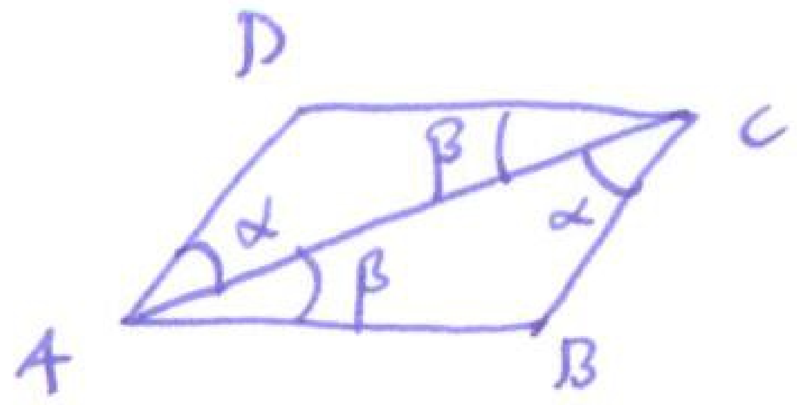
useful consequence:

<u>Def^n</u> A <u>parallelogram</u> is a shape whose sides consist of two parallel lines.

<u>Example</u> 

<u>Th^m</u> Opposite sides of a parallelogram are equal.

<u>Proof</u> 

divide parallelogram into two triangles

<u>claim</u>: the two triangles are congruent.

- common side AC
- common angles α, β (alternate interior angles for parallels)

therefore the triangles are congruent by ASA.

so $|AD| = |BC|$ and $|AB| = |CD|$. $\square$

<u>§2.3 Euclid's common notions</u>

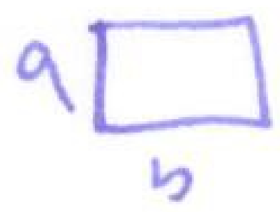
1. $a=b,\ b=c \Rightarrow a=c$
2. $a=b,\ c=d \Rightarrow a+c = b+d$
3. $a=b,\ c=d \Rightarrow a-\cancel{b}c = b-d$
4. $a=a$
5. If $A \subseteq B$ then $|A| \leq |B|$

equal means equal length or angles or equal area (Euclid's area slightly different from ours)

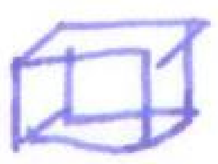
addition of lengths or areas.

(~~can't~~ Euclid never adds length to area).

↑ in algebraic notation!

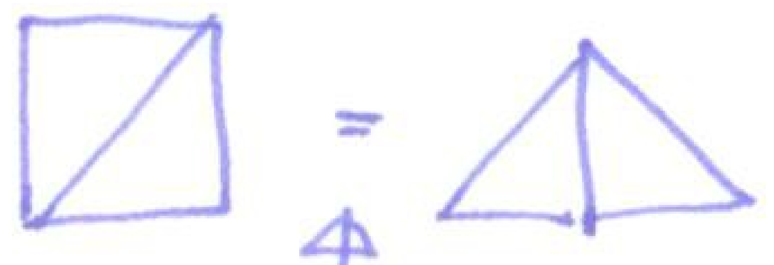
Euclids products: ab 

is area.

abc  is volume.

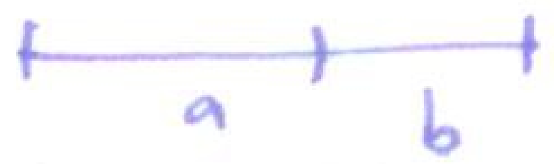
abcd ? didn't do this.

