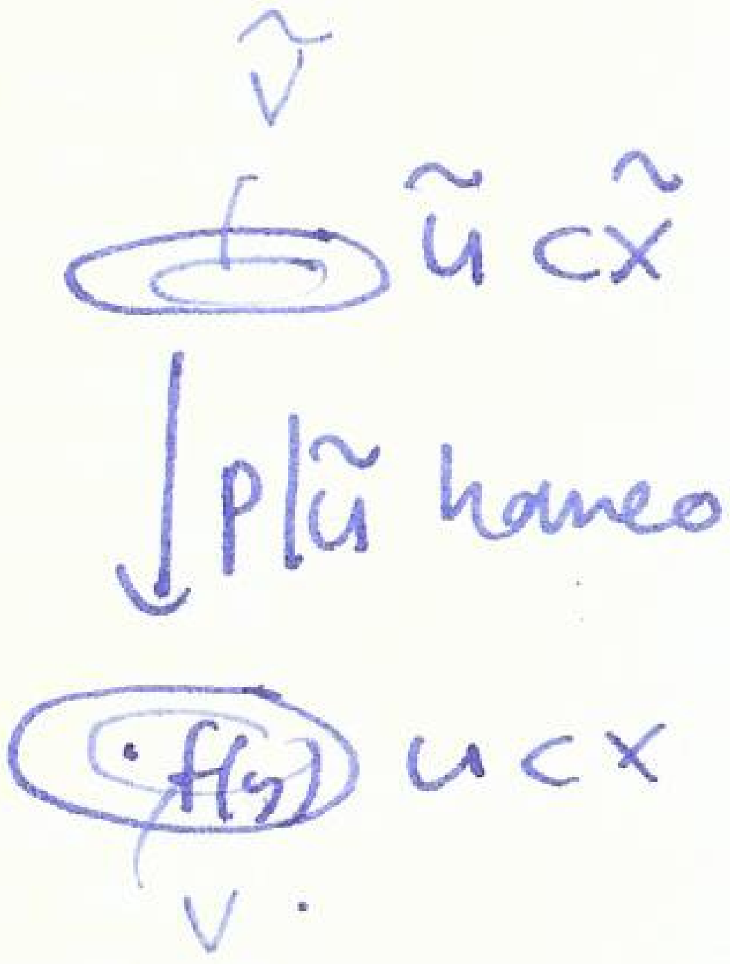
$\tilde{h}_1$ loop at $\tilde{x}_0$, so $\tilde{h}_0$ is a loop at $\tilde{x}_0$

by uniqueness of lifted paths, the first half of $\tilde{h}_0$ is $\widetilde{f\gamma'}$

and the second half is $\widetilde{f\gamma}$ backwards, with common midpoint

$\widetilde{f\gamma'}(1) = \widetilde{f\gamma}(1)$ so $\tilde{f}$ is well defined.



<u>check</u> $\tilde{f}$ is continuous

Let ~~$x \in U \in$~~ U be an open nbd of $f(y)$ s.t. $p|\tilde{U} : \tilde{U} \to U$

is a homeo, and let $V \subset U$ be a path connected nbd

of $f(y)$.

For points y in V, can take fixed path f to $f(y)$, and then path g in ~~$f(y)$~~ V from $f(y)$ to y

, but then g lifts by p^{-1}

so $\widetilde{fg} : V \to \tilde{V}$ by $\widetilde{fg} = p^{-1}$, so cts. $\square$.

