Math 232 Calculus 2 Fall 12 Final b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no phones.
- You may use a 3×5 inch index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Final	
Overall	

(1) (a) Find $\int \frac{x}{x+1}\,dx$.

$$\begin{array}{r} 1 \\ x+1 \,\overline{)\, x} \\ \underline{x+1} \\ -1 \end{array} \qquad \frac{x}{x+1} = 1 - \frac{1}{x+1}$$

$$\int 1 - \frac{1}{x+1}\,dx = x - \ln|x+1| + c$$

(b) Find $\int \frac{1}{x^2-4}\,dx$.

$$\frac{1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$$

$$= \frac{A(x-2) + B(x+2)}{x^2-4}$$

$x=2: \quad 1 = 4B$

$x=-2: \quad 1 = -4A$

$$\int \frac{-1/4}{x+2} + \frac{1/4}{x-2}\,dx = -\frac{1}{4}\ln|x+2| + \frac{1}{4}\ln|x-2| + c$$

(2) Find $\int \frac{1}{4+x^2}\,dx.$ $= \frac{1}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}.$

$= \frac{1}{4}\int \frac{1}{1+u^2}\cdot 2\,du = \frac{1}{2}\tan^{-1}(u) + C = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

(3) Find $\int \cos^3(x)\, dx$. $= \int (1-\sin^2 x)\cos(x)\, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$.

$= \int (1-u^2)\cos(x) \cdot \frac{1}{\cos(x)}\, du = u - \frac{1}{3}u^3 + c$

$= \sin x - \frac{1}{3}\sin^3 x + c$

(4) Find $\int_0^\infty xe^{-2x}\,dx$.

