MTH 232 Calculus 2

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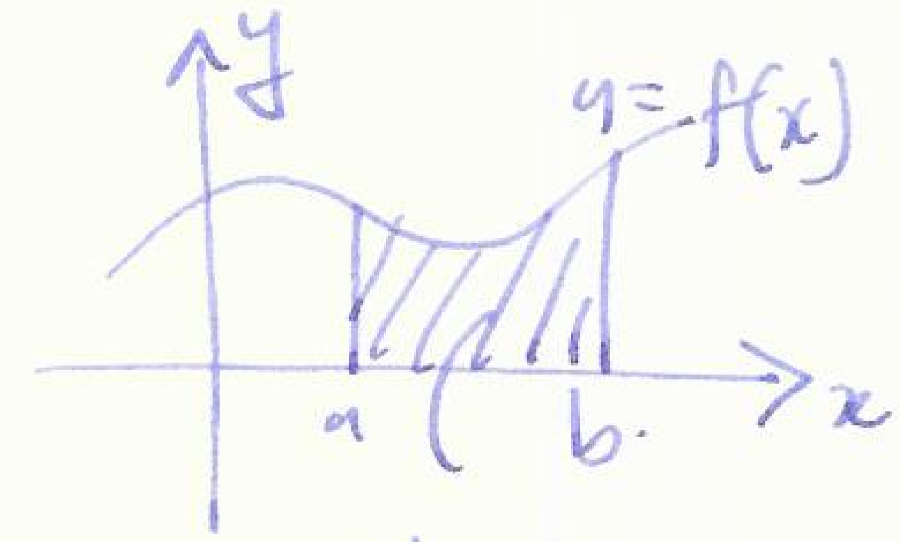
Text: Early Transcendentals, Rogawski

§5.3 Fundamental theorem of calculus

<u>Recall</u> <u>indefinite integral</u>: $\int f(x)\,dx = F(x) + c$

where $F(x)$ is an anti-derivative for $f(x)$, i.e. a function such that $\frac{d}{dx}(F(x)) = f(x)$ alternate notation $F'(x) = f(x)$.

<u>definite integral</u> $\int_a^b f(x)\,dx$ = area under curve

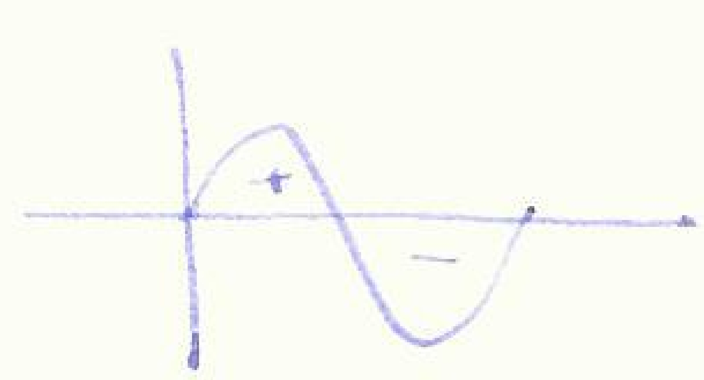


<u>Thm</u> (FTC I) Let $f(x)$ be continuous on the interval $[a,b]$ and let $F(x)$ be an anti-derivative for $f(x)$. $\int_a^b f(x)dx$.

Then
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

alternate notation: $\left[F(x)\right]_a^b = F(b) - F(a)$

<u>Examples</u> $\int_0^1 x^2\,dx$ $\int_0^{2\pi} \sin(x)\,dx$



<u>signed</u> area.

Recall: rules for differentiation / integration.

$f(x)$	$f'(x)$	
x^n	nx^{n-1}	works for $n \in \mathbb{R}$! $\sqrt{x} = x^{1/2}$ ok.
$\sin(x)$	$\cos(x)$	
$\cos(x)$	$-\sin(x)$	
$\tan(x)$	$\sec^2(x)$	
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$	
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$	
e^x	e^x	
$\ln(x)$	$\frac{1}{x}$.	

