Math 329 Geometry

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- students with disabilities

Text: The four pillars of geometry, Stillwell.

Plan

1. Euclidean geometry (constructions) $2^{2^m}+1$
   - compass and straight edge
   - coordinates
   - geometry on Regents exam
2. vectors, transformations, isometries
   - linear algebra (drawing in perspective)
   - classification of plane isometries
   - spherical geometry
3. Non-Euclidean geometry and geometry of surfaces
   - Euler characteristic
   - classification of surfaces

§1 straight edge and compass (see website for java app)

Euclids axioms:

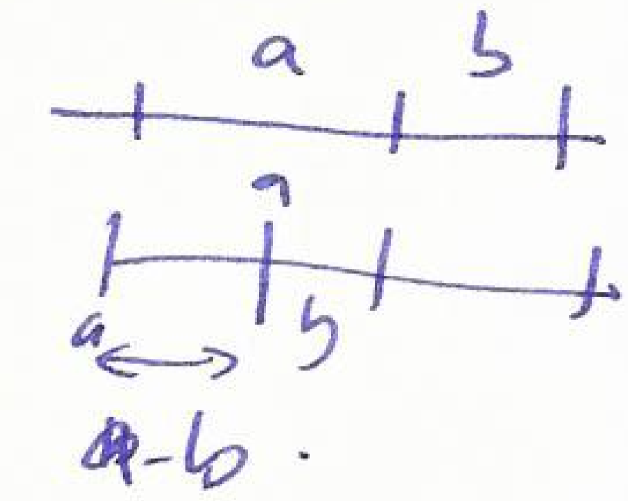
1. You can draw a straight line between any two points } straight edge
2. You can extend a straight line indefinitely }
3. You can draw a circle with a given center and radius } compass
4. Parallel postulate

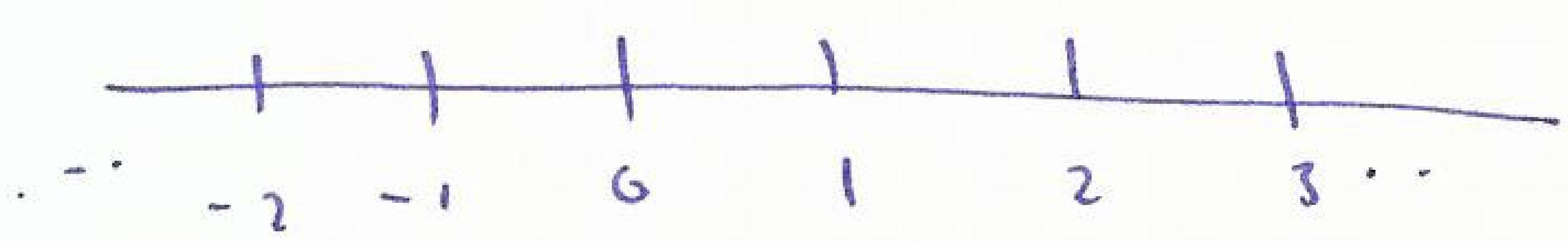
Straight edge vs. ruler

↑ unmarked — ↑ marked with numbers.

important: you can only <u>measure</u> with a compass.

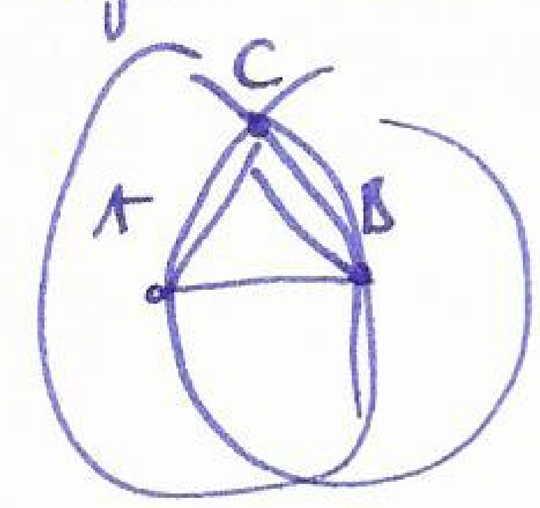
– we can <u>add</u> and <u>subtract</u> lengths.





<u>Q</u>: what numbers can we make? $\frac{1}{2}$, $\sqrt{2}$, π, e?

<u>Euclid Book I, Proposition 1</u>: construct an equilateral triangle with side length AB A•—•B.



