Math 232 Calculus 2 Fall 12 Midterm 1a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no notes or phones.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) Find $\int e^{1-3x}\,dx$.

$u = 1-3x$

$\frac{du}{dx} = -3$

$\int e^u \frac{dx}{du}\,du = \int e^u \cdot \left(\frac{1}{-3}\right) du = \frac{1}{-3}e^u + C$

$= \frac{1}{-3}e^{1-3x} + C$

(2) Find $\int \frac{\cos(x)}{\sqrt{1-\sin(x)}}\, dx$.

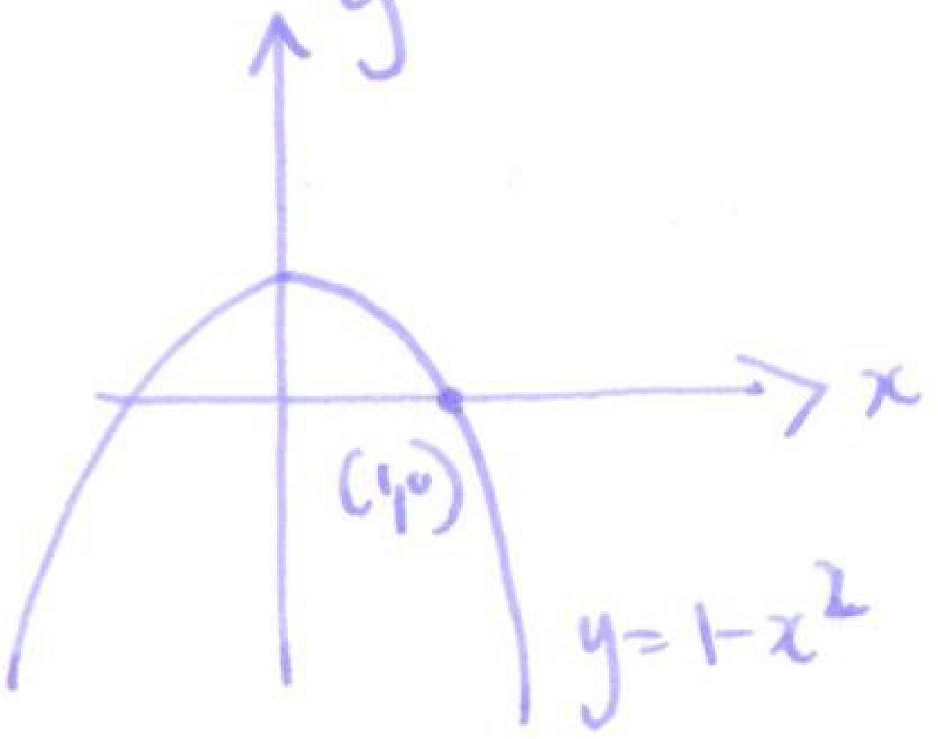
$u = 1 \cancel{xx} \sin(x)$

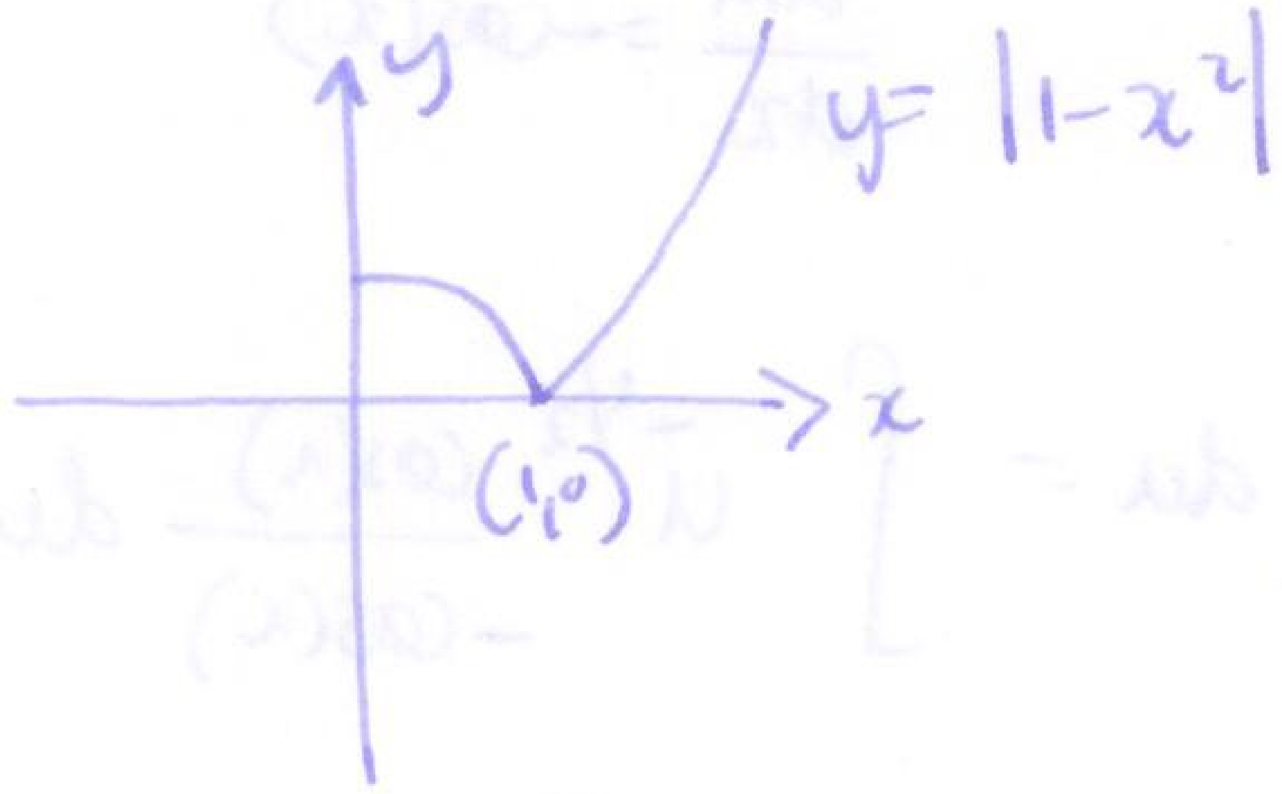
$\frac{du}{dx} = -\cos(x)$

$$\int \frac{\cos(x)}{\sqrt{u}} \frac{dx}{du}\, du = \int u^{-1/2} \frac{\cos(x)}{-\cos(x)}\, du$$

$$= -2u^{1/2} + c = -2\sqrt{1-\sin(x)} + c$$

(3) Find $\int_0^3 |1 - x^2| \, dx$. Draw a picture of the region.