$f(x)$	$\int f(x)\,dx$.
x^n	$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\frac{1}{x} = x^{-1}$	$\ln\lvert x\rvert$
e^x	e^x

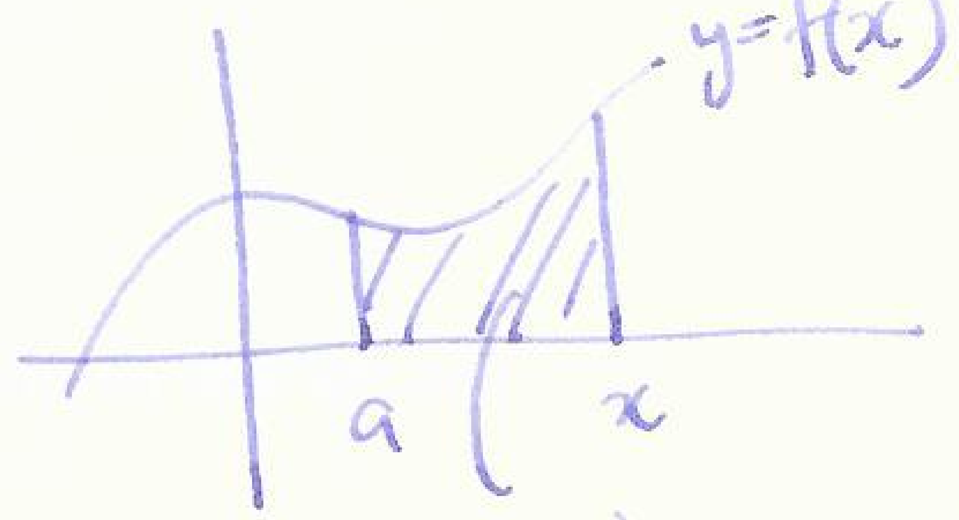
general rules

$kf(x)$	$kf'(x)$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$
$f(g(x))$	$f'(g(x)).g'(x)$

$kf(x)$	$k\int f(x)\,dx$
$f(x)+g(x)$	$\int f(x)\,dx+\int g(x)\,dx$

§5.4 FTC II

area function:



A(x) = area between a and x

$$A(x) = \int_a^x f(t)\,dt$$

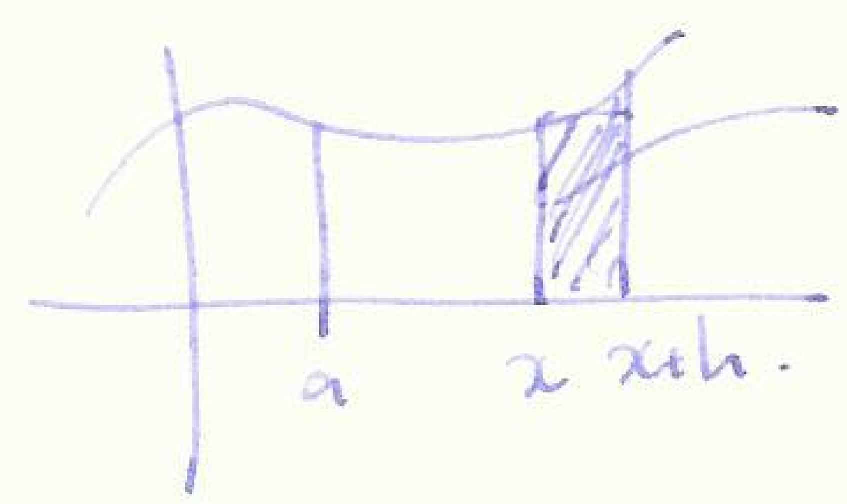
Thm (FTC II) Let $f(x)$ be a continuous function on the interval $[a,b]$

then $A(x) = \int_a^x f(t)\,dt$ is an antiderivative of $f(x)$, i.e.

$A'(x) = f(x)$ alternate notation $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Furthermore $A(0) = 0$.

Intuition $A'(x) = \lim_{h\to 0} \frac{A(x+h)-A(x)}{h}$



$A(x+h) - A(x) \approx h \cdot A(x)$.

Example $\frac{d}{dx}\int_2^x \sqrt{1+t+t^2}\,dt$ $\qquad$ $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$

Aside: when people say "not every function can be integrated" what do they mean?

$f(x)$ cts then $A(x) = \int_0^x f(t)\,dt$ is an integral for $f(x)$, but you might not be able to write it down as an expression involving basic functions.

Analogy: $\sqrt{2}$ is a number, but its not a fraction. $\sqrt{2} = 1.414\ldots$

$x^5 - x - 1$ has 1 real root which can not be written as an expression involving rational numbers and fractional powers. (Galois theory)

e^{-x^2} has an integral that cannot be expressed as a combination of elementary functions. (differential Galois theory)