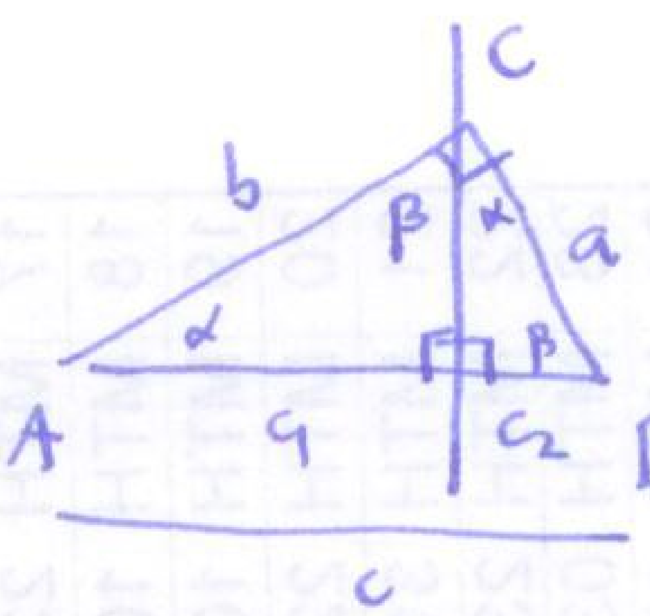
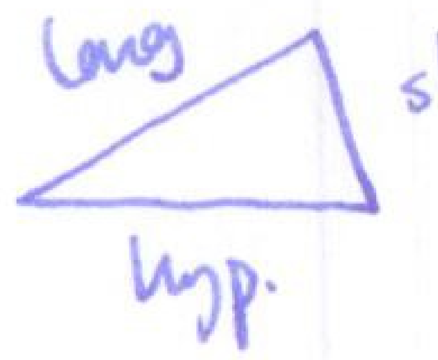
§2.8 Euclid's other proof of pythagoras.

Let ABC be a right angled triangle:



subdivide by a perpendicular through the vertex with the right angle. — get two right angled triangles.

claim: all three triangles are similar (angles $\alpha, \beta, \pi/2$).



$$\frac{\text{long}}{\text{hyp}} = \frac{b}{c} = \frac{c_1}{b} \Rightarrow b^2 = c c_1$$

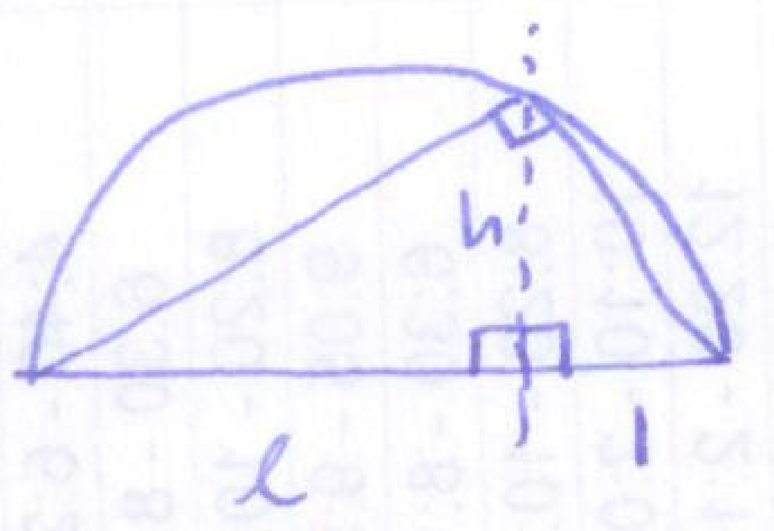
$$\frac{\text{short}}{\text{hyp}} = \frac{a}{c} = \frac{c_2}{a} \Rightarrow a^2 = c c_2$$

so $a^2 + b^2 = c c_1 + c c_2 = c(c_1 + c_2) = c^2$ □

moral: similar triangles are useful.