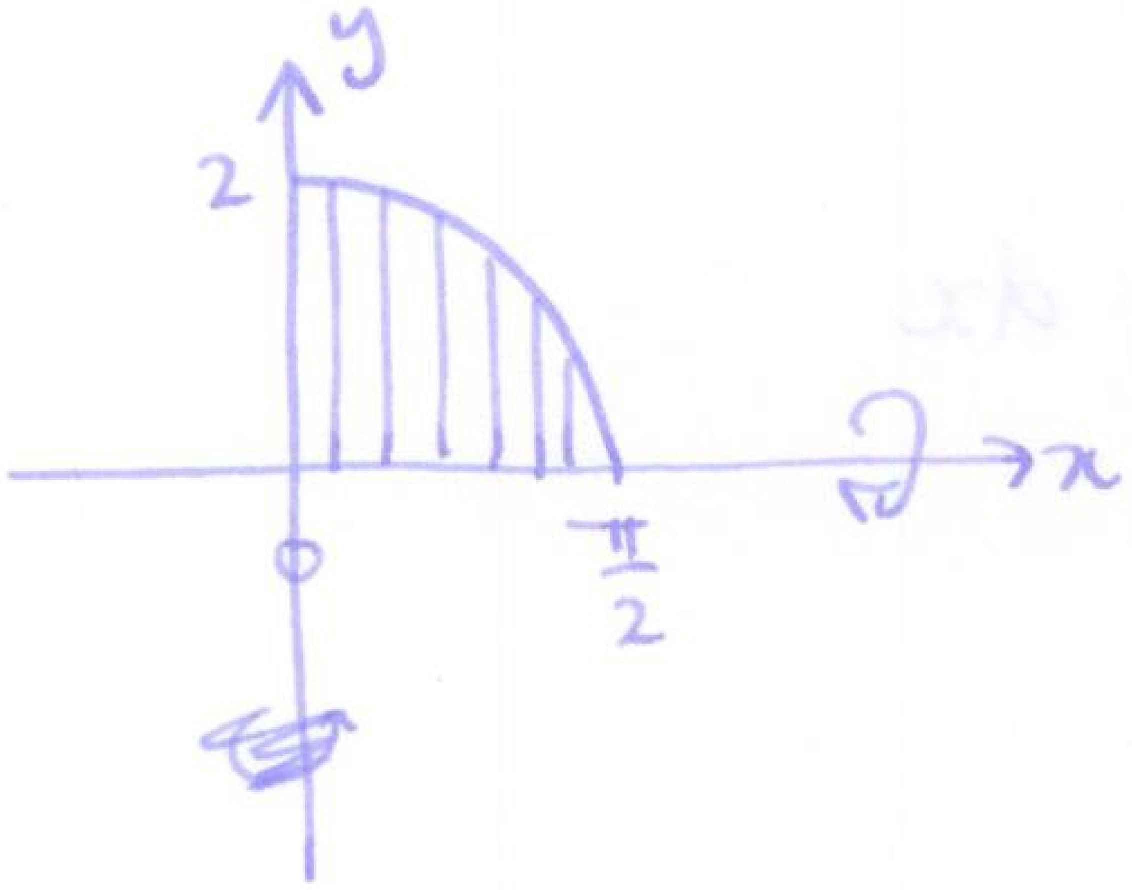


$$\int_0^{1} 1-x^2\,dx + \int_1^3 x^2-1\,dx$$

$$= \left[x - \tfrac{1}{3}x^3\right]_0^1 + \left[\tfrac{1}{3}x^3 - x\right]_1^3$$

