

Coffee

JC

Department of Mathematics College of Staten Island (CUNY)

FINAL SPRING 2012

MTH 232 CALCULUS II

NAME \_\_\_\_\_

Show your work—otherwise no credit. Each part of each question is worth 5 points.

1. Sketch the region under the graph of  $y = e^{-x}$  and above the x-axis on the interval  $0 \leq x < \infty$  and revolve it around the x-axis. Compute the volume of revolution. Volume = \_\_\_\_\_

2. a. Compute the Taylor Series for  $f(x) = e^{-x}$  around  $c = 0$  (i.e. compute the Maclaurin series).

b. What is the radius of convergence for the power series that you computed in 2a?

3. a. Sketch the graph of the polar equation  $r = 3 + \cos(t)$

b. Carefully write the Matlab commands needed to compute the area of the region enclosed by the polar graph of  $r = 3 + \cos(t)$

c. When  $r = 3 + \cos(t)$ , then  $x =$  \_\_\_\_\_ and  $y =$  \_\_\_\_\_

Use these values of  $x$  and  $y$  to compute the slope of the tangent line at  $t = \frac{\pi}{2}$



4. Find the indefinite integrals below:

a.  $\int \frac{x^2+1}{x} dx =$  \_\_\_\_\_

b.  $\int \frac{x^2+1}{x+1} dx =$  \_\_\_\_\_

c.  $\int \frac{x}{x^2+1} dx =$  \_\_\_\_\_

d.  $\int \frac{dx}{x^2+1} =$  \_\_\_\_\_

For questions 5-8 determine if the series converges or diverges. State which test you are using in 5, 7, 8. Explain your answer.

5.  $\sum_{n=0}^{\infty} \left(\frac{2}{e}\right)^n$  Circle your answer: converges or diverges. Test: \_\_\_\_\_

Explanation:

6.  $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$  Circle your answer: converges or diverges. Test: \_\_\_\_\_ Integral Test \_\_\_\_\_

Explanation:

7.  $\sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}$  Circle your answer: converges or diverges. Test: \_\_\_\_\_

Explanation:

8.  $\sum_{n=2}^{\infty} \frac{e^n}{n!}$  Circle your answer: converges or diverges. Test: \_\_\_\_\_

Explanation:



9. a. Does the **SEQUENCE** with the  $n$ th term  $a_n = \left(1 + \frac{1}{n}\right)^n$  converge? Circle  
correct answer: Yes No. If yes, what does it converge to? \_\_\_\_\_ If no, explain why.

b. Does the **SERIES**  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$  converge? Circle  
correct answer: Yes No. If yes, what does it converge to? \_\_\_\_\_ If no, explain why.

10. Let  $\mathbf{v}$  and  $\mathbf{w}$  be vectors where  $\mathbf{v} = \langle 3, 1, -1 \rangle$  and  $\mathbf{w} = \langle 2, 5, 1 \rangle$   
a. Sketch the vectors in 3-dimensional space—label your axes.

b. Compute the parametric equations of the line through the point  $(2,3,-6)$  in the direction of  $\mathbf{v}$ .

c. Compute  $\mathbf{v} \times \mathbf{w}$

d. Compute the equation of the plane that contains the vectors  $\mathbf{v}$  and  $\mathbf{w}$