Constructing square roots:

given a line segment of length l, construct a line segment of length $\sqrt{l}$:



claim length $h = \sqrt{l}$. proof: $\frac{\text{long}}{\text{short}} = \frac{l}{h} = \frac{h}{1} \Rightarrow l = h^2$

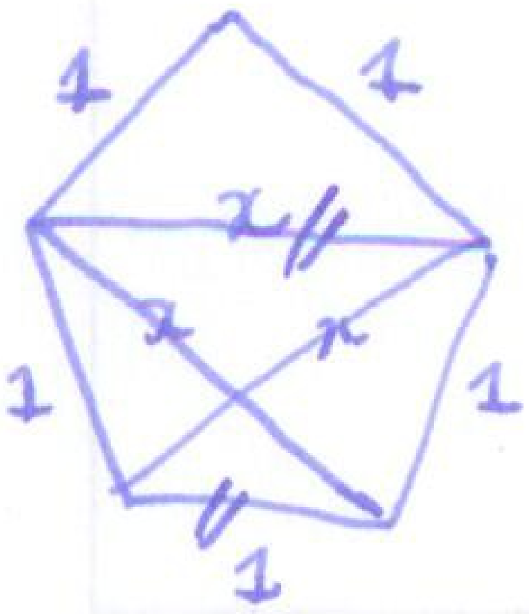
$\sqrt{l} = h$. □.

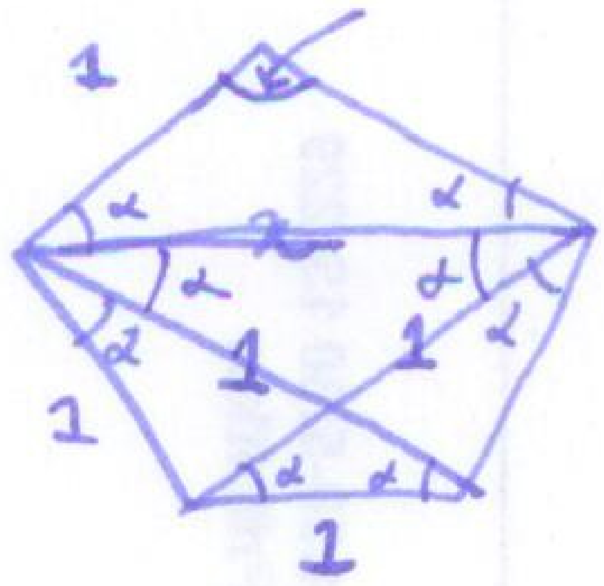
important fact: constructible numbers are precisely those that use $+, -, \times, \div, \sqrt{\cdot}$

e.g. $\sqrt{1 + \frac{1}{1+\sqrt{2}}}$ (pf 1800's using Galois theory).

<u>claim</u> : the regular pentagon is constructible

interior angles sum to $\frac{3\pi}{5}$





so $3\alpha = \frac{3\pi}{5}$ $\alpha = \frac{\pi}{5}$

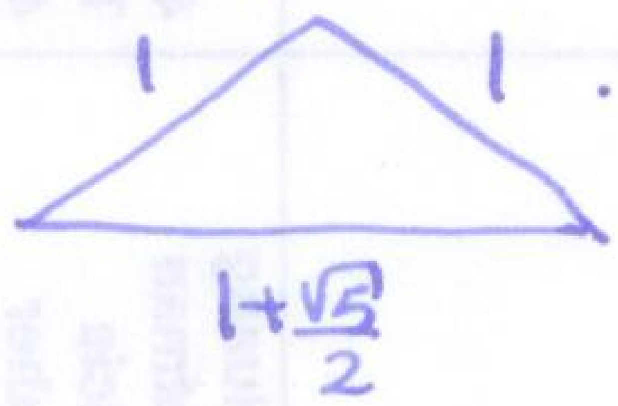
$$\frac{\text{long}}{\text{short}} = \frac{x}{1} = \frac{1}{x-1}$$

$$x^2 - x = 1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$$

exercise: construct a regular pentagon!

(need to construct : ).

<u>Read 2.9.</u>

<u>§3 Coordinates</u>

"arithmetization of geometry" (originally from Fermat / Descartes ~1630)

- describe Euclidean geometry using algebra
- makes it possible to <u>prove</u> that something <u>not</u> constructible
- express "motion" geometrically.

<u>§3.1 Number line and number plane</u>

~~number~~ $\mathbb{R}$ = set of real numbers, obtained by filling the gaps in $\mathbb{Q}$ with all of the irrational numbers ($\sqrt{2}, \pi$, etc...) $\mathbb{R}$ has an <u>order</u> $\frac{x \quad y}{x<y}$

(number line)

can use $\mathbb{R}$ as a model for a geometric line.

<u>First step</u> : build a model for the Euclidean plane: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

i.e. all ordered pairs of numbers ~~(x,y)~~ or (x,y) with $x \in \mathbb{R}$, $y \in \mathbb{R}$.