Math 232 Calculus 2 Fall 12 Midterm 1b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no notes or phones.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) Find $\int e^{1-2x}\, dx$.

$u = 1-2x$

$\frac{du}{dx} = -2$

$$\int e^u \frac{dx}{du}\, du$$

$$= -\frac{1}{2}\int e^u\, du = -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{1-2x} + C$$

(2) Find $\int \frac{\sin(x)}{\sqrt{1+\cos(x)}}\, dx$.

$u = 1+\cos(x)$

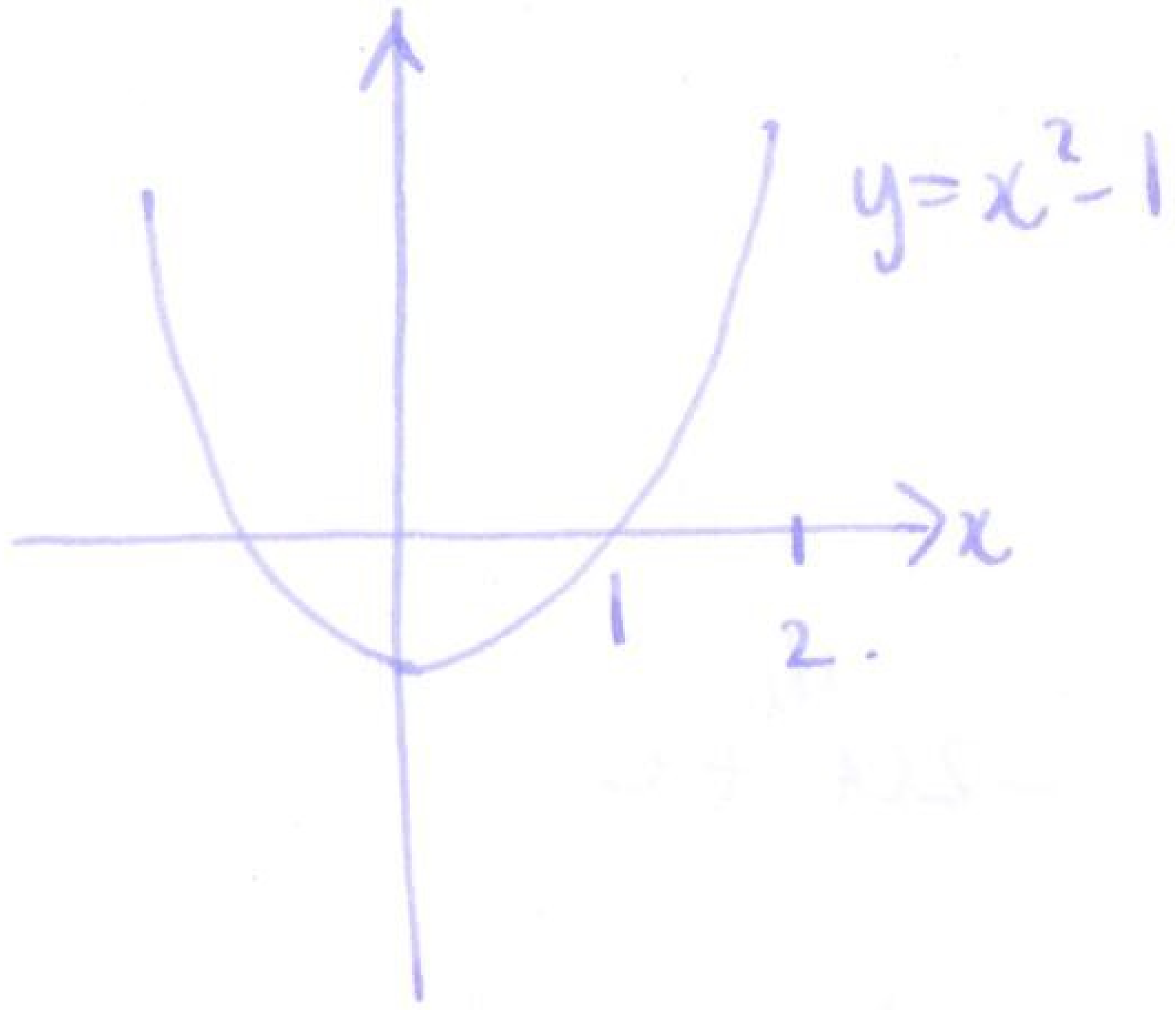
$\frac{du}{dx} = -\sin(x)$

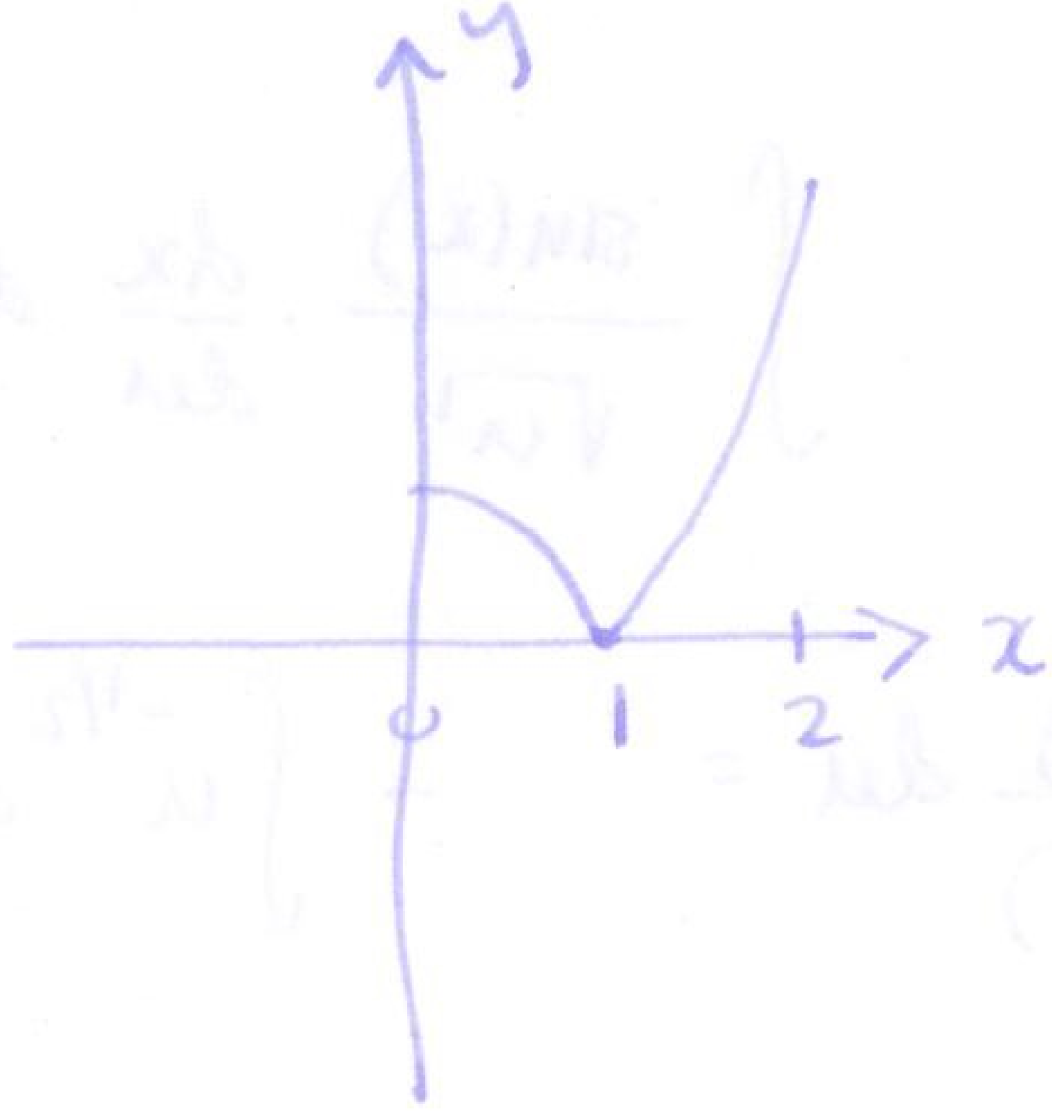
$\int \frac{\sin(x)}{\sqrt{u}} \cdot \frac{dx}{du}\, du$

$= \int u^{-1/2} \frac{\sin(x)}{-\sin(x)}\, du = -\int u^{-1/2}\, du = -2u^{1/2} + c$

$= -2\sqrt{1+\cos(x)} + c$.

