

Math 231 Calculus 1 Spring 12 Sample midterm 1

- (1) The graph of $y = f(x)$ is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

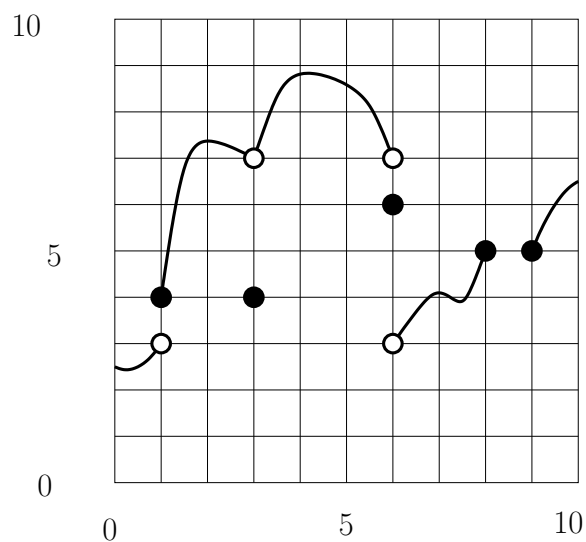


FIGURE 1. $f(x)$

- (a) $\lim_{x \rightarrow 1^-} f(x)$
- (b) $\lim_{x \rightarrow 1^+} f(x)$
- (c) $\lim_{x \rightarrow 3} f(x)$
- (d) $\lim_{x \rightarrow 6^-} f(x)$
- (e) $\lim_{x \rightarrow 6^+} f(x)$
- (f) $\lim_{x \rightarrow 9} f(x)$

- (2) Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - x - 12}{|x + 3|}$

(b) $\lim_{x \rightarrow 6} \frac{\sqrt{x+3} - 3}{x - 6}$

(c) $\lim_{x \rightarrow 0} \frac{\tan 2x}{3x}$

(d) $\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x^2 - 2x}} \right)$

- (3) For what value of c (if any) is the function $f(x)$ continuous at $x = 1$? Justify your answer.

$$f(x) = \begin{cases} x + \frac{4}{x-3} & x < 1 \\ c & x = 1 \\ \frac{\cos(\pi x)}{x} & 2x > 1 \end{cases}$$

- (4) For a sphere of radius r , its surface area $S = 4\pi r^2$. What is the average rate of change of the surface area when the radius increases from $r = 3$ to $r = 4$?
- (5) Show that $e^x = 1/2x$ has a solution for some $x > 0$. You do not need to find this solution.
- (6) Find the derivatives of the following functions
- (a) $\frac{4x}{x+2}$
- (b) $-2x^3e^x$
- (c) $\frac{x^2-1}{x^2+1}$
- (7) Find the second derivatives of the functions above.