<u>warning</u> what about $\int_0^{x^2} \sin t \, dt$?

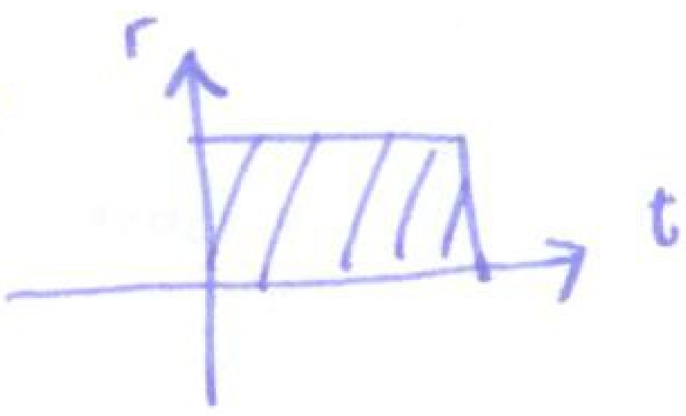
set $A(x) = \int_0^x \sin t \, dt$ then $G(x) = \int_0^{x^2} \sin t \, dt$

so $\frac{d}{dt} \int_0^{x^2} \sin t \, dt = \frac{d}{dt}\left(A(x^2)\right)$ use chain rule!

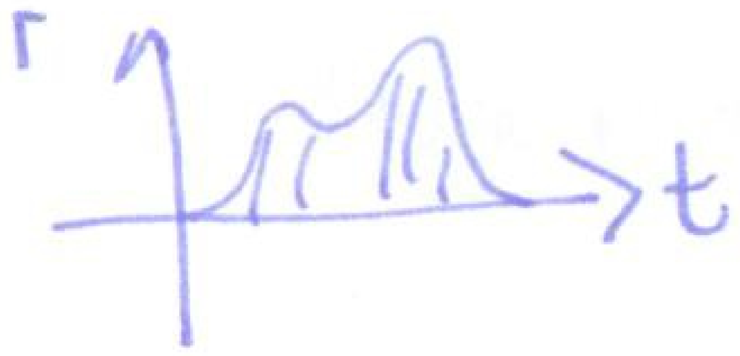
$= A'(x^2) . 2x . = 2x \sin x .$

§5.5 Net change (applications of integrals)

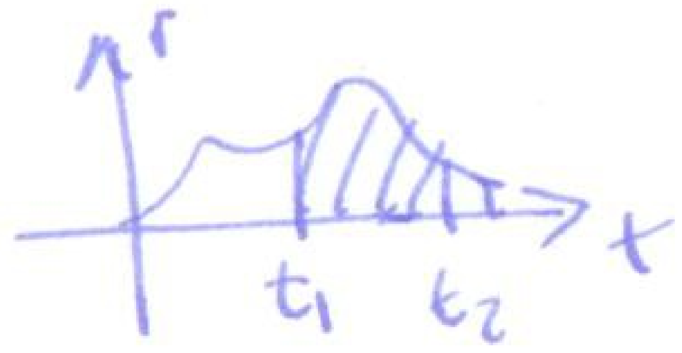
<u>Example</u> you pour water into a bucket at rate $r(t)$ Q: How much water is in the bucket? constant rate: amount = rate×time



vary rate $r(t)$: amount = area under curve $= \int_0^t r(t)\,dt$.

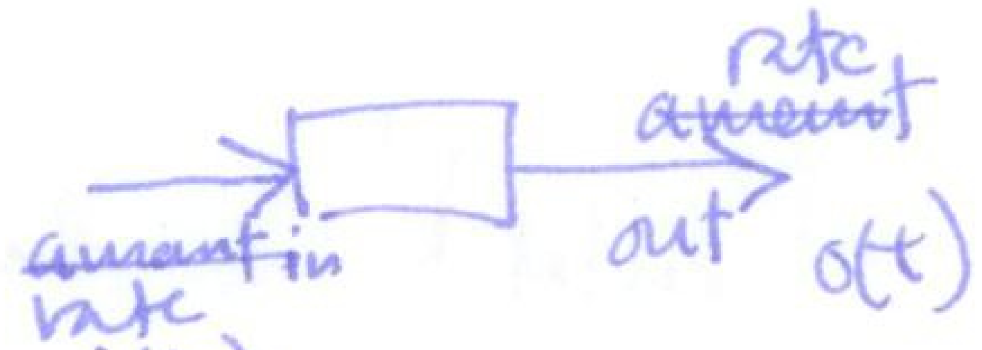


<u>Q</u>: by how much did the amount of water change between times t_1 and t_2?



net change $= \int_{t_1}^{t_2} r(t)\,dt$.

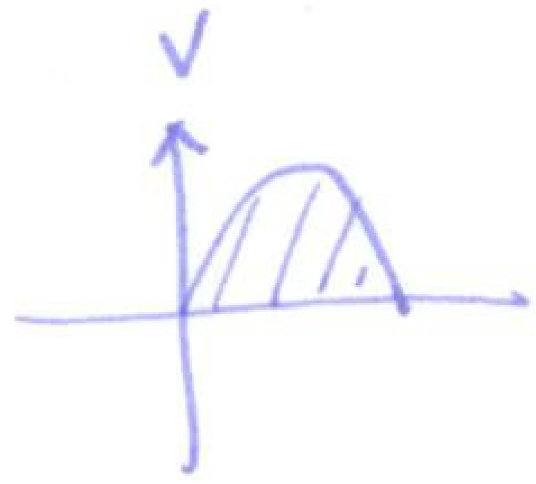
- traffic/queuing



net rate of change: $i(t) - o(t)$

net change: $\int_{t_1}^{t_2} i(t) - o(t) \, dt$

- distance as integral of velocity:



distance travelled: $\int_{t_1}^{t_2} v(t)\,dt$.

Examples

① $\int_0^1 e^{-7x}\,dx$ try: $u = -7x$

$\frac{du}{dx} = -7$ $\frac{dx}{du} = 1 / \frac{du}{dx}$.

$$\int_0^{-7} e^u \frac{dx}{du}\,du = \int_0^{-7} -\frac{1}{7}e^u\,du = -\frac{1}{7}\left[e^u\right]_0^{-7} = -\frac{1}{7}\left(e^{-7}-1\right)$$

② $\int_0^2 x^2\sqrt{x^3+1}\,dx$, $\int_0^{\pi/4} \tan^3 x \sec^2 x\,dx$, $\int_0^1 \frac{x}{x^2+1}\,dx$

§5.7 More integrals

recall: $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ $\int \frac{1}{x}\,dx = \ln|x| + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + c$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + c$

Example

① $\int \frac{1}{9+x^2}\,dx = \frac{1}{9}\int \frac{1}{1+x^2/9}\,dx = \frac{1}{9}\int \frac{1}{1+(x/3)^2}\,dx$

try: $u = x/3$

② $\int \frac{1}{\sqrt{1-4x^2}}\,dx = \int \frac{1}{\sqrt{1-(2x)^2}}\,dx$ try: $u = 2x$.