(3) Find $\int_0^2 |x^2 - 1| \; dx$. Draw a picture of the region.



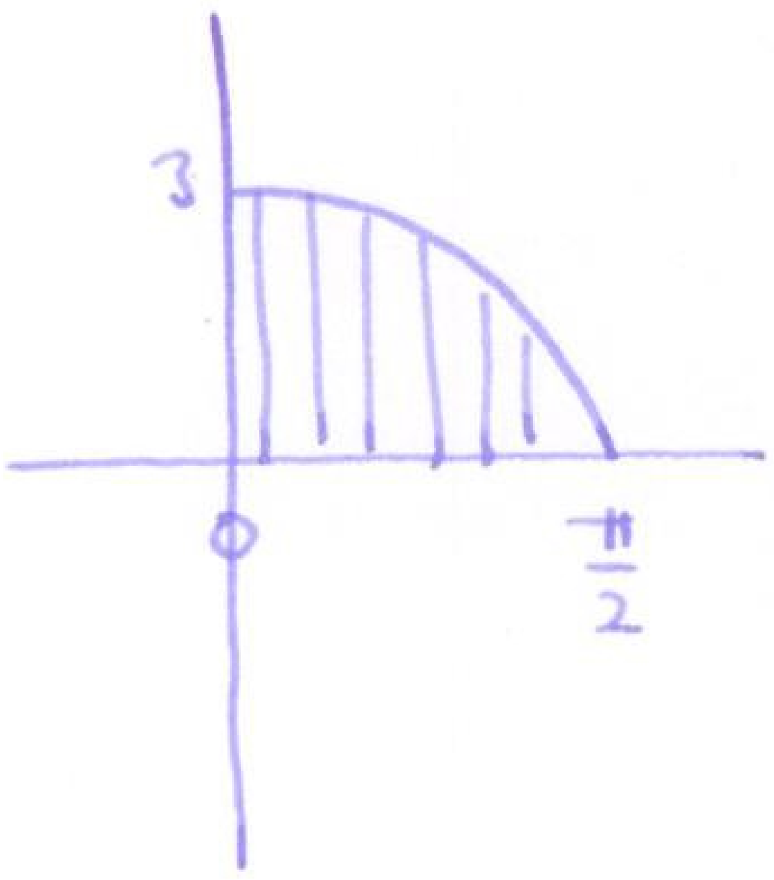


$$\int_0^1 1-x^2\,dx + \int_1^2 x^2-1\,dx$$

$$= \left[x - \tfrac{1}{3}x^3\right]_0^1 + \left[\tfrac{1}{3}x^3 - x\right]_1^2$$

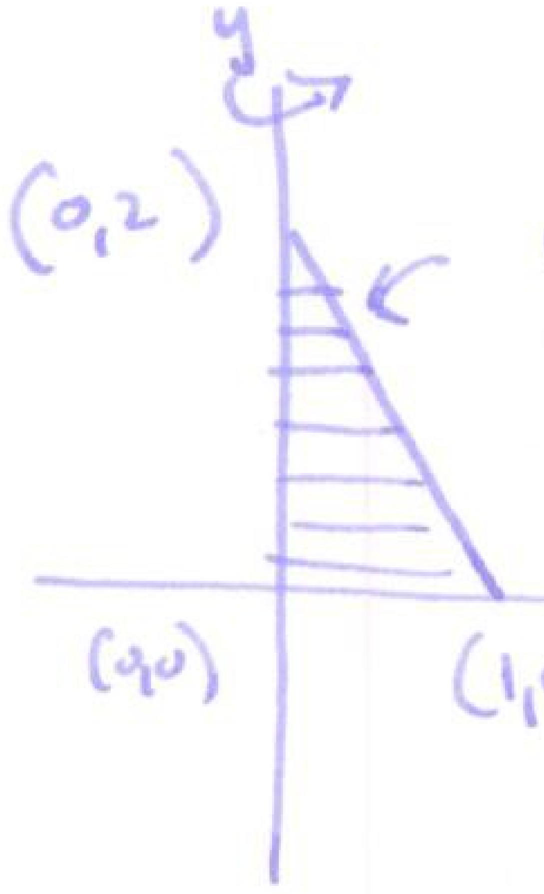
$$= 1 - \tfrac{1}{3} + \tfrac{8}{3} - 2 - \tfrac{1}{3} + 1 = 2$$