Euclid's area comparisons: cut and rearrange:

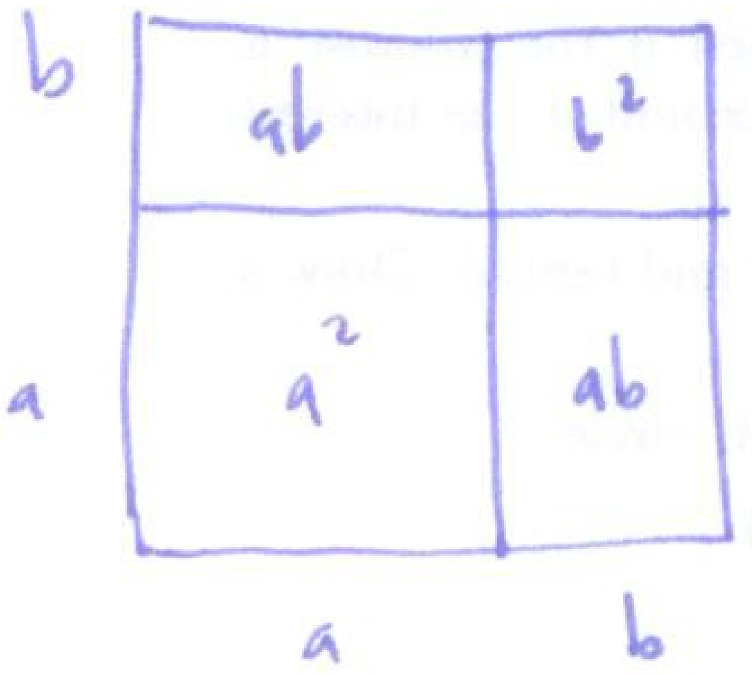


Example square of a sum

$$(a+b)^2 = a^2 + 2ab + b^2$$



"If a line is cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments"



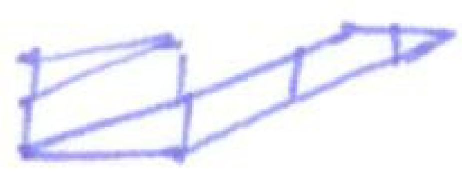
Q. can you show $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$?

§2.4 Areas of parallelograms and triangles

parallelogram:

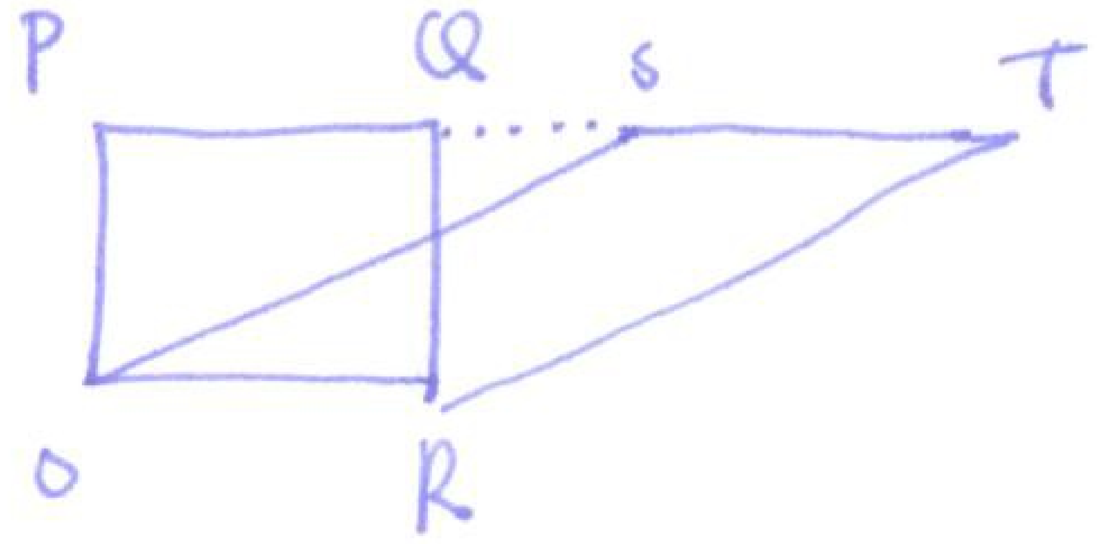
area is ab

~~for~~ shallower slopes:



may need to cut many times

better method using subtraction of area:

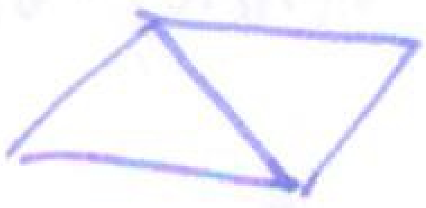


area(OSTR) = area(OPQR) + area(QTR) − area(OPS)

but area(QTR) = area(OPS) as they are congruent triangles (by SSS)

we have shown: area of parallelogram = base × height

Corollary area of triangle $= \frac{1}{2}$ base $\times$ height

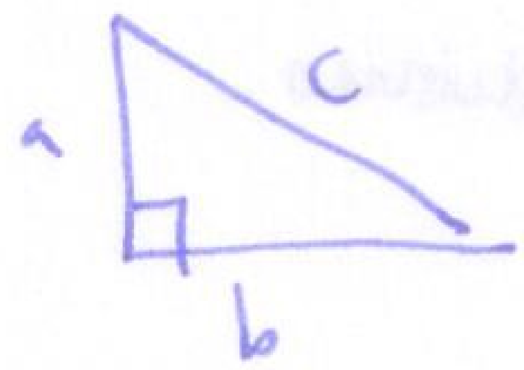
proof  two congruent triangles form a parallelogram $\square$.

Corollary for triangles of the same height, area is proportional to length of base.

Observation all of these triangles have same area!

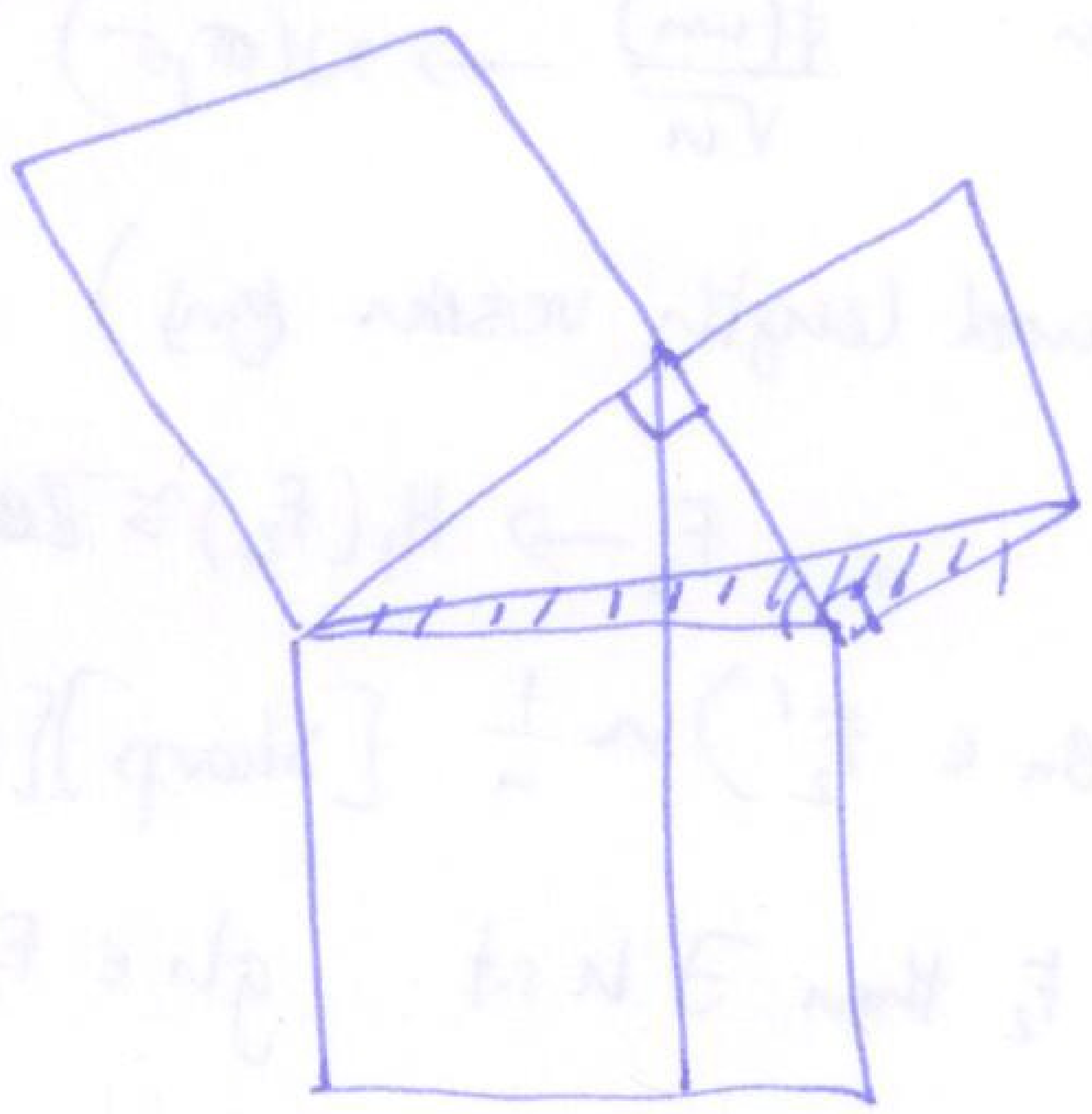
§2.5 Pythagoras' Theorem (vast number of different proofs)

