# §5.8 Exponential growth

Exponential functions: $P_0 e^{kt}$ model various things:

- bacteria growth : # bacteria doubles every five mins.

$$B(t) = P_0 2^{t/5} = P_0 e^{t \ln(2)/5}$$

note : rate of change of # bacteria $\propto$ # bacteria.

$$B'(t) = k\, B(t) \quad \leftarrow \text{differential equation!}$$

check : $B(t) = P_0 e^{kt}$ is a solution : $B'(t) = P_0 k e^{kt} = k P_0 e^{kt}$ ✓

$k > 0$ exponential growth : bacteria, interest rates

$k < 0$ exponential decay : radioactive decay

Compound interest

\$100 for 1 year at 10% : \$110

\$100 for 1 year compounded twice : $(\$100 + 5\%) + 5\%$

$$= 105 + 5\%$$

$$= 100\left(1 + \frac{10}{100 \cdot 2}\right)^2 = 110.25$$

compound 4 times : $\left(\left(\left(100 + \frac{10}{4}\%\right) + \frac{10}{4}\%\right) + \frac{10}{4}\%\right) + \frac{10}{4}\%$.

$$= 100\left(1 + \frac{10}{100 \cdot 4}\right)^4$$

limit as compound more often : $100\left(1 + \frac{10}{100 \cdot n}\right)^n$

more generally : $P_0\left(1 + \frac{r}{n}\right)^{nt}$ $P_0$ = initial amount, $r$ = interest rate, $t$ = # years.

$$\lim_{n\to\infty} P_0\left(1+\frac{r}{n}\right)^n = P_0 e^{rt}$$

<u>Examples:</u> ...