7.4.1 Equidistant set of two points in $\mathbb{R}^3$ is a plane.

(1)

$P \cdot (p_1, p_2, p_3)$

$X \cdot$ ~~(x_1, x_2, x_3)~~ (x_1, x_2, x_3)

$\cdot Q \ (q_1, q_2, q_3)$

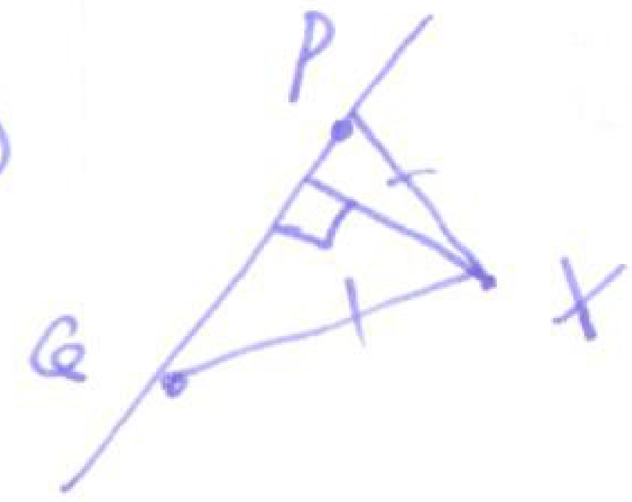
$$d(x,P)^2 = (x_1-p_1)^2 + (x_2-p_2)^2 + (x_3-p_3)^2$$

$$d(x,Q) = (x_1-q_1)^2 + (x_2-q_2^2) + (x_3-p_3)^2.$$

$$d(x,P)^2 = d(x,Q)^2 \Rightarrow -2x_1p_1 + p_1^2 - 2x_2p_2 + p_2^2 - 2x_3p_3 + p_3^2 = -2x_1q_1 + q_1^2 + \cdots$$

linear in $x_1\, x_2\, x_3$ equation of plane.

(2)



$d(x,P) = d(Q,X) \Rightarrow$ triangle isoceles $\Rightarrow$ altitude bisects QP

$\Rightarrow$ X lies on bisecting plane to PQ.

7.4.2

same Euclidean distance $\Leftrightarrow$ same angular distance.

$\Rightarrow$ same distance on $S^2 \Rightarrow$ same distance in $\mathbb{R}^3$.

so equidistant set $\subset$ plane

origin is equidistant from P, Q $\Rightarrow$ plane goes through origin.

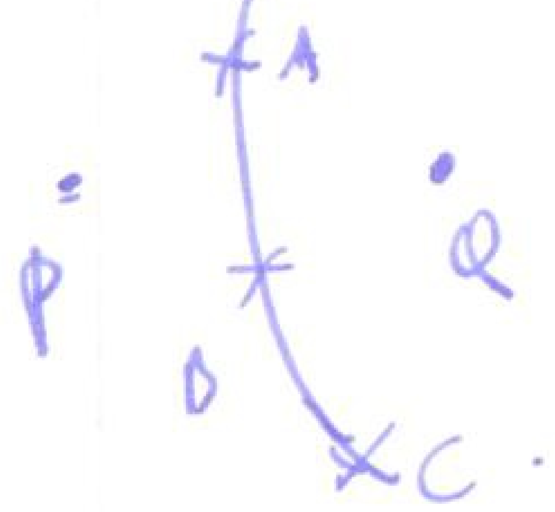
so equidistant set is a great circle.

7.4.3

spose P, Q have same distances to A, B, C not colinear.

A, B, C ~~...~~ lie on equidistant sets. to P, Q

~~$E_{AB} \cap E_{BC}$~~ $= 2$ points.



$\Rightarrow$ A, B C lie on a line.

7.4.4 isometry preserves distance so give $d(P,A)$ $d(P,B)$ $d(P,C)$ know images. not on a line preserved by isometry. so $f(P)$ unique.

7.4.5 same as 3.7.

§7.5 The rotation group of the sphere

$\mathrm{Isom}^+(S^2) \subset \mathrm{Isom}(S^2)$

$\uparrow$ orientation preserving isometries $\Leftrightarrow$ even number of reflections.

product of two reflections is a rotation.

product of two rotations ?

<u>claim</u>: product of two rotations is a rotation.

rotation of θ about P $\Leftrightarrow$ product of reflections in two lines L and M through P which meet at angle $\theta/2$.

choose M to go through P and Q.

rotation of ϕ about Q $\Leftrightarrow$ product of reflections in two lines M and N through Q meeting at angle $\phi/2$.

rotation of θ about P $\Leftrightarrow =$ $r_M r_L$

rotation of ϕ about Q $=$ $r_N r_M$

product is $\quad r_N r_M r_M r_L = r_N r_L =$ rotation about R intersection of N and L.