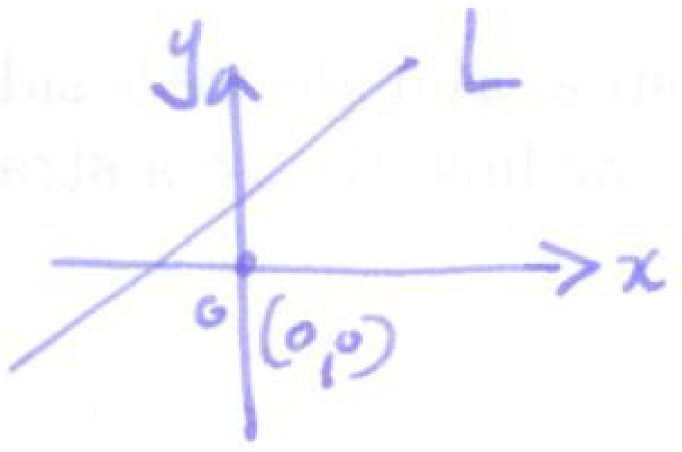
where does this formula come from?

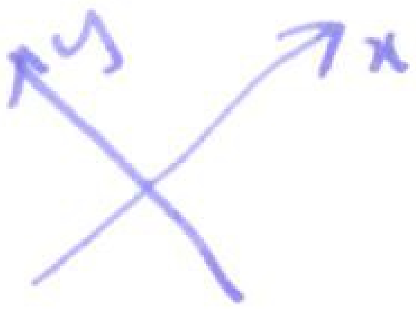
recall: $\tan(\theta_1 - \theta_2) = \dfrac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$ (choose $\tan\theta_1 = t_1$, $\tan\theta_2 = t_2$)

§3.6 Isometries

Euclidean plane:  vs. $\mathbb{R}^2$ 

no special points or lines

special point (0,0)
special lines
x-axis
y-axis

Q: why didn't we draw the axes like this  ?

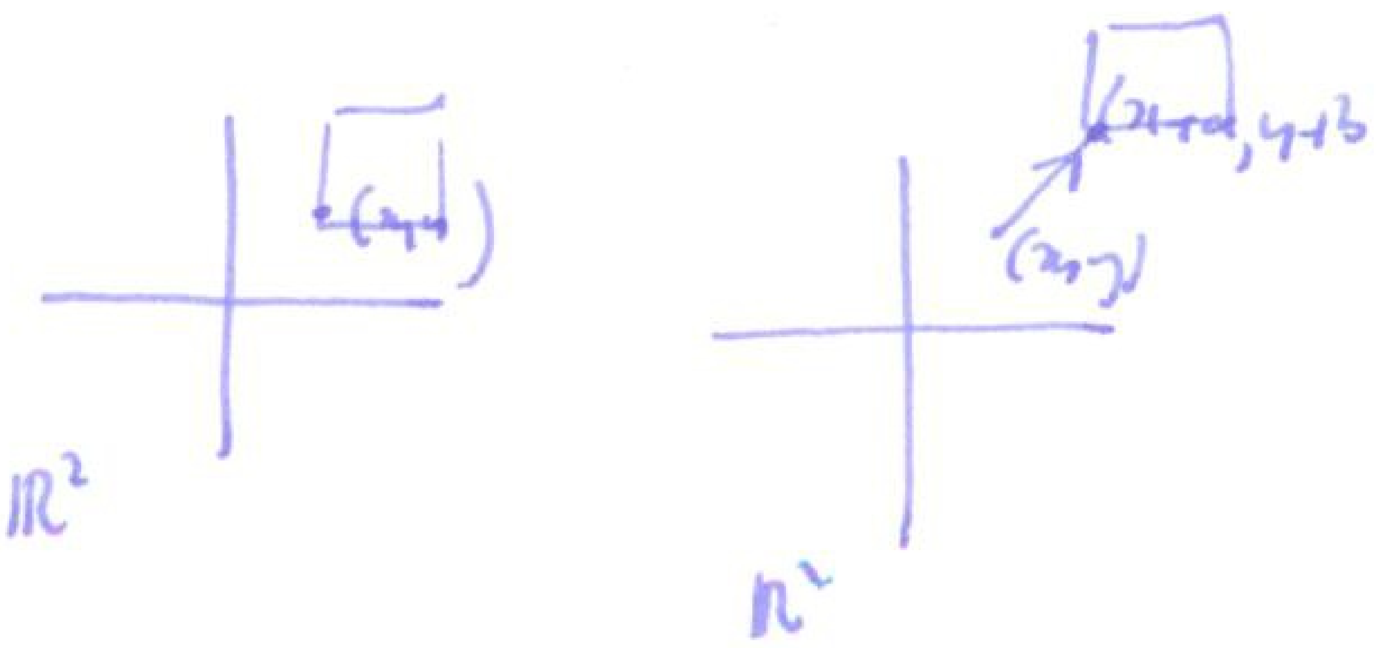
Defn: a transformation is a function $f: \mathbb{R}^2 \to \mathbb{R}^2$

an isometry is a transformation which preserves distances,

i.e. for any points P, Q, $|f(P)f(Q)| = |PQ|$

also called rigid motions.

