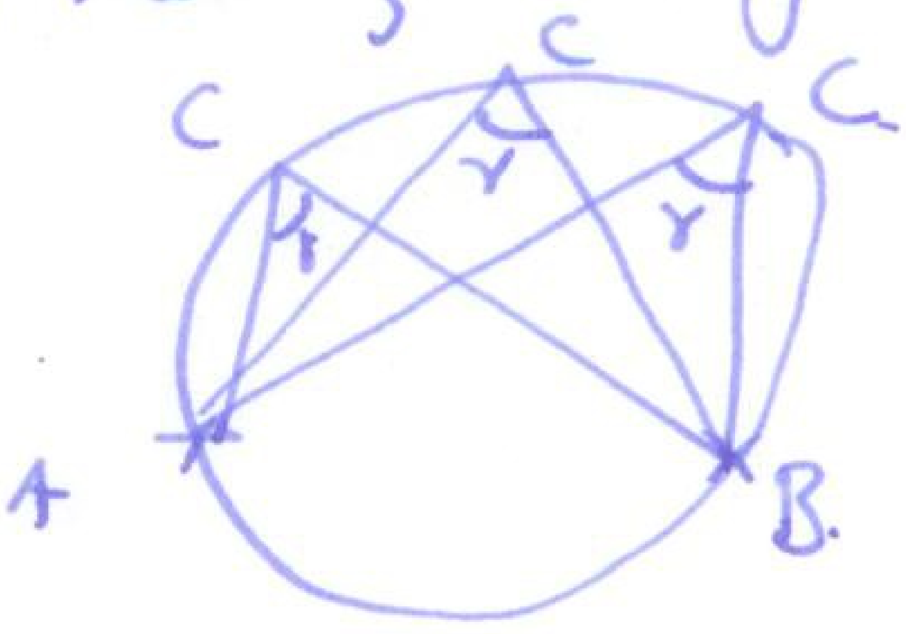
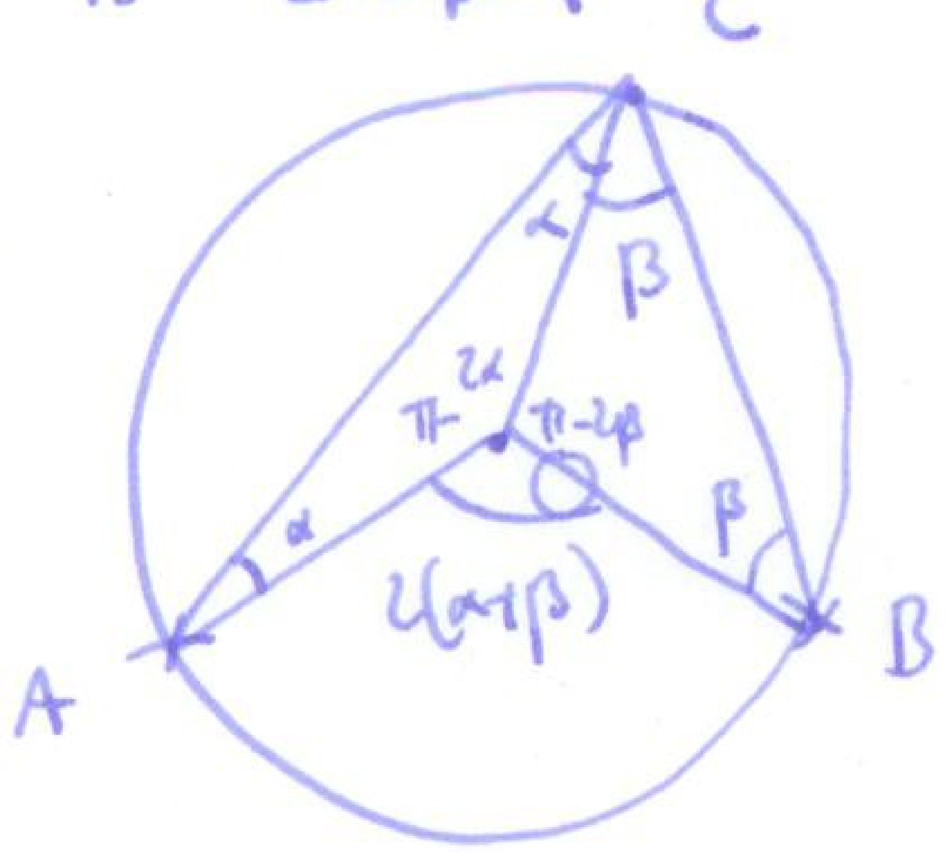
§2.7 Angles in a circle

Thm (Invariance of angles in a circle) If A, B are two points on a circle, then for all points C on one of the arcs connecting them, the angle ACB is constant.



Proof



Let O be the center of the circle. Then AOC and BCO are isoceles triangles.

so $\angle ACO = \alpha = \angle CAO$, so $\angle AOC = \pi - 2\alpha$

$\angle CBO = \beta = \angle OCB$, so $\angle BOC = \pi - 2\beta$

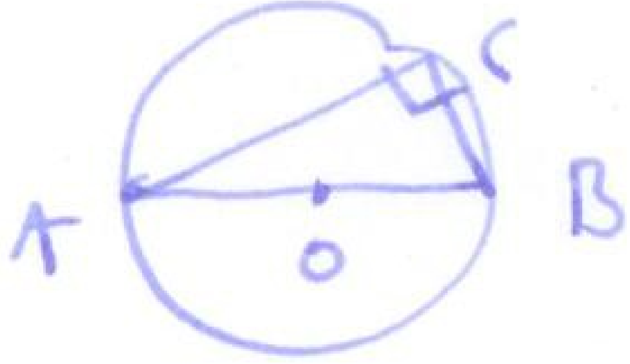
so $\angle AOB = 2\pi - (\pi - 2\alpha) - (\pi - 2\beta) = 2\pi - \pi + 2\alpha - \pi + 2\beta = 2(\alpha+\beta)$

angle $\angle AOB$ is constant so $2(\alpha+\beta)$ is constant so $\alpha+\beta$ is constant so angle $\angle ACB$ is constant. $\square$.

Note α, β not constant!

Corollary angle at center is twice angle at circumference.

Special case Thm if AB lie on a diameter, then angle at C is a right angle



Observation we can now construct squares with same area as rectangles / triangles / polygons.

case: rectangle:

or bisect rectangle:...

triangle:

half area of rectangle with same base/height.

half = area of triangle.

polygon: divide into triangles:

now add up areas:

sum.