Thm For any right angled triangle, the sum of the squares of the two shorter sides is equal to the square of longer side (called the hypotenuse)



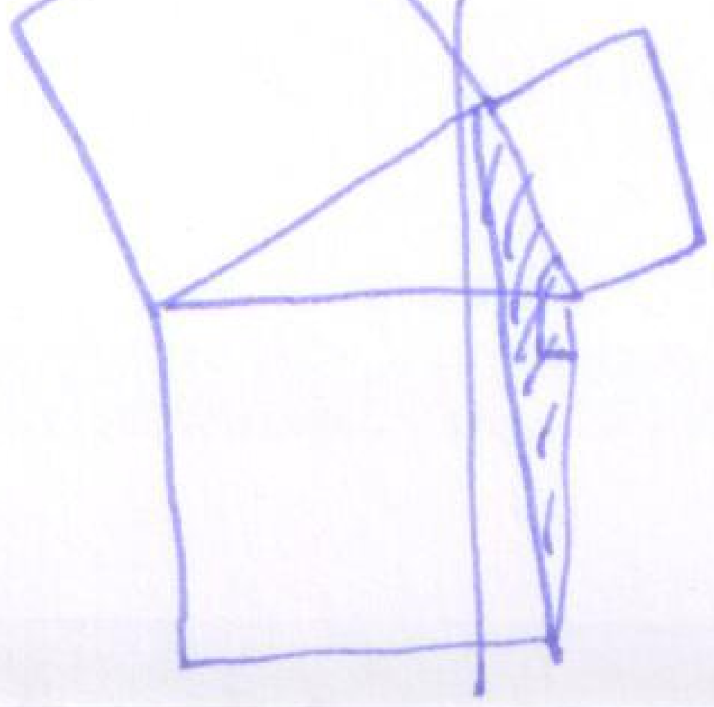
$a^2+b^2=c^2$

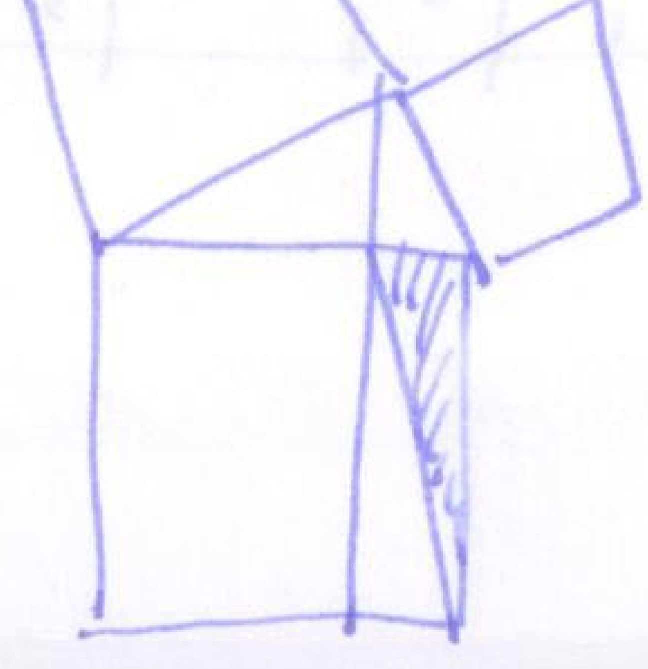
Euclid's proof:



same area as same base and height

same area as congruent triangles (SAS)



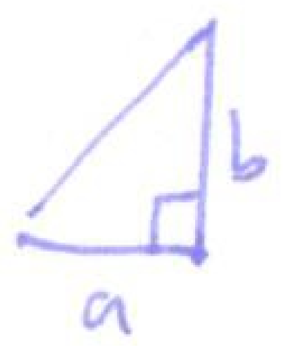


same area as same base and height.

same argument works on other side. $\square$.