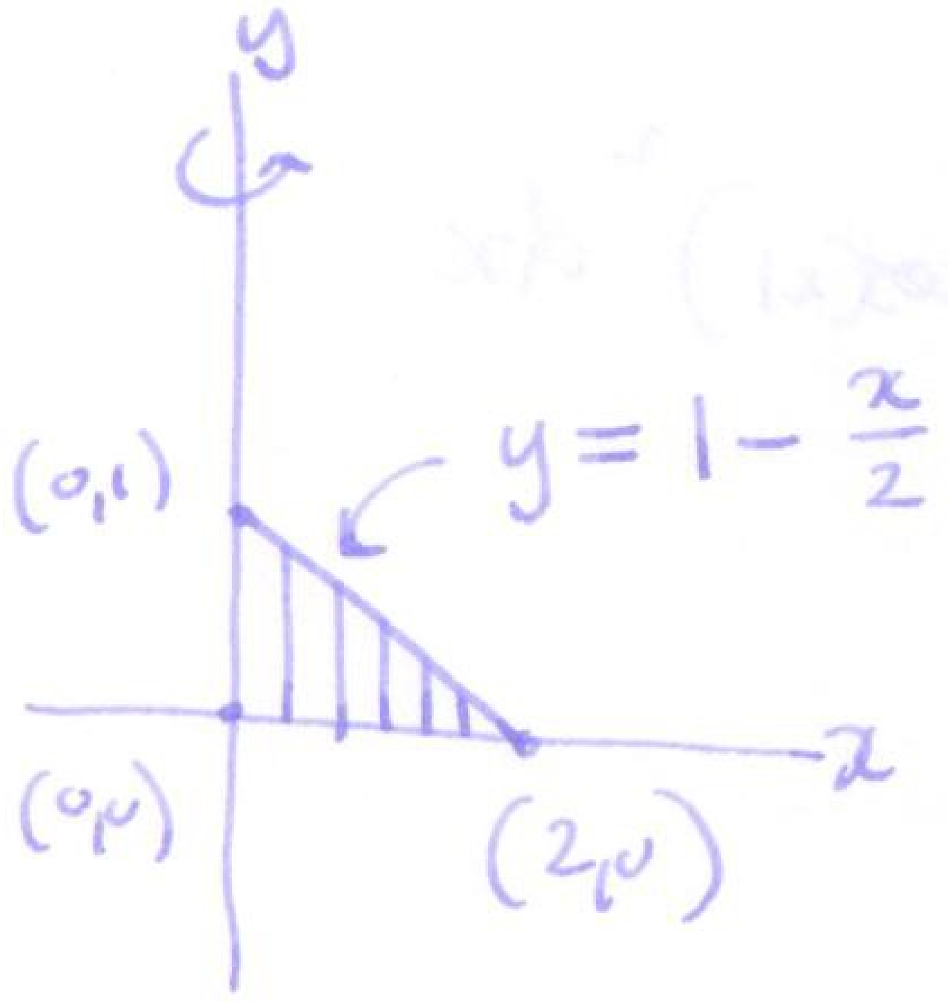
$$= 1 - \tfrac{1}{3} + 9 - 3 - \tfrac{1}{3} + 1 \quad = 7\tfrac{1}{3} = \tfrac{22}{3}$$