$\int uv'\,dx = uv - \int u'v\,dx$

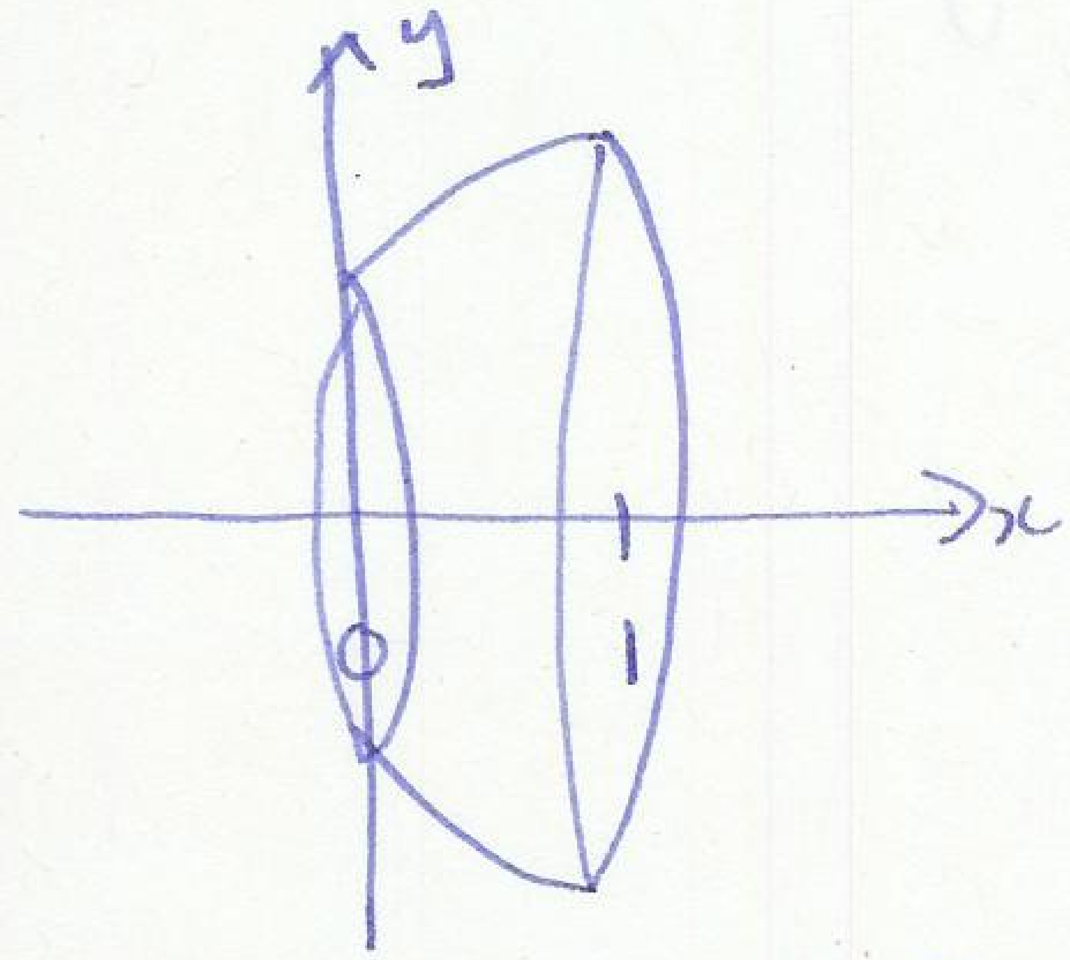
$u = x \qquad u' = 1$

$v' = e^{-2x} \qquad v = -\frac{1}{2}e^{-2x}$

$= \lim_{R\to\infty} \left[-\frac{1}{2}xe^{-2x}\right]_0^R + \frac{1}{2}\int_0^R e^{-2x}\,dx$

$= \lim_{R\to\infty} \underbrace{-\frac{1}{2}Re^{-2R}}_{\to 0} + \frac{1}{2}\left[-\frac{1}{2}e^{-2x}\right]_0^R = \lim_{R\to\infty} \underbrace{-\frac{1}{2}e^{-2R}}_{\to 0} + \frac{1}{4} = \frac{1}{4}$

(5) Find the volume of revolution of the curve $y = 2 + \sqrt{x}$ above the interval $[0, 1]$, rotated about the x-axis.



$$\pi \int_0^1 (2+\sqrt{x})^2 \, dx$$

$$= \pi \int_0^1 4 + 4\sqrt{x} + x \, dx$$

$$= \pi \left[4x + \frac{8}{3} x^{3/2} + \frac{1}{2} x^2 \right]_0^1 = \pi \left(4 + \frac{8}{3} + \frac{1}{2} \right)$$

$$= \pi \cdot \frac{43}{6}$$

(6) Does the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ converge? Justify your answer.

Geometric series : converges to $\frac{1}{1+1/2} = \frac{2}{3}$

(7) Does the series $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^3+3}$ converge? Justify your answer.

comparison test: $\frac{\sqrt{n}}{n^3+3} \leq \frac{n}{n^3} = \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series $p>1$) $\Rightarrow \sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^3+3}$ converges.

(8) Find a power series for the function $f(x) = e^{-x} - e^{x}$ centered at $c = 0$. What is the radius of convergence of the power series?

$f(x) = e^{-x} - e^{+x}$ $\qquad f(0) = 0$

$f'(x) = -e^{-x} \cancel{+} e^{+x}$ $\qquad f'(0) = -2$

$f''(x) = e^{-x} - e^{x}$ $\qquad f''(0) = 0$

$f^{(n)}(x) = (-1)^n e^{-x} - e^{-x}$ $\qquad f^{(n)}(x) = (-1)^n - 1$

power series $\quad \sum_{n=0}^{\infty} \frac{(-1)^n - 1}{n!} x^n = -2x - \frac{2x^3}{3!} - \frac{2x^5}{5!} - \dots$

ratio test: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{-2x^{2n+1}}{(2n+1)!} \frac{(2n-1)!}{-2x^{2n-1}} \right|$

$= \lim_{n\to\infty} \left| \frac{x^2}{2n(2n+1)} \right| = 0 < 1$ so radius of convergence is $R = \infty$.

(9) Find the angle between the vectors $\mathbf{v} = \langle -2, 1, 3\rangle$ and $\mathbf{w} = \langle 3, -1, 2\rangle$. Find the equation of the plane through the origin containing the two vectors.

