§7.2 Integration by parts

"reverse product rule".

product rule: $(uv)' = u'v + uv'$

$$\int (uv)'\,dx = \int u'v + uv'\,dx.$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx.}$$

__Example__. $\int \underbrace{x}_{u}\,\underbrace{\sin(x)}_{v'}\,dx$ $\quad u = x \quad u' = 1$

$\quad v' = \sin(x) \quad v = -\cos(x)$

$$\int \underbrace{x}_{u}\,\underbrace{\sin(x)}_{v'}\,dx = \underbrace{x}_{u}\cdot\underbrace{(-\cos(x))}_{v} - \int (1)\cdot(-\cos(x))\,dx$$

$$\int x\sin(x)\,dx = -x\cos(x) + \int \cos(x)\,dx$$

$$= -x\cos(x) + \sin(x) + C$$

__check!__ $(-x\cos(x) + \sin(x) + C)' = -x\cdot -\sin(x) - \cos(x) + \cos(x)$

$$= x\sin(x)\ \checkmark.$$

Examples. 1 $\displaystyle\int \underbrace{x}_{u}\underbrace{e^x}_{v'}\,dx$ $\qquad u = x \quad u' = 1 \quad v' = e^x \quad v = e^x$ } Q: spose chose this other way round?

$$= \overset{uv}{xe^x} - \int \overset{u'v}{(1).e^x}\,dx = xe^x - e^x + c.$$ check!

2 observation $\displaystyle\int_a^b uv'\,dx = \Big[uv\Big]_b^a - \int_a^b u'v\,dx$

2 $\displaystyle\int_1^2 \ln(x)\,dx = \int_1^2 \underbrace{1}_{v'}.\underbrace{\ln(x)}_{u}\,dx$ $\qquad u = \ln(x) \quad u' = \frac{1}{x} \quad v' = 1 \quad v = x.$

$$\int_1^2 1.\ln(x)\,dx = \overset{uv}{x\ln(x)} - \int \overset{u'v}{\frac{1}{x}x}\,dx = x\ln x - \int 1\,dx$$
$$= x\ln(x) - x + c$$ check!

3 $$\int \underbrace{x^2}_{u}\underbrace{\cos(x)}_{v'}\,dx = \overset{u}{x^2}\overset{v}{(\sin(x))} - \int \underbrace{2x}_{u}\underbrace{\sin(x)}_{v'}\,dx.$$

$$= x^2\sin(x) - \overset{uv}{2x(-\cos(x))} + \int \overset{u'v}{2(-\cos(x))}\,dx$$

$$= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + c.$$ check!

4 $$\int \underbrace{e^x}_{u}\underbrace{\sin(x)}_{v'}\,dx = \overset{uv}{e^x(-\cos(x))} - \int \underbrace{e^x}_{u}.\underbrace{(-\cos(x))}_{v'}\,dx$$

$$= -e^x\cos(x) - \Big\{ \overset{uv}{e^x.(-\sin(x))} + \int \overset{u'v}{e^x.(-\sin(x))}\,dx$$

$$\int e^x\sin(x)\,dx = e^x\cos(x) + e^x\sin(x) - \int e^x\sin(x)\,dx$$

$$2\int e^x\sin(x)\,dx = e^x(\sin(x) - \cos(x)) + c$$

$$\int e^x\sin(x)\,dx = \frac{1}{2}e^x(\sin(x) - \cos(x)) + c$$ check!