(4) Draw a picture of the region bounded by the curve $y = 2\cos(x)$, for $0 \leq x \leq \pi/2$ and $y \geq 0$. Write down an integral to give you the volume of revolution of this region about the x-axis. DO NOT EVALUATE THIS INTEGRAL.



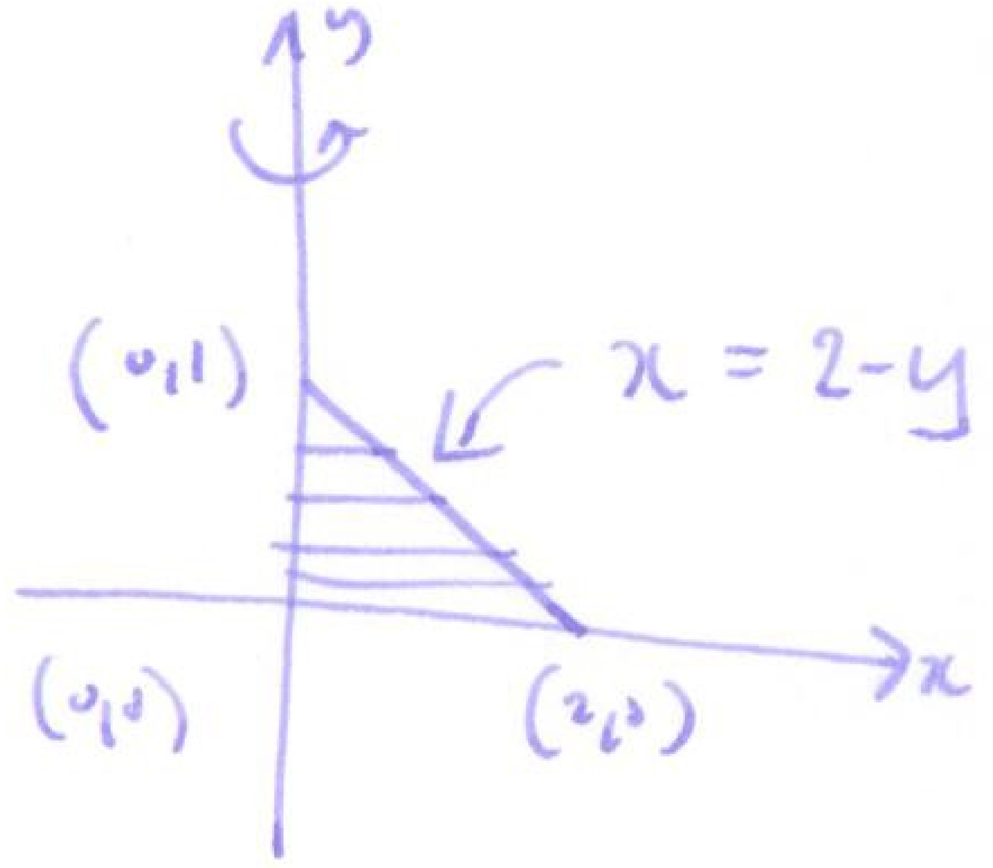
discs : $\int_0^{\frac{\pi}{2}} \pi \left(2\cos(x)\right)^2 dx$

(5) Use shells to write down an integral for the volume of the cone formed by rotating the triangle with vertices $(0,0)$, $(0,1)$ and $(2,0)$ about the y-axis. DO NOT EVALUATE THIS INTEGRAL.