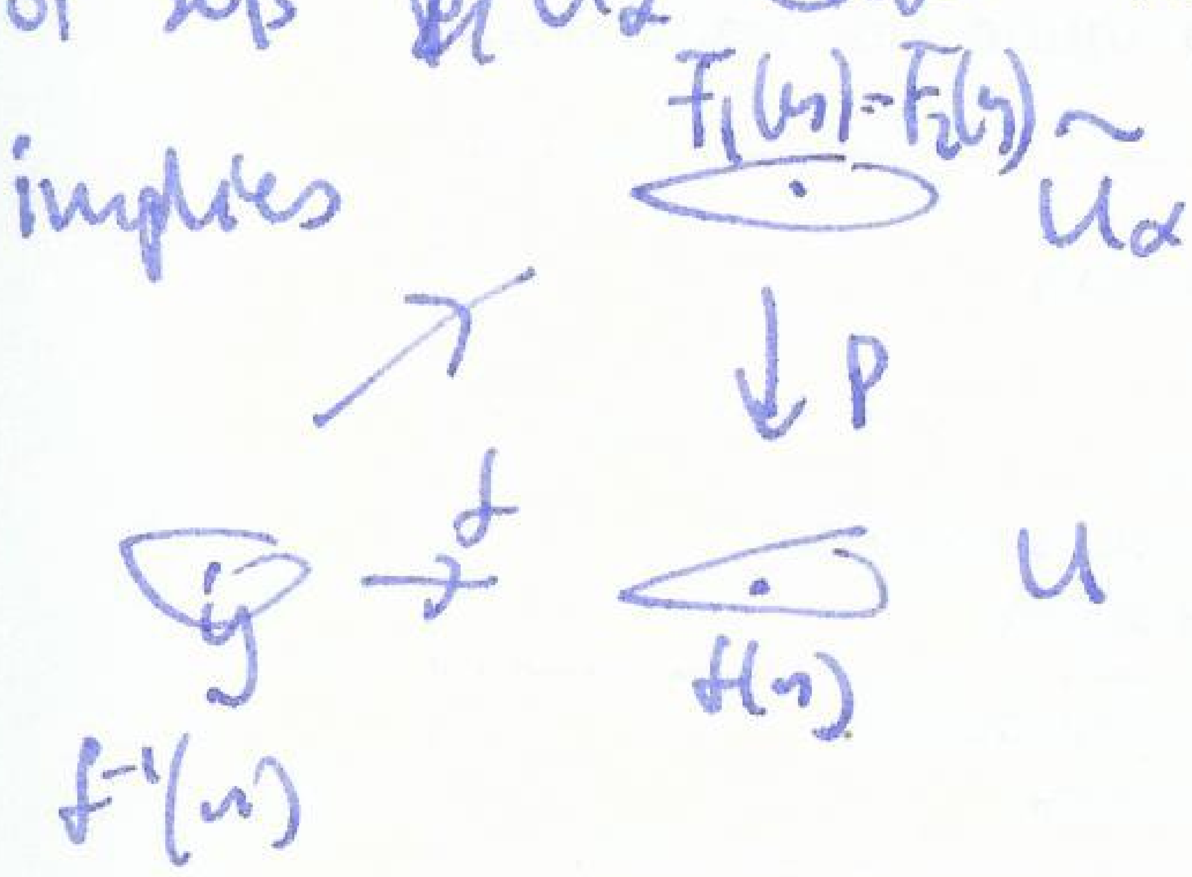
<u>unique lifting property</u>

<u>Prop^n</u> $p : \tilde{X} \to X$ covering space, $f : Y \to X$ with two lifts $\tilde{f}_1, \tilde{f}_2 : Y \to \tilde{X}$

s.t. there is a point $y \in Y$ with $\tilde{f}_1(y) = \tilde{f}_2(y)$, then $\tilde{f}_1 = \tilde{f}_2$ on all of Y.

<u>Proof</u> Let U be an open nbd of y s.t. $p^{-1}(U)$ is a disjoint union of sets ~~$\tilde{U}$~~ $\tilde{U}_\alpha$ each homeomorphic to U. Then $\tilde{f}_1(y) = \tilde{f}_2(y)$

implies



so $p\tilde{f}_1 = p\tilde{f}_2 \Rightarrow \tilde{f}_1 = \tilde{f}_2$ on $f^{-1}(U)$

now extend over any cover U_i of X. $\square$

Classification of covering spaces

X path connected, locally path connected, semilocally simply connected

<u>Def</u>

Def$^{\underline{n}}$ X is <u>semilocally simply connected</u> if each $x \in X$ has a neighbourhood U s.t. $\pi_1(U,x) \xrightarrow{i_*} \pi_1(X,x)$ is trivial

<u>Remark</u> locally simply connected $\Rightarrow$ semilocally simply connected
locally contractible $\Rightarrow$ locally simply connected
CW-complexes are locally contractible.

Th$^{\underline{m}}$ Let X be path connected, locally path connected, semilocally simply connected. Then there is a bijection between the set of basepoint preserving isomorphism classes of covering spaces $p:(\tilde{X},\tilde{x}_0) \to (X,x_0)$ and the set of subgroups of $\pi_1(X,x_0)$, bijection given by

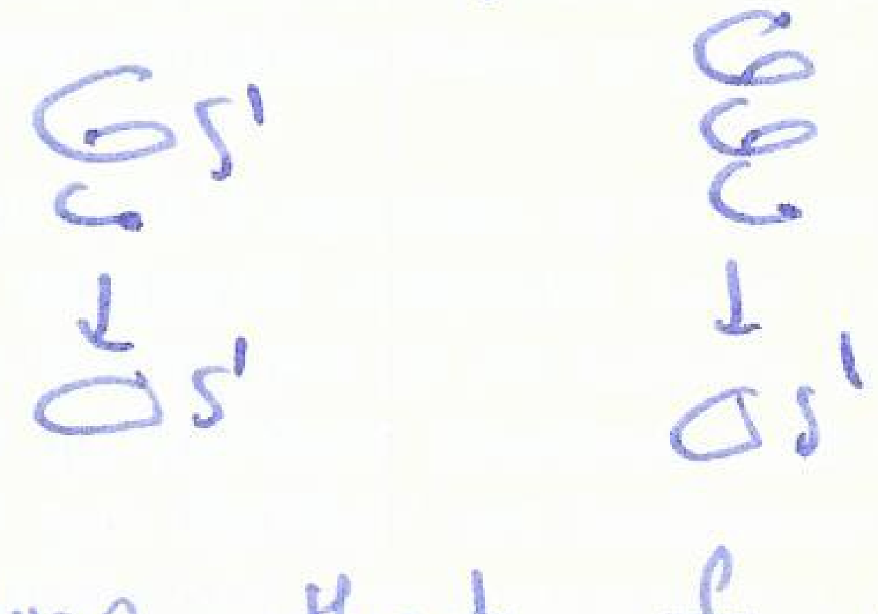
$$p:(\tilde{X},\tilde{x}_0) \to (X,x_0) \longleftrightarrow p_*(\pi_1(\tilde{X},\tilde{x}_0)).$$

If we ignore basepoints, this gives a correspondence between isomorphism classes of path connected covering spaces $p:\tilde{X} \to X$ and conjugacy classes of subgroups of $\pi_1(X,x_0)$.

Def$^{\underline{n}}$ Two covering spaces $p_1:\tilde{X}_1 \to X$, $p_2:\tilde{X}_2 \to X$ are <u>isomorphic</u> if there is a homeomorphism $f:\tilde{X}_1 \to \tilde{X}_2$ such that $p_1 = p_2 f$ ⊛

<u>Exercise</u>: this gives an equivalence relation on covering space.

<u>Non-example</u>



<u>not</u> isomorphic even though $S^1 \cong S^1$.

<u>Remark</u> ⊛ implies that f preserves the covering space structure, i.e. it takes $p_1^{-1}(x)$ to $p_2^{-1}(x)$ for each $x \in X$.