Translations $t_{a,b} : (x,y) \mapsto (x+a, y+b)$



check preserves distances

$P = (x_1, y_1)$ $Q = (x_2, y_2)$

$t_{a,b}(P) = (x_1+a, y_1+b)$ $t_{a,b}(Q) = (x_2+a, y_2+b)$

$$|t_{a,b}(P)\, t_{a,b}(Q)| = \sqrt{(x_2+a-x_1-a)^2 + (y_2+b-y_1-b)^2}$$

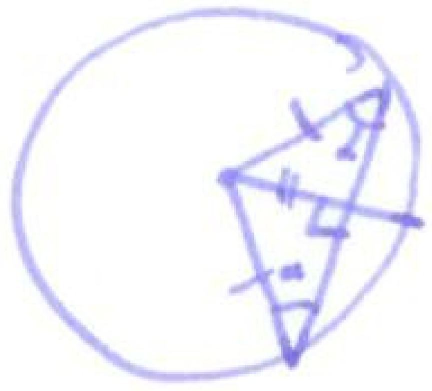
$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = |PQ| \text{ as required } \square.$$

observation: we can take any point to any other point by a translation.

Chords in circles

chord.

useful facts:

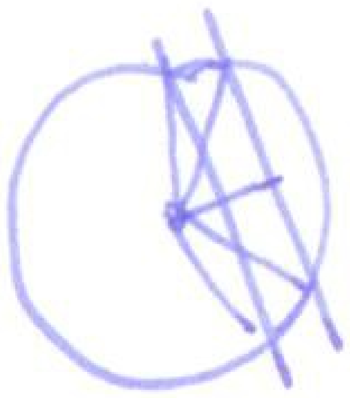
- 

  the perpendicular radius bisects the chord.

Proof: big triangle iscoceles => smaller triangles congruent □.

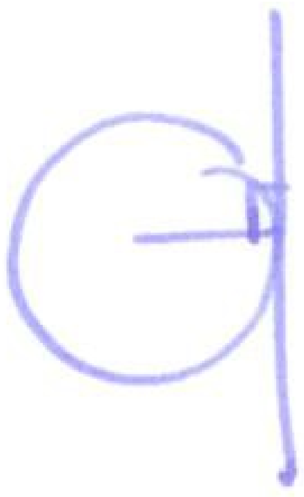
- 

  chords have same length => same distance from the centre

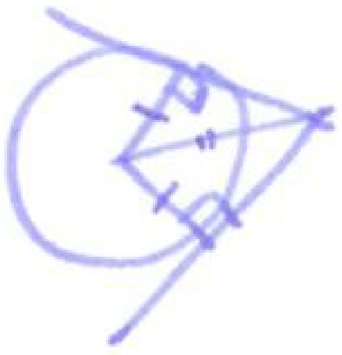
- 

  parallel chords => arcs have same length.

- tangents are perpendicular to radius:

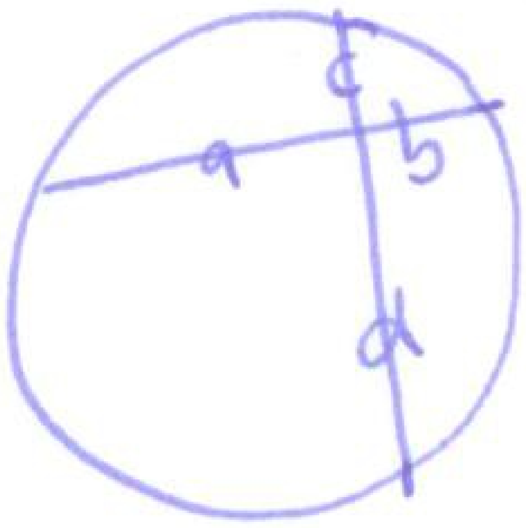
  

- two tangents from the same point are congruent:

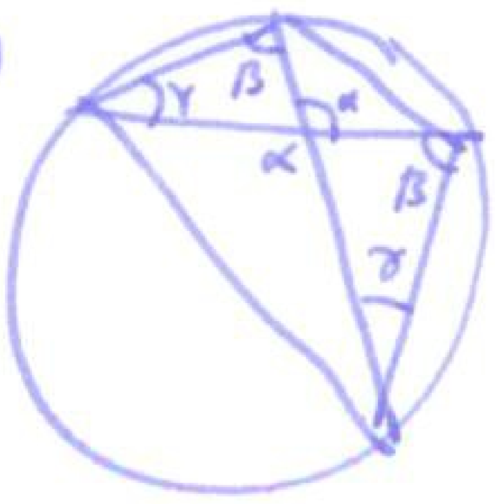
  (common side and radius same, 3rd side determined by Pythagoras!)