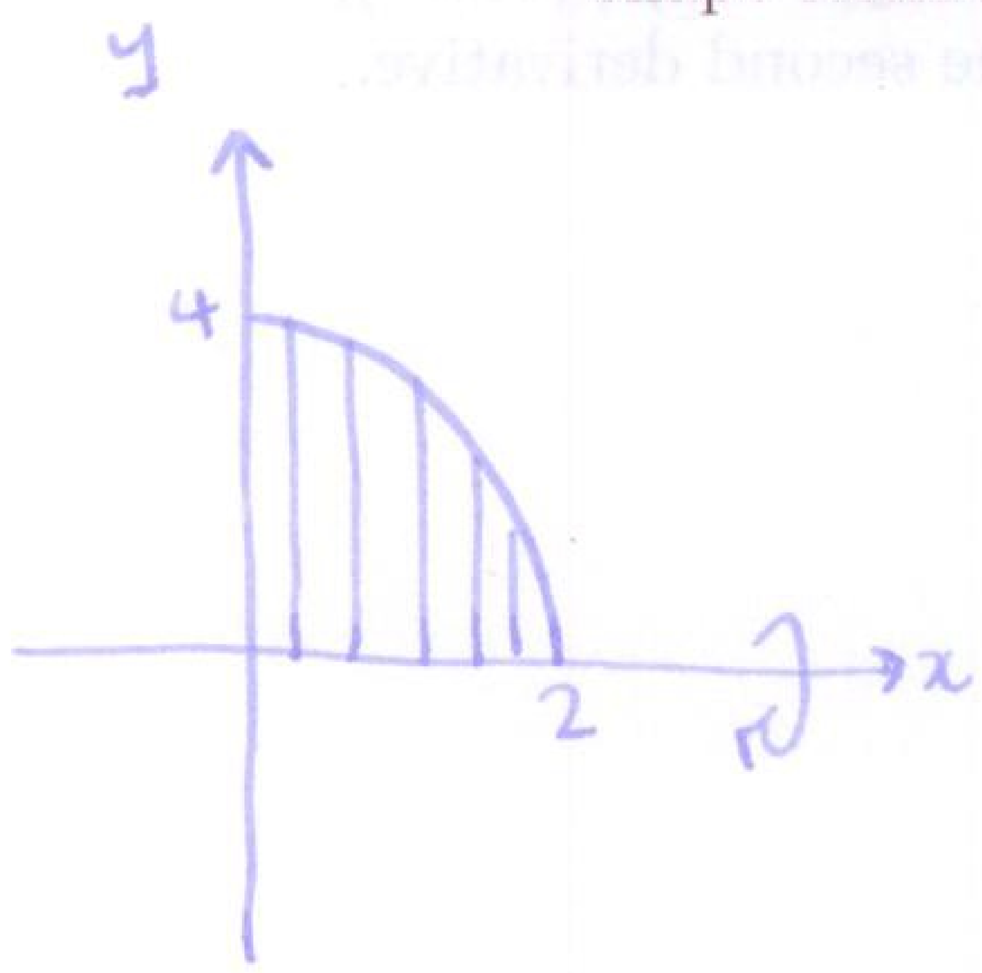
shells $\displaystyle\int_0^2 2\pi x\left(1-\frac{x}{2}\right)dx$

discs



$\displaystyle\int_0^1 \pi(2-y)^2\,dy$

(6) Consider the subset of the plane bounded by $y = 4 - x^2$ in the first quadrant (i.e. $x \geq 0$ and $y \geq 0$). Find the volume of revolution of the 3-dimensional shape formed by rotating this region around the x-axis.