(4) Draw a picture of the region bounded by the curve $y = 3\cos(x)$, for $0 \le x \le \pi/2$ and $y \ge 0$. Write down an integral to give you the volume of revolution of this region about the x-axis. DO NOT EVALUATE THIS INTEGRAL.



discs : $\int_0^{\frac{\pi}{2}} \pi \left(3\cos(x)\right)^2 dx$

(5) Use shells to write down an integral for the volume of the cone formed by rotating the triangle with vertices $(0,0),(0,2)$ and $(1,0)$ about the y-axis. DO NOT EVALUATE THIS INTEGRAL.

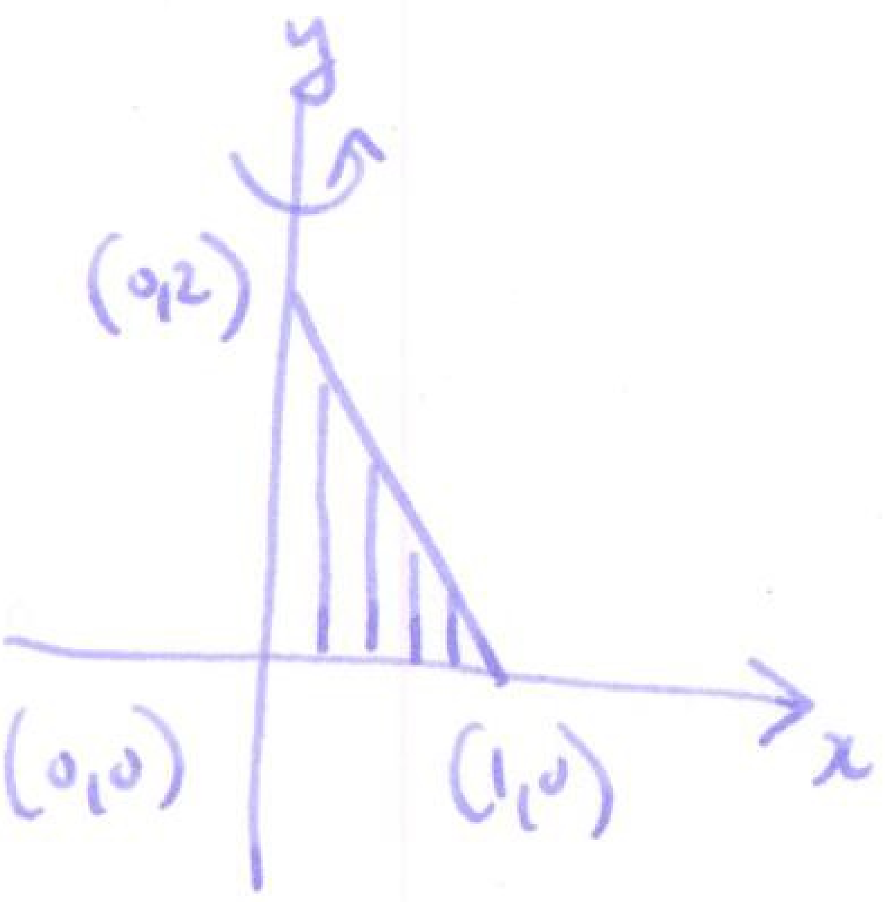