Method:
1. draw circle radius AB centered at A
2. " AB " B
3. draw line segments to point of intersection.

claim: all three sides have equal length:

$|AB| = |BC|$ as radii of circle centered at B

$|AB| = |AC|$ " " A

$$\left.\begin{array}{l}|AB| = |BC| \\ |AB| = |AC|\end{array}\right\} \Rightarrow |AC| = |BC|$$ □

<u>Discussion of the proof</u>

– construction axioms (existence of circles, line segments)

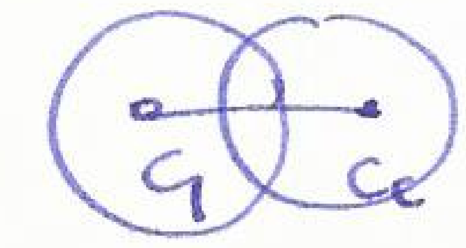
– geometric axioms (the two circles intersect at point C)

– logic ($a=b$, $b=c \Rightarrow a=c$)

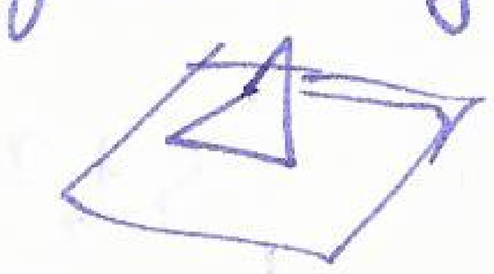
Euclid states no axiom about intersection of circles!

Problems with Euclid's argument:

1. Why does C exist? Euclid gave no axiom that guarantees two circles intersect if centres c_1, c_2 have $|c_1 c_2| < r_1 + r_2$



2. Why is ABC a plane figure? e.g. 4 line segments need not lie in a plane.



(see book XI for solid geometry)

3. Why does ABC contain an equilateral triangle?

Suppose AC and BC meet at E before they reach C
ABE isoceles but not equilateral.

(need to assume that if two straight lines have a common line segment then they are part of the same straight line).

In fact there are geometries which satisfy 1-3 in which any of the above can happen (and 4 can fail).

<u>Defn</u> A regular polygon is a shape with equal sides and equal angles.

<u>Example</u> an equilateral triangle is a regular 3-gon.

<u>Q</u>: for which n is the regular n-gon constructible by compass and straight edge?

<u>A</u>: not known! Depends on when $F_n = 2^{2^n} + 1$ is prime (Fermat primes)

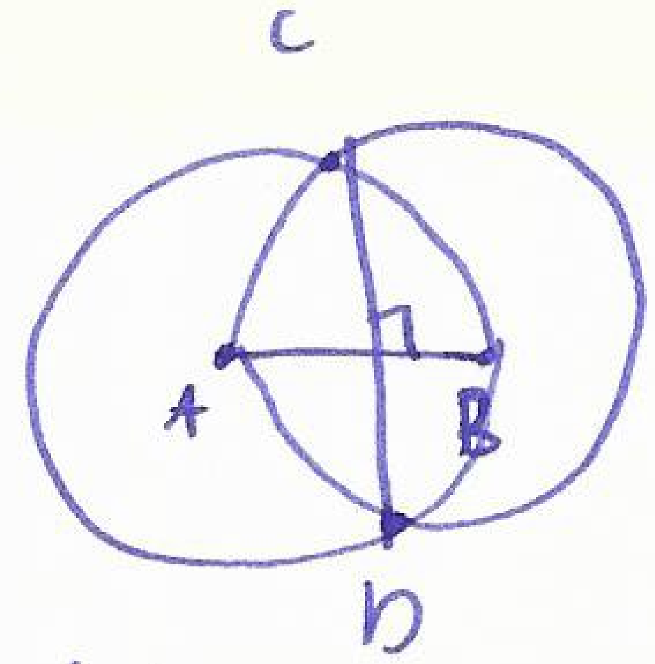
n	0	1	2	3	4	5 — 12	13?
F_n	3	5	17	257	65537	composite (not prime).	

[Gauss/Wantzel]

<u>Thm</u> A regular n-gon is constructible $\iff$ $n = 2^m \cdot$(product of distinct Fermat primes)

<u>Examples</u> 3, 5, 15, 17, 51, 85, 255, 257, ...