Converse holds: if sides of a triangle satisfy $a^2+b^2=c^2$ then it is a right angled triangle.

<u>Proof</u> construct right angled triangle with short side lengths a and b



then length of third side is $\sqrt{a^2+b^2}=c$, but then the two triangles are congruent by SSS so both have right angles.

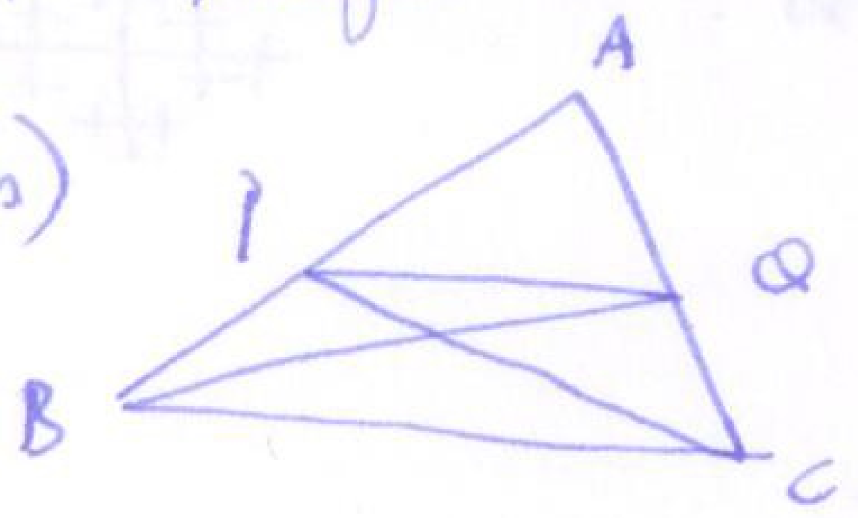
<u>observation</u>: can "square a rectangle" i.e. construct a square with the same area as a rectangle.

<u>§2.6 Proof of Thales theorem</u>

<u>Thm</u> A line drawn parallel to one side of a triangle cuts the others proportionally i.e. $\dfrac{|AP|}{|PB|}=\dfrac{|AQ|}{|QC|}$

<u>recall</u>: for triangles of the same height, area is proportional to base.

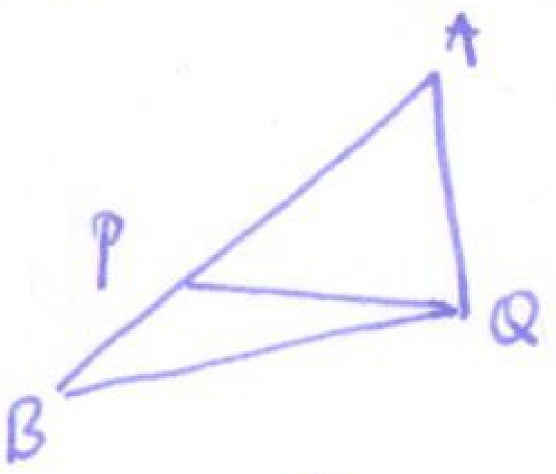
<u>Proof</u> (thales)



· PQ parallel to BC.

<u>note</u> $\triangle PQC$ $\triangle PQB$ have same base and height, so same area.

$\Rightarrow \triangle ABQ$ and ACP have same area.



~~AQB~~ APQ, BPQ } same height so areas proportional to their bases,

i.e. $\dfrac{|AP|}{|PB|}=\dfrac{\text{area } APQ}{\text{area } PBQ}$ } equal! $\Rightarrow \dfrac{|AP|}{|PB|}=\dfrac{|AQ|}{|CQ|}$ $\square$.

similarly $\dfrac{|AQ|}{|QC|}=\dfrac{\text{area } AQP}{\text{area } QCP}$