discs

$y = 2-2x \Leftrightarrow \frac{y}{2}-1 = x$

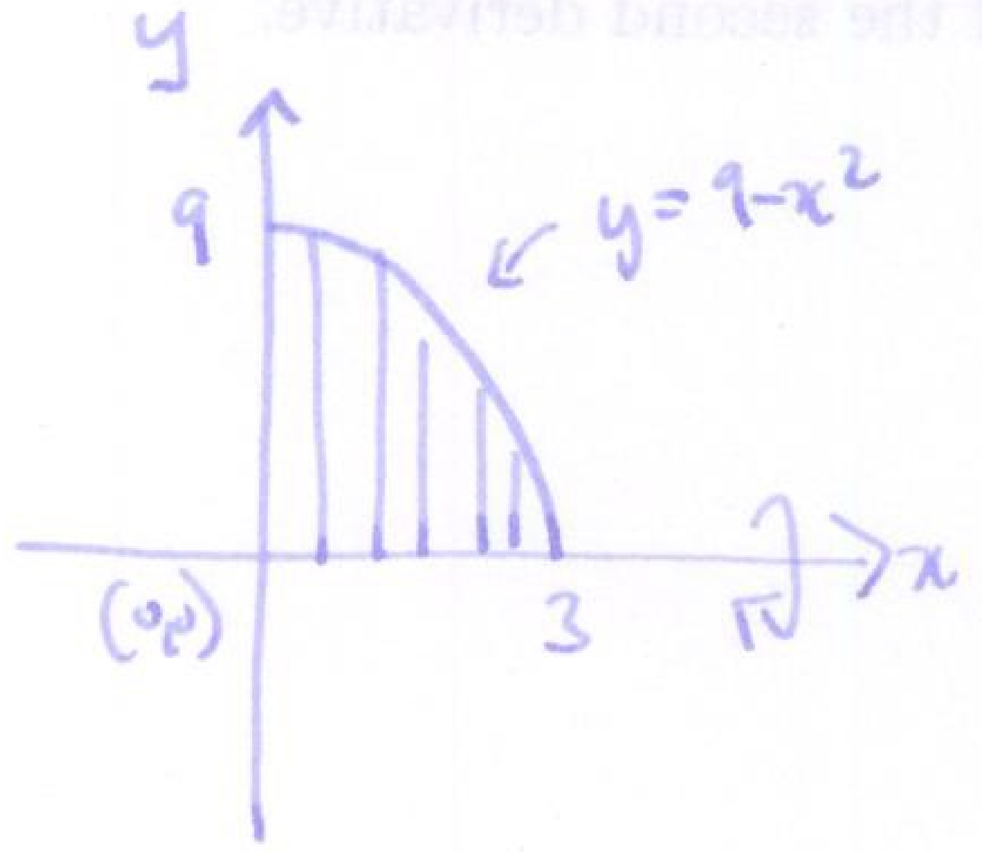
$$\int_0^2 \pi\left(\frac{y}{2}-1\right)^2 dy$$

shells



$$\int_0^1 2\pi x\,(2-2x)\,dx$$

(6) Consider the subset of the plane bounded by $y = 9 - x^2$ in the first quadrant (i.e. $x \geq 0$ and $y \geq 0$). Find the volume of revolution of the 3-dimensional shape formed by rotating this region around the x-axis.



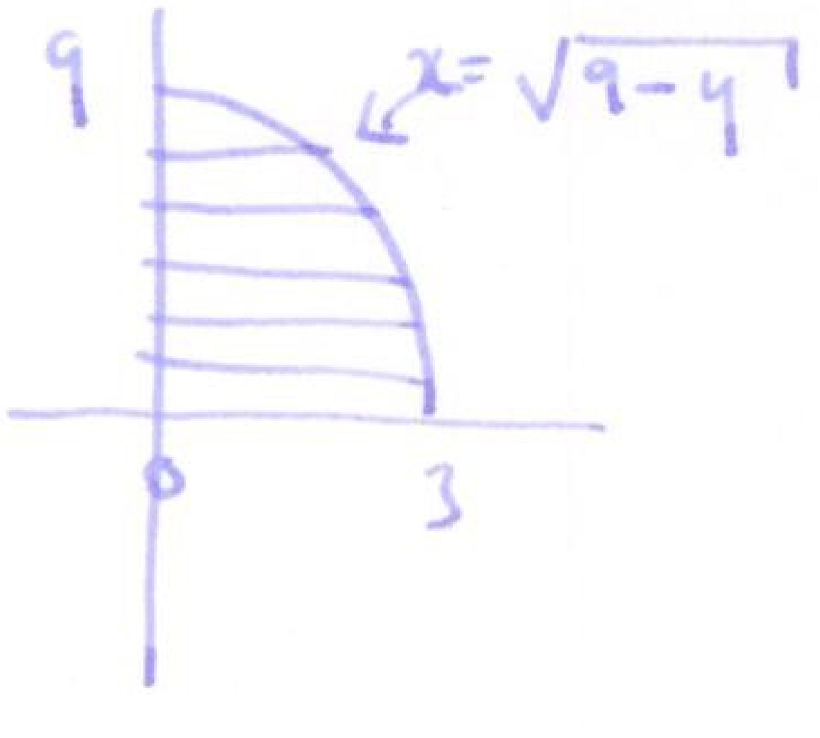
discs: $\int_0^3 \pi \left(9-x^2\right)^2 dx$