§1.3 Basic constructions

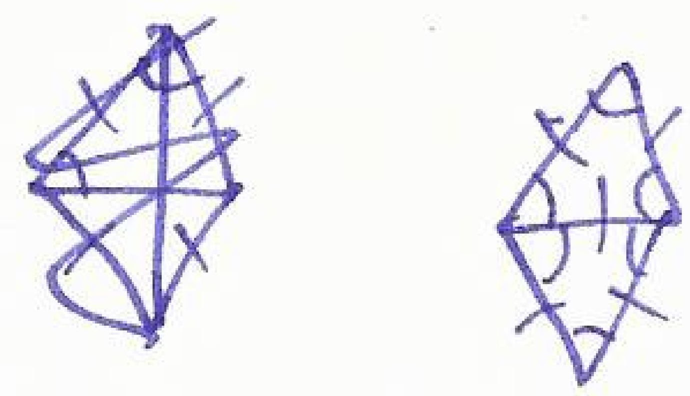


- **Bisecting a line segment**

CD bisects AB, its also perpendicular, so its called the perpendicular bisector.

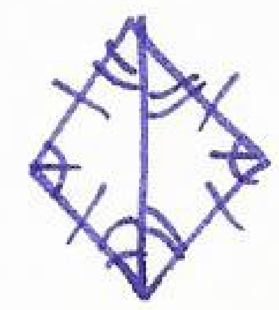
Proof (complicated!)

equilateral triangles:

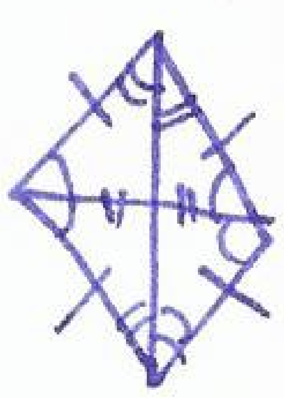


equal sides, equal angles.

iscoceles triangles:

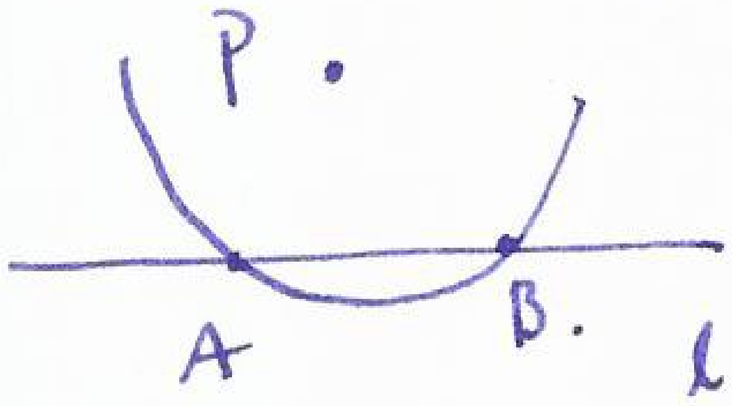


two equal sides, two equal angles



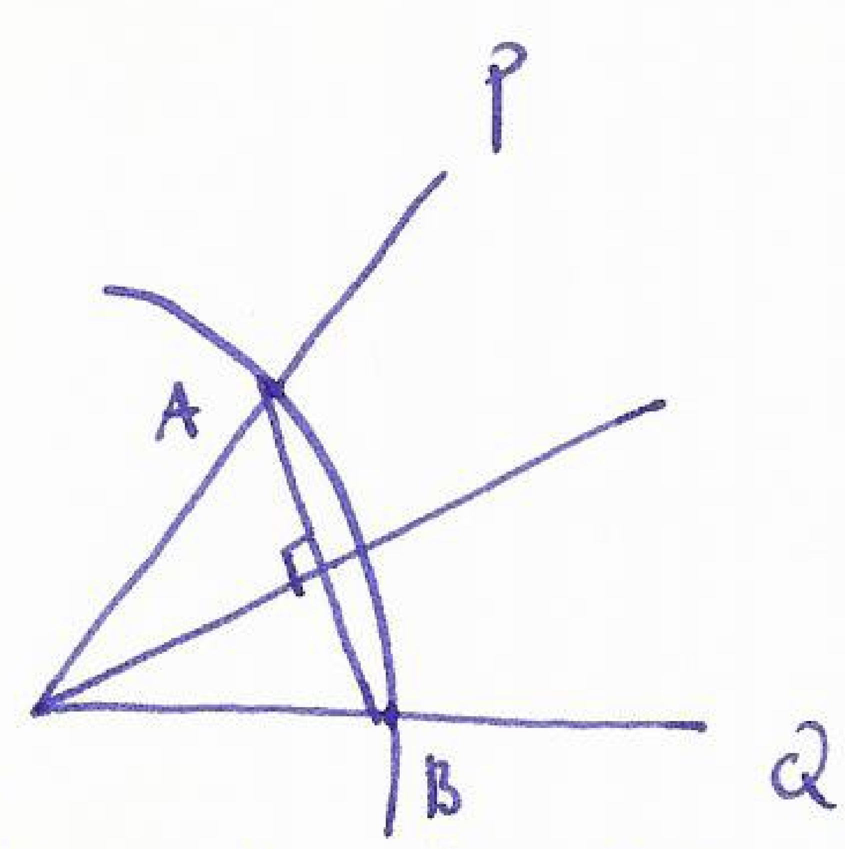
ASA determines triangle! so all 4 triangles congruent – pairs of edges same.
3rd angle at center makes $360°$, so must be $90°$, right angle □

