(x) = \frac{x}{8 - x^3}$$

- (a) Find all vertical and horizontal asymptotes of the function.
- (b) Find all critical points of the function.
- (c) Determine the intervals where  $f(x)$  is increasing and decreasing.
- (d) Use the 2nd derivative test to attempt to identify all local maxima and minima.
- (e) Sketch the function and label all relative maxima and minima.

(4) Consider the following function:

$$g(x) = (x^2 - 2x)e^{-2x}$$

- (a) Find, if they exist, the coordinates of all relative maxima and minima.
- (b) Determine the interval(s) where  $g$  is increasing and those where  $g$  is decreasing.
- (c) Find, if they exist, the coordinates of all points of inflection.
- (d) Determine the intervals where  $g$  is concave up and those where  $g$  is concave down.
- (e) Sketch the curve as accurately as possible.

(5) A function  $f(x)$  has derivative

$$f'(x) = \frac{1}{e^{2x} + 1}.$$

Where on the interval  $[1, 4]$  does it take its maximum value?

(6) Take a circular piece of paper, and remove a sector of angle  $\theta$ , and fold the remainder into a cone shape. Which angle  $\theta$  gives the largest volume?

(7) Compute the following limits. Show all work.

(a)

$$\lim_{x \rightarrow -\infty} \frac{12x + 1}{\sqrt{4x + 4}}$$

(b)

$$\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$$

(c)

$$\lim_{x \rightarrow 0} \left( \frac{e^x}{e^x - 1} - \frac{1}{x} \right)$$

(d)

$$\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{\sin x - x \cos x}$$

(8) Evaluate the following

(a)

$$\int \frac{x^2 + x + 1}{x} dx$$

(b)

$$\int e^x - 4 \sin(x) dx$$

(c)

$$\int_1^2 2\sqrt{x} dx$$

(d)

$$\int_0^t \frac{1}{x+1} dx$$

(9) Approximate the area under the graph of  $y = e^{-x}$  between 0 and 2 using four rectangles. Use the right hand endpoints to find the heights of the rectangles. Can you say whether this is an under- or over-estimate?