observations

① choice of antiderivative doesn't matter in definite integral

spose $F(x)$, $F(x)+c$ are anti-derivatives for $f(x)$

then $\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b)-F(a)$

$\int_a^b f(x)\,dx = \Big[F(x)+c\Big]_a^b = F(b)+c-F(a)-c = F(b)-F(a)$.

② $\int_a^b f(x)\,dx$ is a function of a and b. x is a dummy variable.

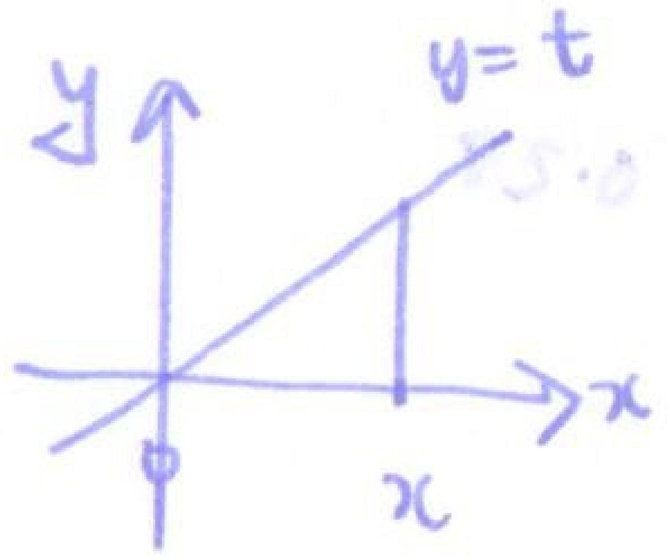
$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$.

§5.4 Fundamental theorem of calculus ②

Thm FTC② Let $f(x)$ be cts on $[a,b]$ then $A(x) = \int_a^x f(t)\,dt$ is an anti-derivative for $f(x)$, i.e. $A'(x) = \frac{dA}{dx} = f(x)$.

i.e. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$, and furthermore $A(a)=0$.

Examples



$\int_0^x t\,dt = \Big[\frac{1}{2}t^2\Big]_0^x = \frac{1}{2}x^2 - 0 = \frac{1}{2}x^2$

$\int_0^x e^{-t^2}\,dt \leftarrow$ a function with derivative e^{-t^2}

observation:

$f(x) \xrightarrow{\text{integrate}} F(x) \xrightarrow{\text{differentiate}} \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

$\int_a^x f(t)\,dt$

$f(x) \xrightarrow{\text{differentiate}} f'(x) \xrightarrow{\text{integrate}} \int_a^x f'(t)\,dt = f(x) - f(a)$ only equal to $f(x)$ up to a constant

<u>warning</u> what about $\int_0^{x^2} \sin t \, dt$?

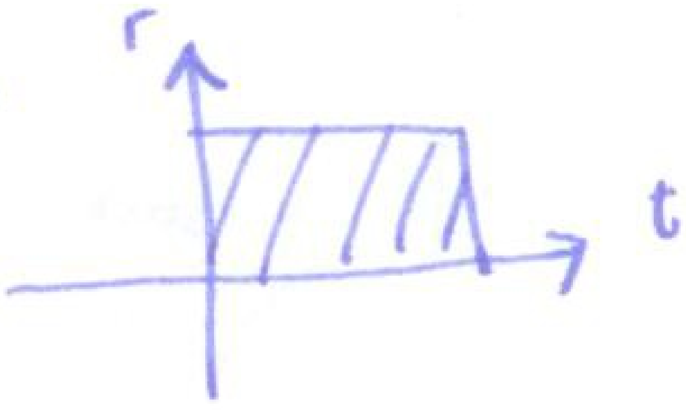
set $A(x) = \int_0^x \sin t \, dt$ then $G(x) = \int_0^{x^2} \sin t \, dt$

so $\frac{d}{dt} \int_0^{x^2} \sin t \, dt = \frac{d}{dt}\left(A(x^2)\right)$ use chain rule!

$= A'(x^2) . 2x . = 2x \sin x .$

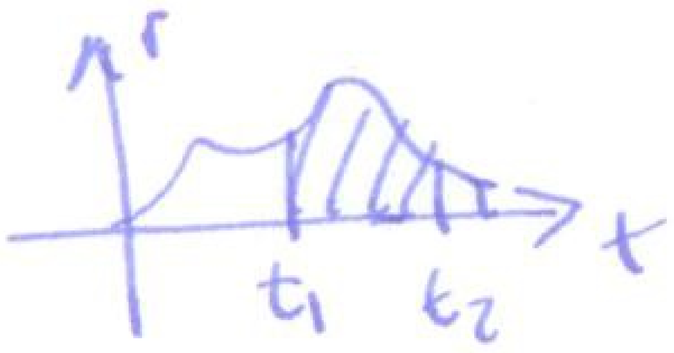
§5.5 Net change (applications of integrals)

<u>Example</u> you pour water into a bucket at rate $r(t)$ Q: How much water is in the bucket? constant rate: amount = rate × time



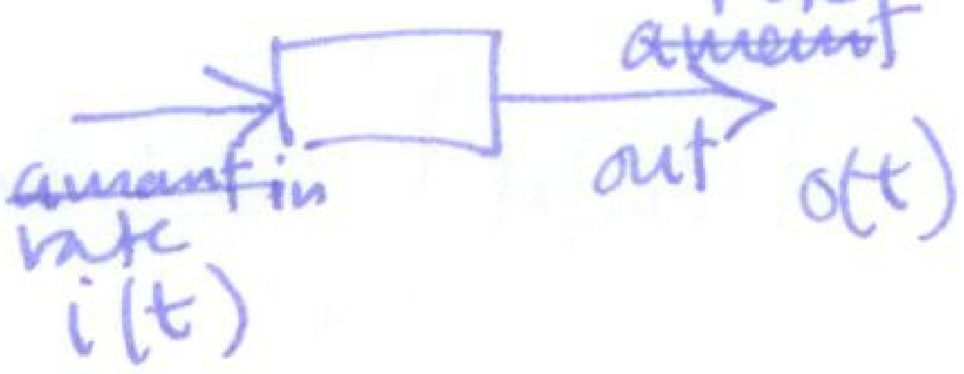
vary rate $r(t)$: amount = area under curve = $\int_0^t r(t) dt$.

<u>Q</u>: by how much did the amount of water change between times t_1 and t_2?



net change = $\int_{t_1}^{t_2} r(t) dt$.

- traffic / queuing



net rate of change : $i(t) - o(t)$

net change : $\int_{t_1}^{t_2} i(t) - o(t) \, dt$

- distance as integral of velocity:

distance travelled : $\int_{t_1}^{t_2} v(t) dt$.