$\underline{v} \cdot \underline{w} = \|\underline{v}\| \|\underline{w}\| \cos\theta$ $\qquad -6-1+6 = \sqrt{4+1+9}\sqrt{9+1+4}\cos\theta$

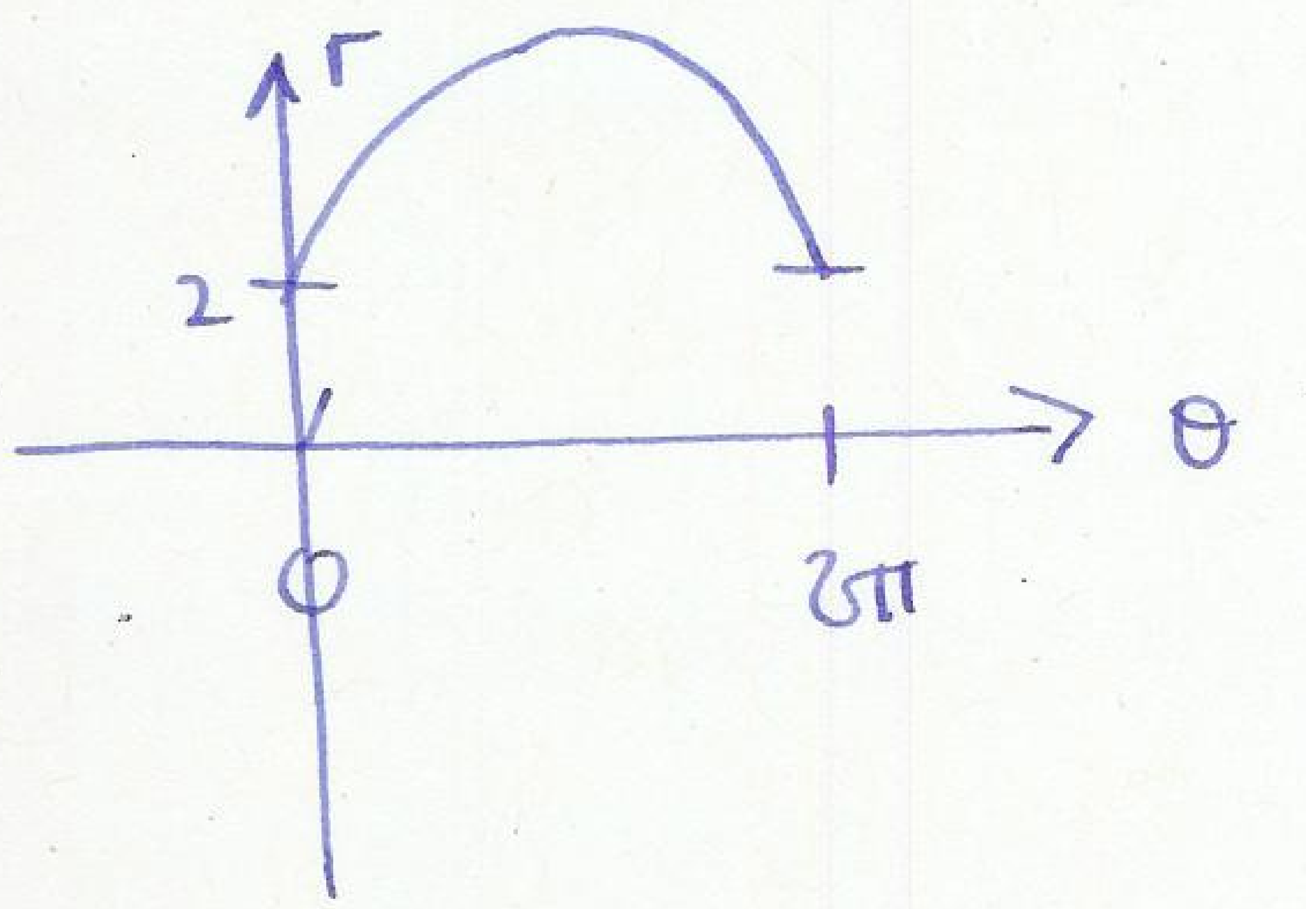
$\cos\theta = \frac{-1}{14}.$

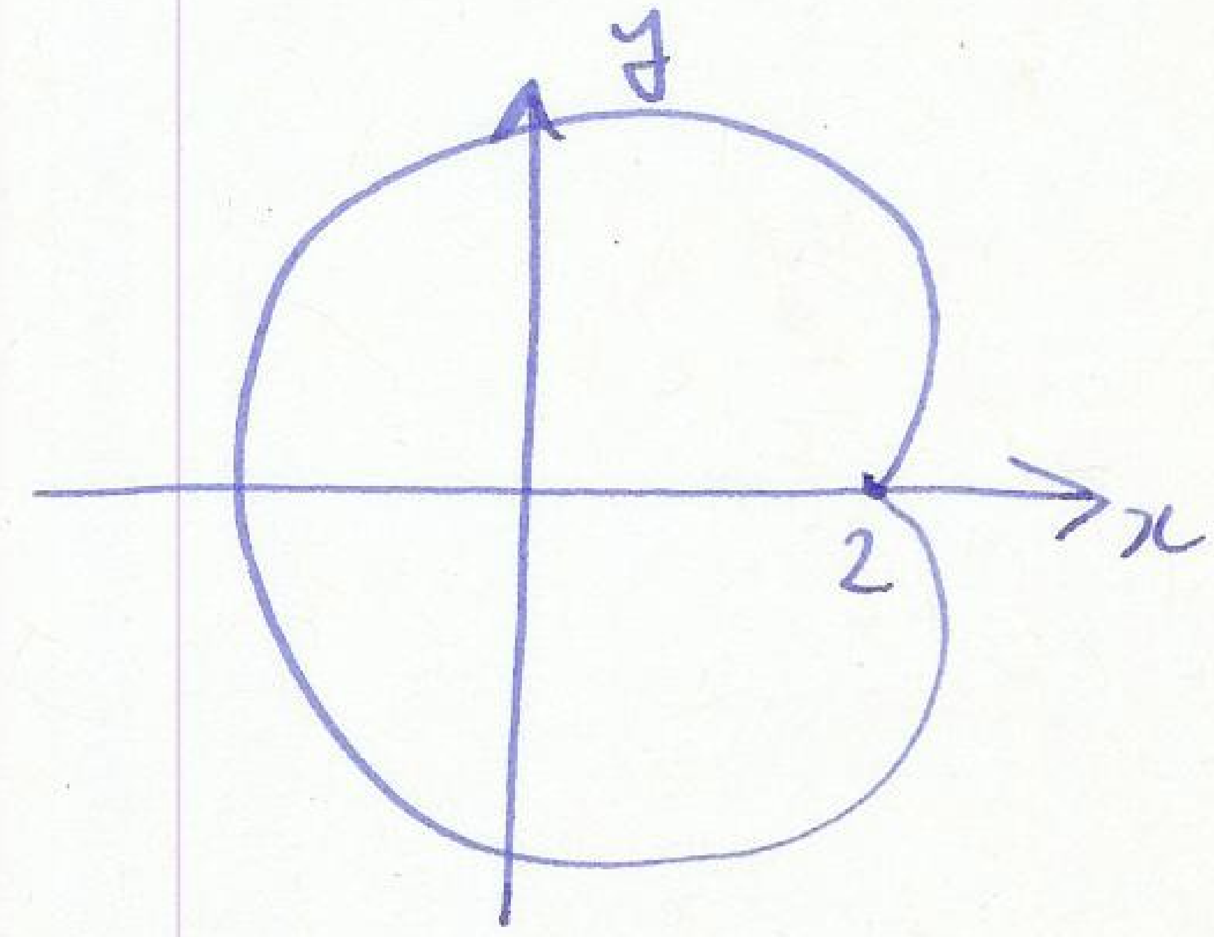
$$\underline{v \times w} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2 & 1 & 3 \\ 3 & -1 & 2 \end{vmatrix} = \underline{i}\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} - \underline{j}\begin{vmatrix} -2 & 3 \\ 3 & 2 \end{vmatrix} + \underline{k}\begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix}$$

$$= \langle 5, 13, -1 \rangle$$

$5x + 13y - z = 0$

(10) Sketch the polar curve $r = 2 + \theta(2\pi - \theta)$, for $0 \leq \theta \leq 2\pi$. Find the slope $\dfrac{dy}{dx}$ at $\theta = \pi/2$.





$$x = r\cos\theta = (2 + 2\pi\theta - \theta^2)\cos\theta$$

$$y = r\sin\theta = (2 + 2\pi\theta - \theta^2)\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{(2\pi - 2\theta)\sin\theta + (2 + 2\pi\theta - \theta^2)\cos\theta}{(2\pi - 2\theta)\cos\theta + (2 + 2\pi\theta - \theta^2)(-\sin\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{2}} = \frac{\pi + 0}{0 + \left(2 + \pi - \frac{\pi^2}{4}\right)} = \frac{\pi}{2 + \pi - \frac{\pi^2}{4}}.$$