$= \pi \int_0^3 81 - 18x^2 + x^4 \, dx$

$= \pi \left[81x - 6x^3 + \frac{1}{5}x^5 \right]_0^3$

$= \pi \left(243 - 162 + \frac{243}{5} \right)$

shells



$\cancel{\int_0^9 \pi \left(\sqrt{9-y}\right)^2 dy = \pi \int_0^9 9 - y \, dy}$

$\cancel{= \pi \left[9y - \frac{1}{2}y^2 \right]_0^9 = \pi \left(81 - \frac{81}{2} \right) = \frac{81\pi}{2}}$

$\int_0^9 2\pi y \sqrt{9-y} \, dy$

(7) How many trapezoids to you need to use to estimate $\int_0^4 \sin(x^2)dx$ to 2 decimal places?

Recall: the error bound for the trapezoid method is $K_2(b-a)^3/(12N^2)$, where K_2 is an upper bound on the absolute value of the second derivative.

$f(x) = \sin(x^2)$

$f'(x) = \cos(x^2).2x$

$f''(x) = \underbrace{-\sin(x^2)}_{\leq 1}.\underbrace{4x^2}_{\leq 64} + \underbrace{2\cos(x^2)}_{\leq 1}$ $\quad K_2 = 66$

on $[0,4]$

$$\frac{66 \cdot 4^3}{12 N^2} \leq \frac{10^{-2}}{2}$$

$N^2 \geq 11 \cdot 32 \cdot 2 \cdot 100$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(8) Find $\int 3xe^{-2x}\,dx$.

$u = 3x \qquad u' = 3$

$v' = e^{-2x} \qquad v = -\frac{1}{2}e^{-2x}$

$$\int 3xe^{-2x}\,dx = -\frac{3}{2}xe^{-2x} + \int \frac{3}{2}e^{-2x}\,dx$$

$$= -\frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x} + c$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(9) Find $\int \sqrt{x}\ln(3x)\ dx$.

$u = \ln(3x)$ $\quad u' = \frac{1}{3x}\cdot 3 = \frac{1}{x}$

$v' = x^{1/2}$ $\quad v = \frac{2x^{3/2}}{3}$

$$\int \sqrt{x}\ln(3x)\,dx = \frac{2}{3}x^{3/2}\ln(3x) - \int \frac{2}{3}x^{1/2}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln(3x) - \frac{4}{9}x^{3/2} + C$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(10) Find $\int e^{-x}\cos(2x)\,dx$.

$$\int e^{-x}\cos(2x)\,dx = e^{-x}\cdot\frac{1}{2}\sin(2x) + \int e^{-x}\frac{1}{2}\sin(2x)\,dx$$

$u = e^{-x}$ $\quad u' = -e^{-x}$ $\qquad\qquad u = e^{-x}$ $\quad u' = -e^{-x}$

$v' = \cos(2x)$ $\quad v = \frac{1}{2}\sin(2x)$ $\qquad\qquad v' = \frac{1}{2}\sin(2x)$ $\quad v = -\frac{1}{4}\cos(2x)$

$$\int e^{-x}\cos(2x)\,dx = \frac{1}{2}e^{-x}\sin(2x) - \frac{1}{4}e^{-x}\cos(2x) + \int(-e^{-x})\cdot\left(-\frac{1}{4}\cos(2x)\right)dx$$

$$\left(1+\frac{1}{4}\right)\int e^{-x}\cos(2x)\,dx = \frac{1}{2}e^{-x}\sin(2x) - \frac{1}{4}e^{-x}\cos(2x) + C.$$

$$\int e^{-x}\cos(2x)\,dx = \frac{2}{3}e^{-x}\sin(2x) - \frac{1}{3}e^{-x}\cos(2x) + c$$