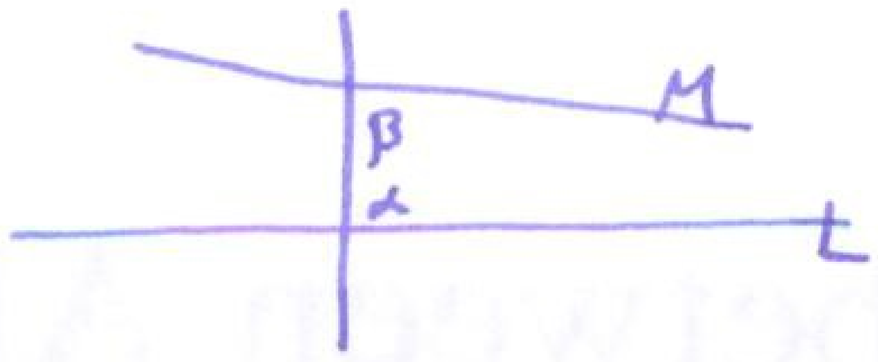
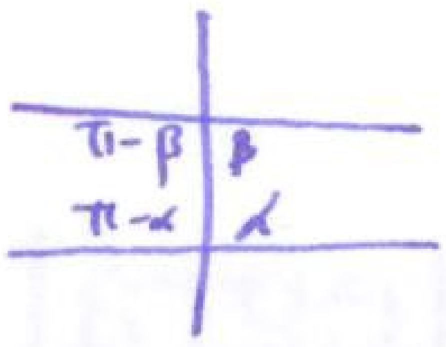
§2 Euclid's parallel postulate



Euclid's parallel axiom (#4 axiom).

"If a straight line crossing two straight lines makes the interior angles on one side less than two right angles, then the two straight lines will meet on that side".

i.e. if $\alpha+\beta<\pi$ then the two lines meet.

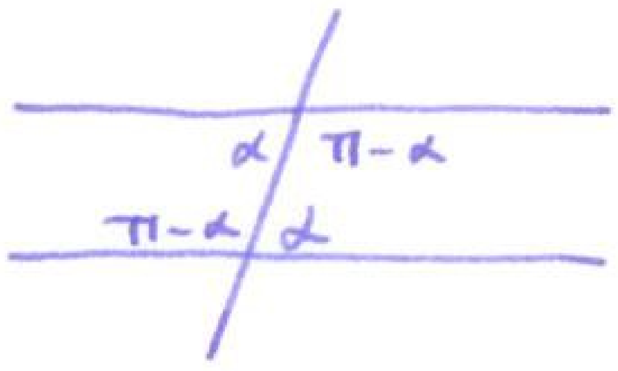


$\alpha+\beta \geq \pi$

$(\pi-\alpha)+(\pi-\beta) \geq \pi$

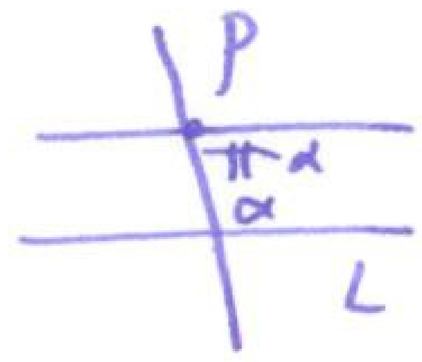
$\alpha+\beta \leq \pi$

$\Rightarrow$ if M and L do not meet on either side then $\alpha+\beta=\pi$.



"alternate interior angles" $\Leftrightarrow$ lines are parallel.

Consequences

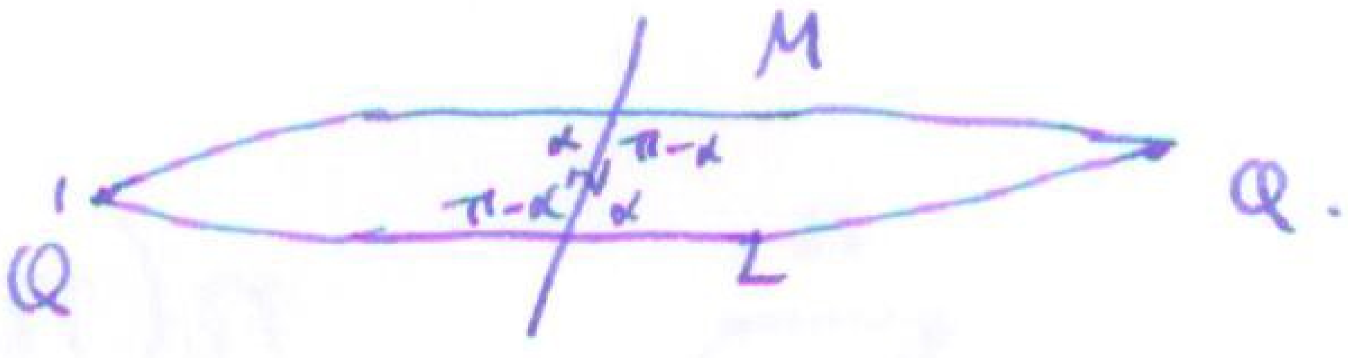


- If a parallel line to ~~ℓ~~ L exists through P, then it's unique.

  (lines have same point and angle)

- converse is true: i.e. if M meets common line with angles $\pi-\alpha$, α, then M, L parallel.