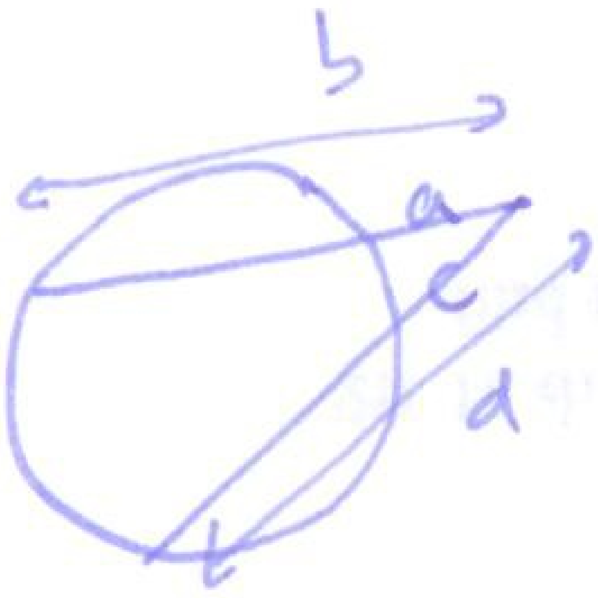
- product results:

  $ab = cd$

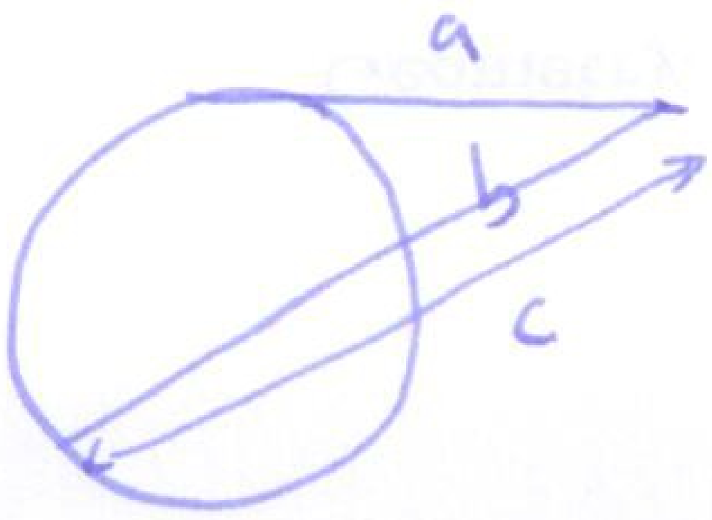
  similar triangles (angles subtended are constant)

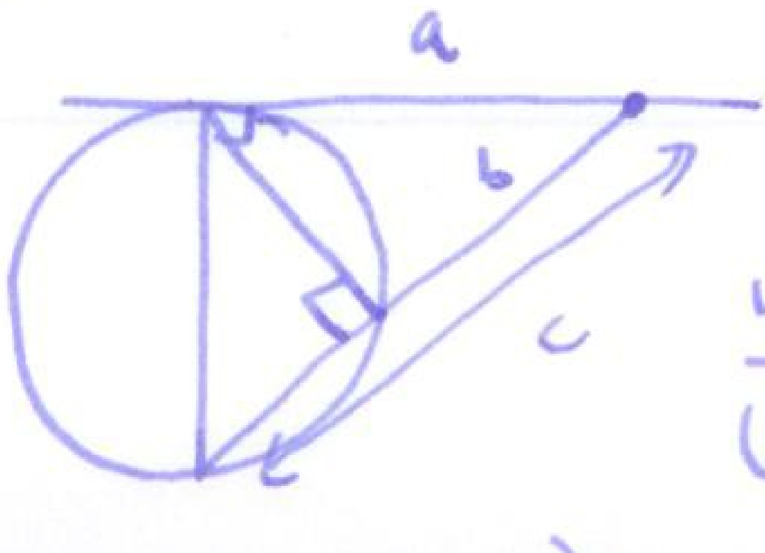


$ab = cd$

similar triangles.



$a^2 = bc$



$\frac{\text{hyp}}{\text{long}} = \frac{c}{a} = \frac{a}{b}$

(then use previous case).