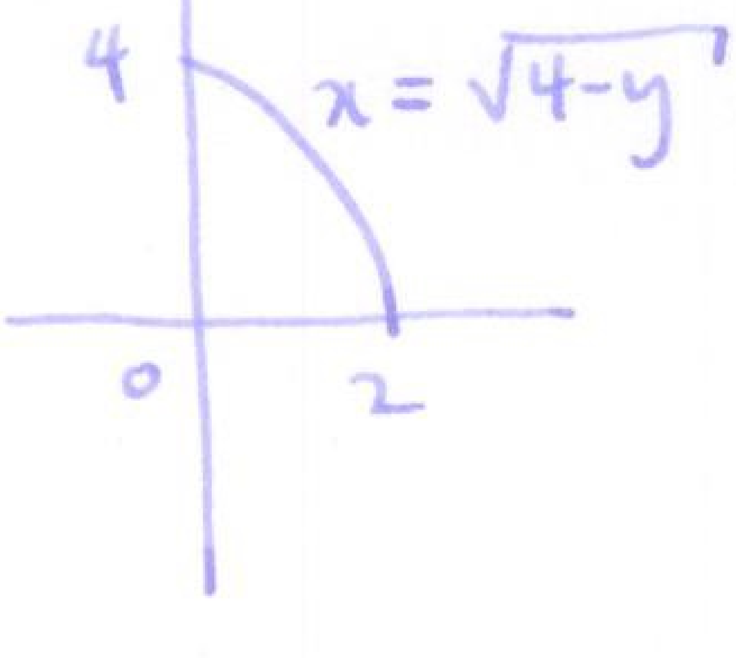


discs : $\int_0^2 \pi (4-x^2)^2 \, dx$

$= \pi \int_0^2 16 - 8x^2 + x^4 \, dx$

$= \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2 = \pi \left(32 - \frac{64}{3} + \frac{32}{5} \right) =$

shells



$\int_0^4 \pi y \sqrt{4-y} \, dy$

(7) How many trapezoids to you need to use to estimate $\int_0^3 \cos(x^2)dx$ to 2 decimal places?

Recall: the error bound for the trapezoid method is $K_2(b-a)^3/(12N^2)$, where K_2 is an upper bound on the absolute value of the second derivative.

$f(x) = \cos(x^2)$

$f'(x) = -\sin(x^2) \cdot 2x$

$f''(x) = -\cos(x^2) \cdot 4x^2 - 2\sin(x^2)$

$|f''(x)| \leq 1 \cdot 4 \cdot 9 + 2 \cdot 1 = 38$

on $[0,3]$

$$\frac{38 \cdot 3^3}{12 N^2} < \frac{10^{-2}}{2}$$

$$N^2 > \frac{38 \cdot 3^3}{12} \cdot 2 \cdot 100$$

(8) Find $\int 2xe^{-3x}\,dx$.

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = 2x \qquad u' = 2$

$v' = e^{-3x} \qquad v = -\frac{1}{3}e^{-3x}$

$$\int 2xe^{-3x}\,dx = -\frac{2}{3}xe^{-3x} + \int \frac{2}{3}e^{-3x}\,dx$$

$$= -\frac{2}{3}xe^{-3x} - \frac{2}{9}e^{-3x} + c$$

(9) Find $\int \sqrt{x}\ln(2x)\ dx$.

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = \ln(2x)$ $\quad u' = \frac{1}{2x}\cdot 2 = \frac{1}{x}$

$v' = x^{1/2}$ $\quad v = \frac{2}{3}x^{3/2}$

$$\int \sqrt{x}\ln(2x)\,dx = \frac{2}{3}x^{3/2}\ln(2x) - \int \frac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln(2x) - \int \frac{2}{3}x^{1/2}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln(2x) - \frac{4}{9}x^{3/2} + c$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(10) Find $\int e^{-x}\sin(2x)\,dx$.

$u = e^{-x}$ $\quad u' = -e^{-x}$

$v' = \sin(2x)$ $\quad v = -\frac{1}{2}\cos(2x)$

$$\int e^{-x}\sin(2x)\,dx = -\frac{1}{2}e^{-x}\cos(2x) - \frac{1}{2}\int e^{-x}\cos(2x)\,dx$$

$u = e^{-x}$ $\quad u' = -e^{-x}$

$v' = \cos(2x)$ $\quad v = \frac{1}{2}\sin(2x)$

$$= -\frac{1}{2}e^{-x}\cos(2x) - \frac{1}{4}e^{-x}\sin(2x) - \frac{1}{4}\int e^{-x}\sin(2x)$$

$$\left(1+\frac{1}{4}\right)\int e^{-x}\sin(2x)\,dx = -\frac{1}{2}e^{-x}\cos(2x) - \frac{1}{4}e^{-x}\sin(2x).$$

$$\int e^{-x}\sin(2x)\,dx = -\frac{2}{5}e^{-x}\cos(2x) - \frac{1}{5}e^{-x}\sin(2x) + C.$$