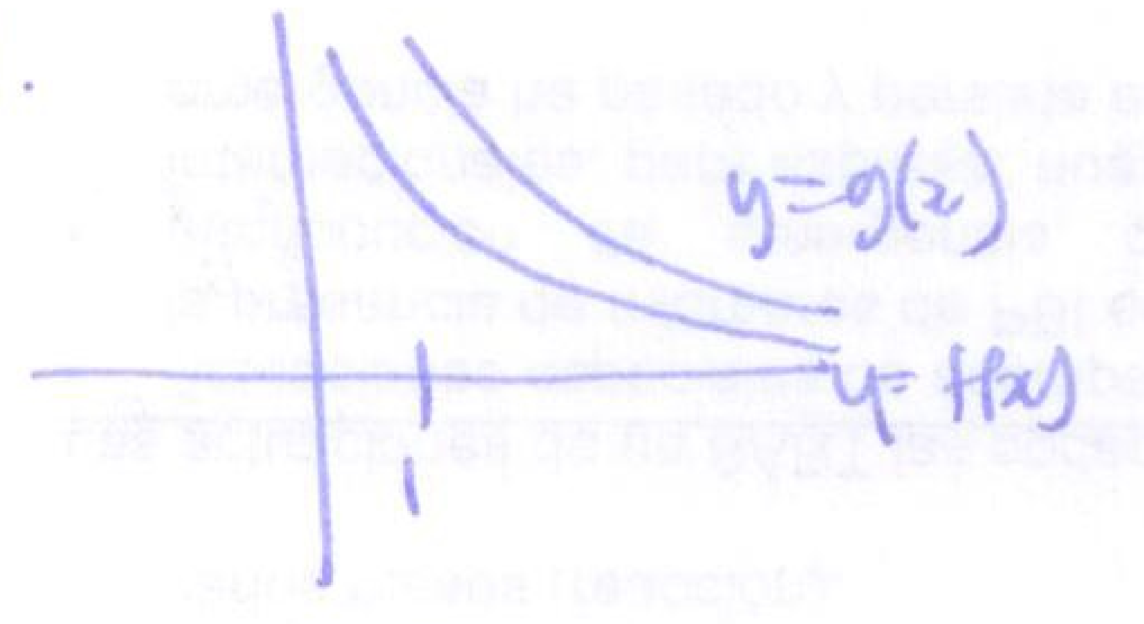
(30)

so $\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx$ converges $\Rightarrow \int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$ converges (but don't know exact value).

other way.



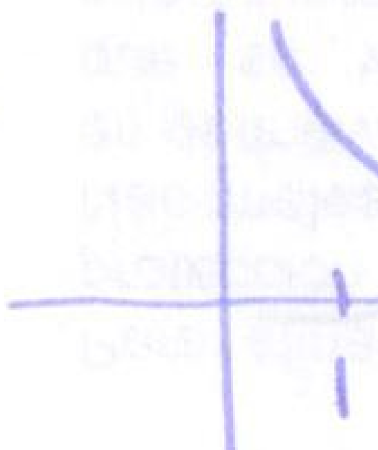
$0 \le f(x) \le g(x)$ on $(1, \infty)$

if $\int_1^\infty f(x)\,dx$ diverges $\Rightarrow \int_1^\infty g(x)\,dx$ diverges.

note $\int_1^\infty g(x)\,dx$ diverges $\not\Rightarrow \int_1^\infty f(x)\,dx$ diverges

$\int_1^\infty f(x)\,dx$ converges $\not\Rightarrow \int_1^\infty g(x)\,dx$ converges.

Example



show $\int_1^\infty \frac{1}{x-\sqrt{x}}\,dx$ diverges.

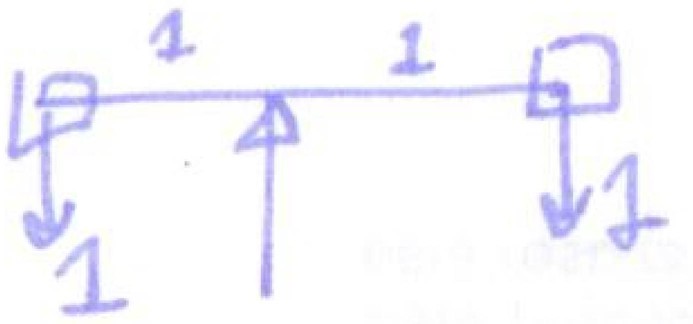
$-\sqrt{x} < \cancel{\sqrt{x}+4}$ for x (for $x > 1$).

