Example $\int_0^\infty x e^{-x}\,dx = \lim_{R\to\infty} \int_0^R \underbrace{x}_{u}\underbrace{e^{-x}}_{v'}\,dx \qquad \int uv'\,dx = uv - \int u'v\,dx$

$u = x \qquad u' = 1$

$v' = e^{-x} \qquad v = -e^{-x}$

$$= \lim_{R\to\infty} \left[-xe^{-x}\right]_0^R - \int_0^R 1\cdot(-e^{-x})\,dx$$

$$= \lim_{R\to\infty} -Re^{-R} + 0 + \int_0^R e^{-x}\,dx = \lim_{R\to\infty} -Re^{-R} + \left[-e^{-x}\right]_0^R$$

$$= \lim_{R\to\infty} -Re^{-R} - e^{-R} + 1 = \lim_{R\to\infty} e^{-R}(-R-1) + 1$$

l'Hôpital $\lim_{R\to\infty} \frac{-R-1}{e^R} = \lim_{R\to\infty} \frac{-1}{e^R} = 0.$

so $\int_0^\infty xe^{-x}\,dx = 1$

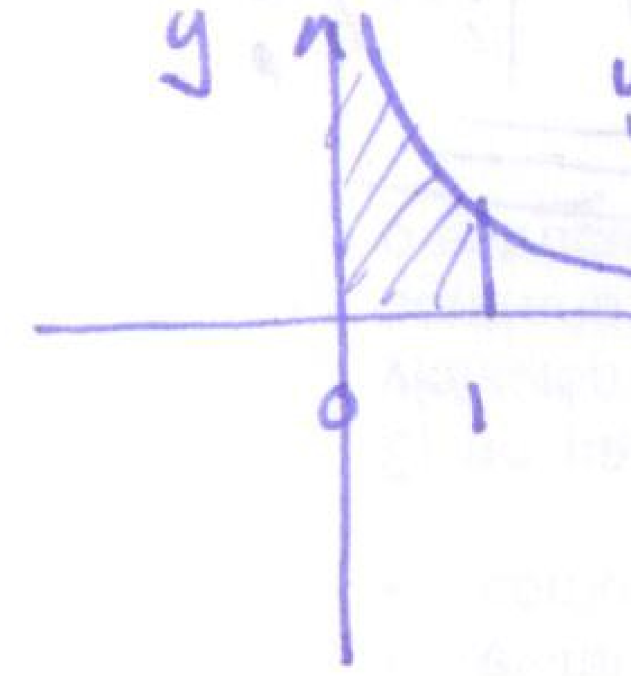
Example when does $\int_1^\infty \frac{1}{x^p}\,dx$ converge? know $p=1$ no, $p=2$ yes.

$$\lim_{R\to\infty} \int_1^R x^{-p}\,dx = \lim_{R\to\infty} \left[\frac{x^{-p+1}}{-p+1}\right]_1^R = \lim_{R\to\infty} \left(\frac{R^{-p+1}}{-p+1} - \frac{1}{-p+1}\right).$$

$\lim R^{-p+1} = 0$ if $p > 1$

∞ if $p < 1$ (in fact $p = 1$ by $\ln|R| \to \infty$).

Intervals containing discontinuities.



$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{R\to 0}\int_R^1 \frac{1}{\sqrt{x}}\,dx \qquad x^{-1/2}.$$

$$= \lim_{R\to 0}\left[2x^{1/2}\right]_R^1 = \lim_{R\to 0}\ 2-2\sqrt{R}. = 2$$

Watch out! $\int_{-1}^{1}\frac{1}{x}\,dx = \left[\ln|x|\right]_{-1}^{1} = \ln|1|-\ln|-1| = 0$

wrong

interval contains discontinuity!

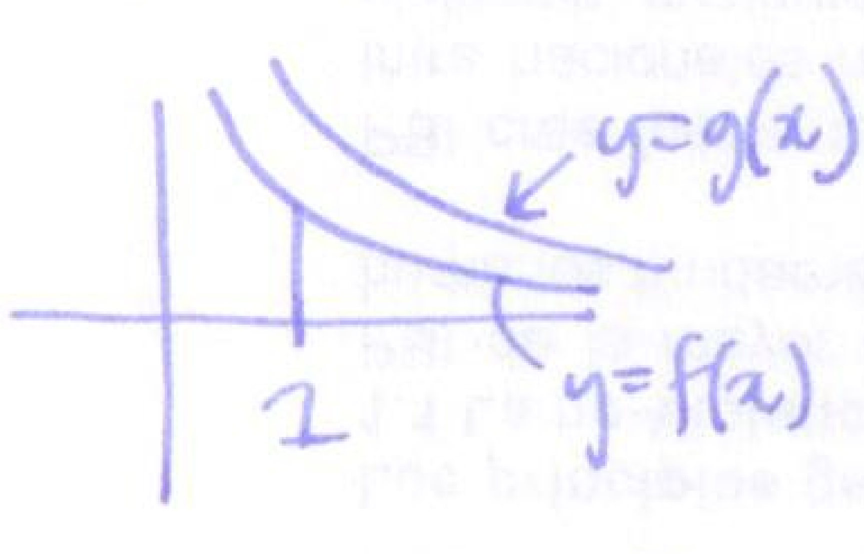
$$\int_{-1}^{1}\frac{1}{x}\,dx = \int_{-1}^{0}\frac{1}{x}\,dx + \int_{0}^{1}\frac{1}{x}\,dx$$

$$= \lim_{R\to 0}\int_{-1}^{R}\frac{1}{x}\,dx + \lim_{R\to 0}\int_{R}^{1}\frac{1}{x}\,dx.$$

$$= \lim_{R\to 0}\left[\ln|x|\right]_{-1}^{R} = \lim_{R\to 0}\ \ln|R| - \underbrace{\ln|-1|}_{0} = -\infty.$$

(DNE).

Comparison test



$0 \le f(x) \le g(x)$. on $[1,\infty)$.

if $\int_1^\infty g(x)\,dx$ converges then $\int_1^\infty f(x)\,dx$ converges.

Example $\int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx$

Note: $\sqrt{x^3+1} \ge \sqrt{x^3}$.

so $\frac{1}{\sqrt{x^3+1}} < \frac{1}{\sqrt{x^3}}$.

try: $\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx = \int_1^\infty x^{-3/2}\,dx = \lim_{R\to\infty}\left[-2x^{-1/2}\right]_1^R = \lim_{R\to\infty}\frac{-2}{\sqrt{R}}+2 = 2.$