Proof (of converse)



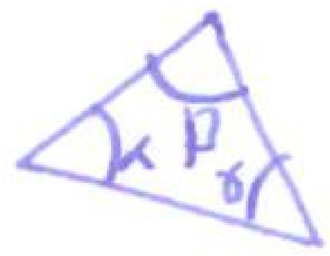
uses ASA: determines a unique triangle, and get a similar triangle on the other side. But then there are two distinct lines connecting Q and Q'. Contradicts: Euclid's assumption that there is a unique line connecting any two points.

so we've shown

Euclid's parallel axiom equivalent to modern parallel axiom:

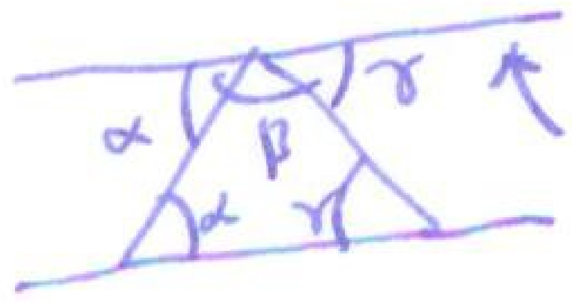
"For any line L and a point P not on L there is a unique line through P that does not meet L".

Corollary to parallel axiom: Angles in a triangle add up to π



$\alpha+\beta+\gamma=\pi$

Proof:

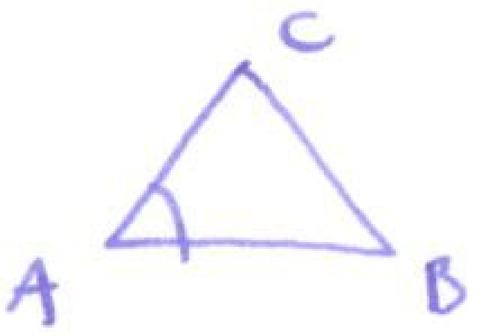


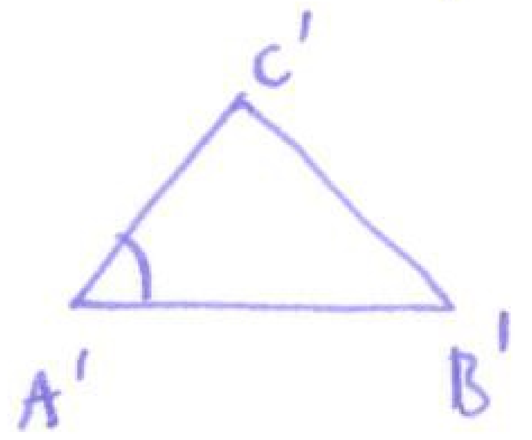
construct unique parallel

$\alpha+\beta+\gamma=\pi$. $\square$

§2.2 Congruence axioms

SAS axiom: If two triangles have two corresponding sides equal, and the angle between them is equal, then their third sides, and the two angles are equal.

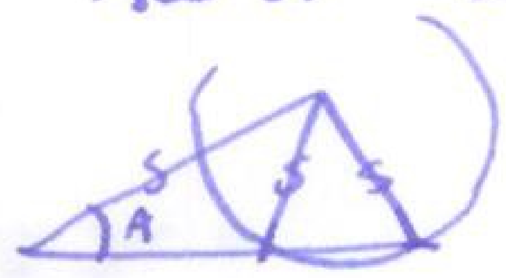




$$\left.\begin{aligned} |AB| &= |A'B'| \\ |AC| &= |A'C| \\ \angle CAB &= \angle C'A'B' \\ \angle A &= \angle A' \end{aligned}\right\} \text{ then } \begin{aligned} |BC| &= |B'C'| \\ \angle B &= \angle B' \\ \angle C &= \angle C'. \end{aligned}$$

write $\Delta ABC \cong \Delta A'B'C'$

also ASA, SSS determine triangles but not ASS or SSA

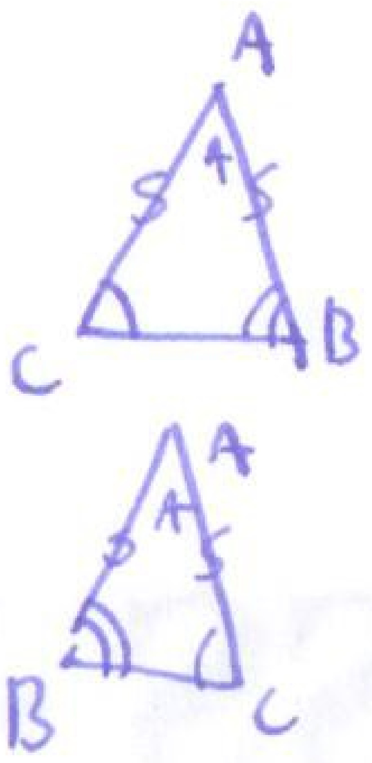


(in fact SAS $\Rightarrow$ ASA and SSS, but we ~~can~~ will just assume them)

Thm (Isoceles triangle theorem) If a triangle has two equal sides, then the angles opposite these sides are also equal.

i.e. 

Proof



$|AB|=|AC|$, then ΔABC and ΔACB are congruent by SAS $\Rightarrow$ ~~eq~~ same angles

i.e. $\angle C = \angle B$ and $\angle B = \angle C$ as required. $\square$