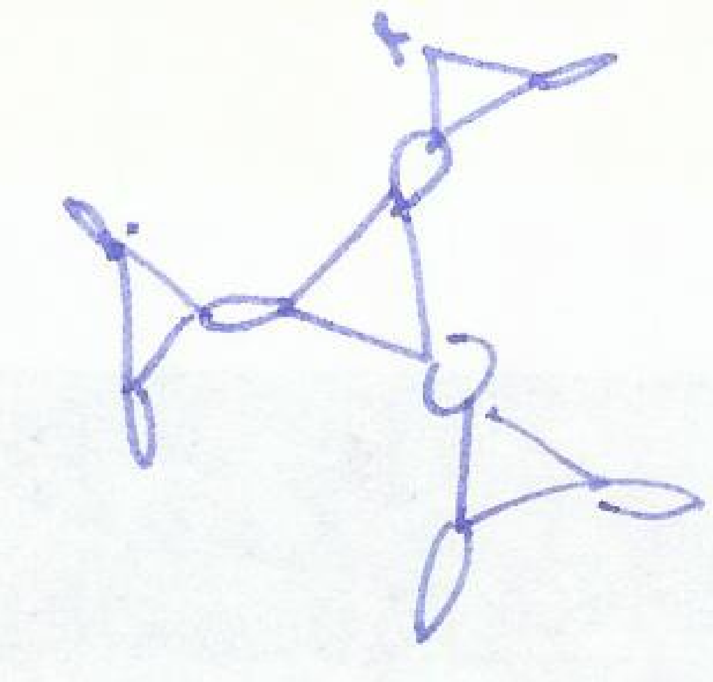
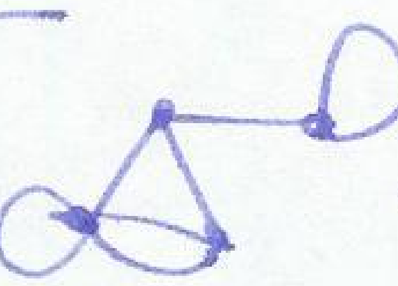
$PSL_2\mathbb{Z} \cong \mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/3\mathbb{Z}$.



Graphs and free groups

graphs : edges
vertices



X.

Prop^n $\pi_1(X, x_0)$ is free if X is a graph, basis $[f_\alpha]$ corresponding to α edge in $X \setminus T$ maximal tree.

Proof Let T be a maximal tree, then apply van-Kampen. $\square$.

Lemma Every covering space of a graph is also a graph

Proof

— edge has pre-image $\equiv$ edges

$\times$ vertex has pre-im $\times$ $\times$ $\times$ vertices.

Th^m Every subgroup of a free group is free.

Proof subgroup $\leftrightarrow$ cover = graph, so π_1 free $\square$.

§1.B $K(G,1)$'s ~~and graphs of groups~~

Def^n If $\pi_1(X, x_0) = G$ and $\tilde{X}$ universal cover is contractible, then we say X is a $K(G,1)$ space.

Examples : S^1, graphs, surfaces, $\mathbb{RP}^\infty = K(\mathbb{Z}/2\mathbb{Z}, 1)$, products.

Prop^n Every group G has a $K(G,1)$.

Proof : EG Δ-complex n simplices: $[g_0, g_1, \cdots g_n]$ elements of G

$\partial(n\text{-simplex}) = \bigcup [g_0, g_1, \cdots \hat{g}_i \cdots g_n]$