to construct perpendicular lines:



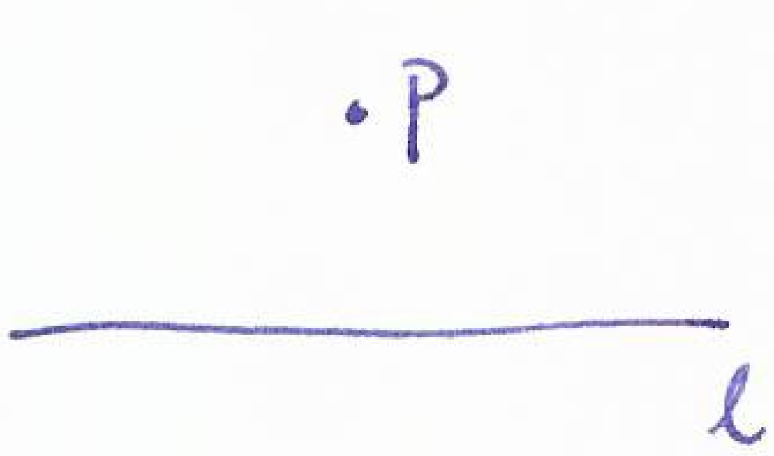
- **Bisecting an angle**

angle POQ.

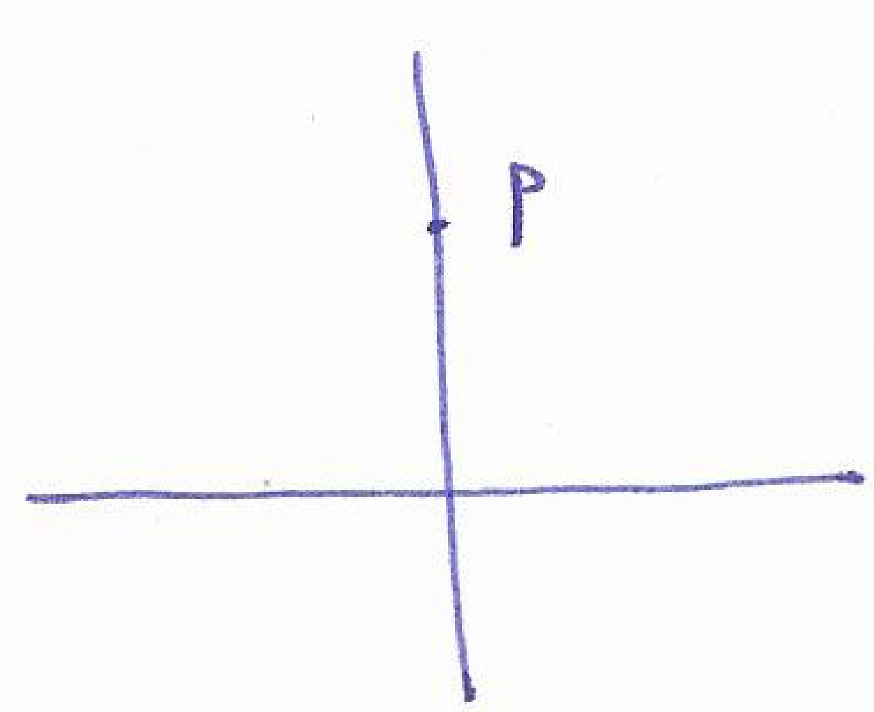


draw circle through O, cuts OP and OQ at A, B say
construct perpendicular bisector to AB – also bisects POQ.

- **Constructing parallel lines**



construct perpendicular line



construct perpendicular through P