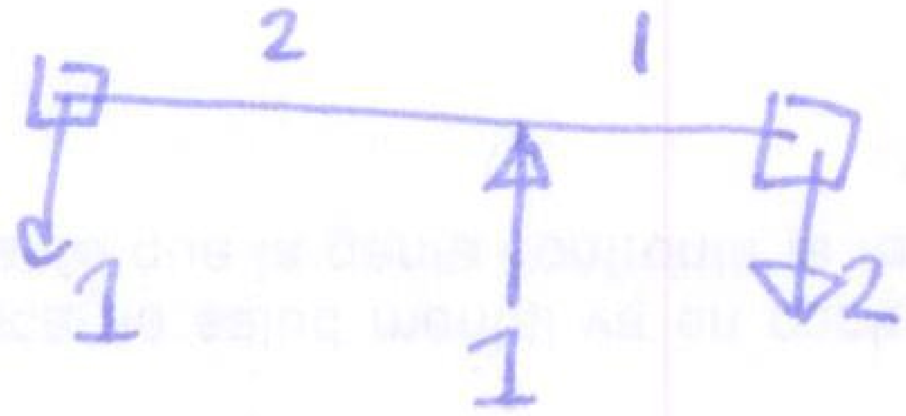
$\frac{1}{x-\sqrt{x}} > \frac{1}{x}$.

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{R\to\infty} \int_1^R \frac{1}{x}\,dx = \lim_{R\to\infty} \Big[\ln|x|\Big]_1^R = \lim_{R\to\infty} \ln|R| - 0 \Rightarrow \infty$$

so $\int_1^\infty \frac{1}{x-\sqrt{x}}\,dx$ diverges.

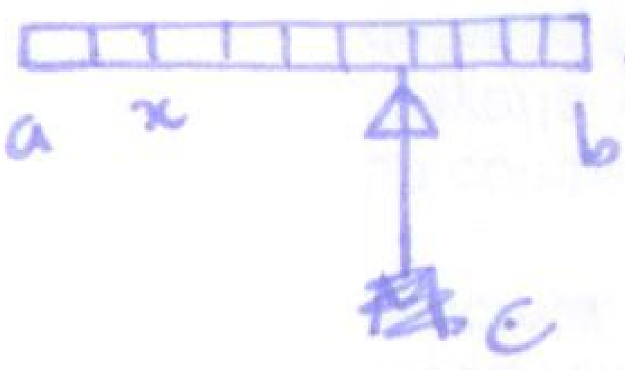
8.3 Center of mass





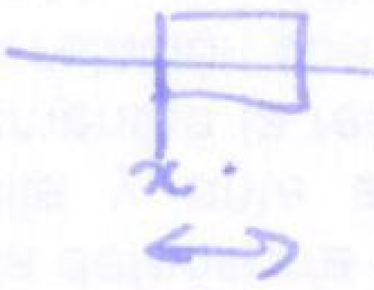
turning force / moment = weight × distance.

1×1   1×1   2×1   1×2



← varying density $\rho(x)$ total mass $M = \int_a^b \rho(x)\,dx$

turning force at x:



$\approx$ weight × distance

$\rho(x)\Delta x \quad (c-x)$

total turning force: $\sum \rho(x)(c-x)\Delta x \rightsquigarrow \int_a^b \rho(x)(c-x)\,dx$

balanced: when total turning force = 0

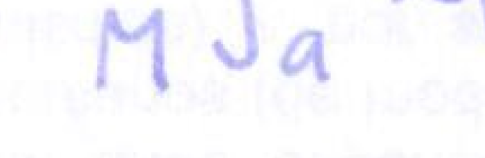
i.e. $\int_a^b \rho(x)(c-x)\,dx = 0$

$$\int_a^b c\rho(x) - x\rho(x)\,dx = \int_a^b c\rho(x)\,dx - \int_a^b x\rho(x)\,dx = 0$$

$$= c\underbrace{\int_a^b \rho(x)\,dx}_{=\text{total mass } M} - \int_a^b x\rho(x)\,dx = 0$$ solve for c:

$$c = \frac{1}{M}\int_a^b x\rho(x)\,dx$$

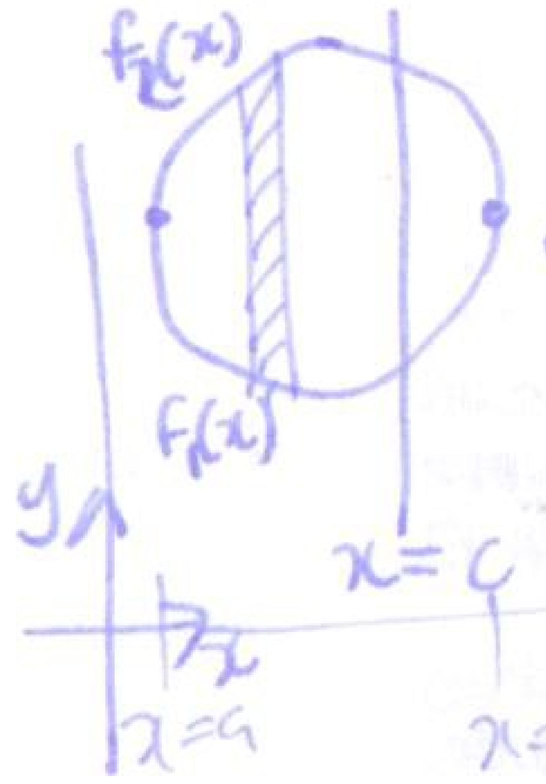
Example



$\rho(x) = 2x$ total mass is $\int_0^1 2x\,dx = \left[x^2\right]_0^1 = 1.$

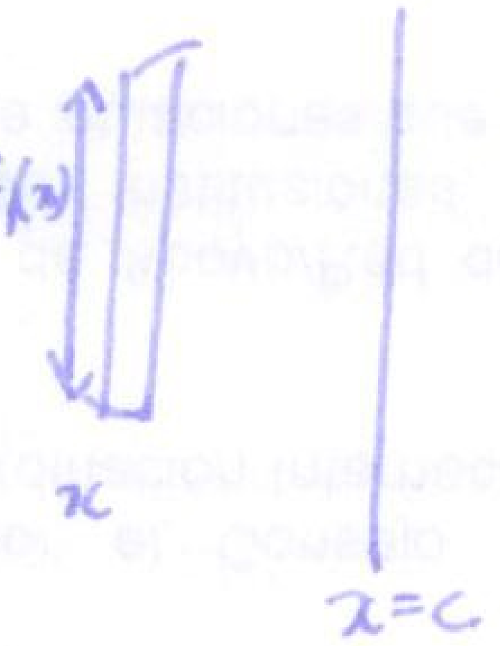
center of mass is $c = \frac{1}{M}\int_0^1 x \cdot 2x\,dx = \left[\frac{2}{3}x^3\right]_0^1 = \frac{2}{3}$

2d case: (uniform density ρ).



where is the center of mass? Look for turning force about a vertical/horizontal lines.

find turning force at x:



$$= \text{weight} \times \text{distance}$$

$$\rho\left(f_2(x)-f_1(x)\right)\Delta x\,(c-x)$$

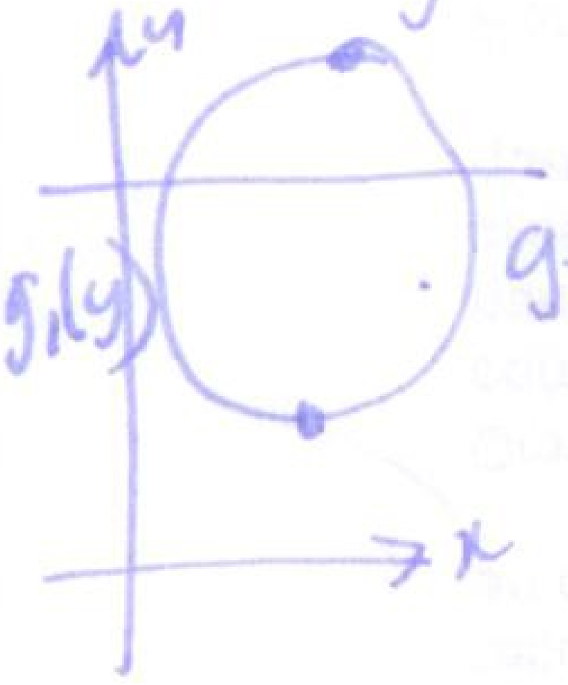
total turning force around $x=c$ is $\sum \left(f_2(x)-f_1(x)\right)\Delta x\,(c-x) \rightarrow \int_a^b \left(f_2(x)-f_1(x)\right)(c-x)\,dx$

balance when total turning force $=0$

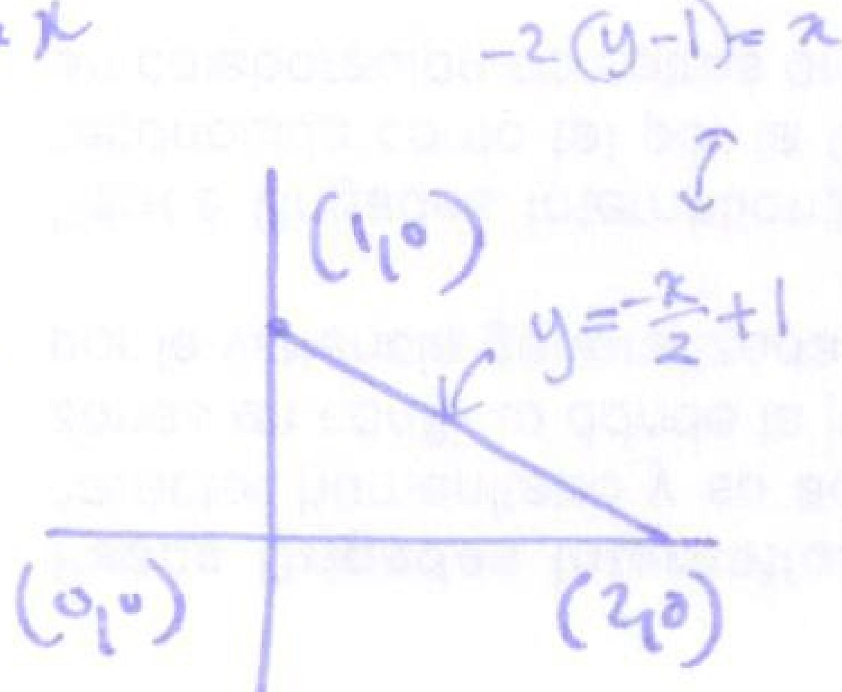
i.e. $$c\underbrace{\rho\int_a^b f_2(x)-f_1(x)\,dx}_{=\rho\,\text{Area}=\text{mass}} = \int_a^b x\left(f_2(x)-f_1(x)\right)dx$$

so x-balance point is $$M_x = \frac{1}{\rho M}\int_a^b x\left(f_2(x)-f_1(x)\right)dx.$$

Similarly y-balance point is $$M_y = \frac{1}{M}\int_c^d y\left(g_2(y)-g_1(y)\right)dy$$



Example ($\rho=1$)



total mass $$M = \int_0^2 -\frac{x}{2}+1\,dx$$

$$= \left[-\frac{x^2}{4}+x\right]_0^2 = -1+2 = 1$$

$f_2(x) = -\frac{x}{2}+1$ $\quad$ $g_2(x) = -2y+2$

$f_1(x) = 0$ $\quad$ $g_1(x) = 0$

$$M_x = \frac{1}{1}\int_0^2 x\left(-\frac{x}{2}+1\right)dx.$$

$$M_y = \frac{1}{1}\int_0^1 y\,(-2y+2)\,dy$$