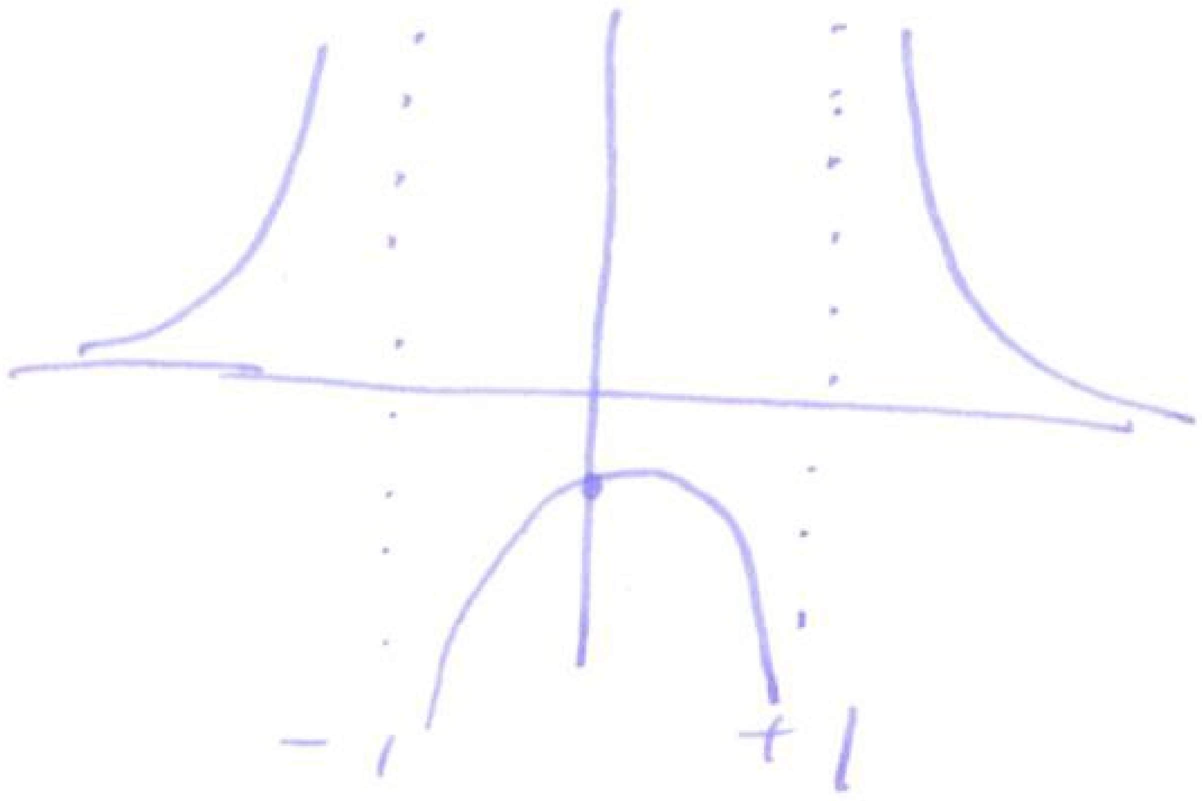
<u>Example</u> $f(x) = \dfrac{1}{x^2-1} = \dfrac{1}{(x+1)(x-1)}$




vertical asymptotes at $x = \pm 1$

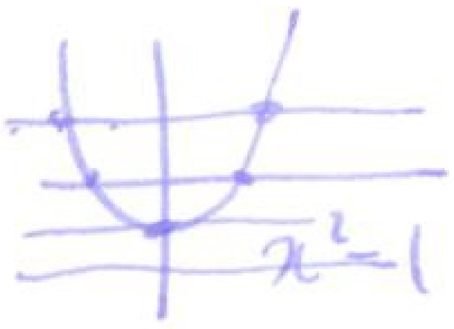
horizontal asymptote at $y = 0$.

$f'(x) = -(x^2-1)^{-2} \cdot 2x = \dfrac{-2x}{(x^2-1)^2}$

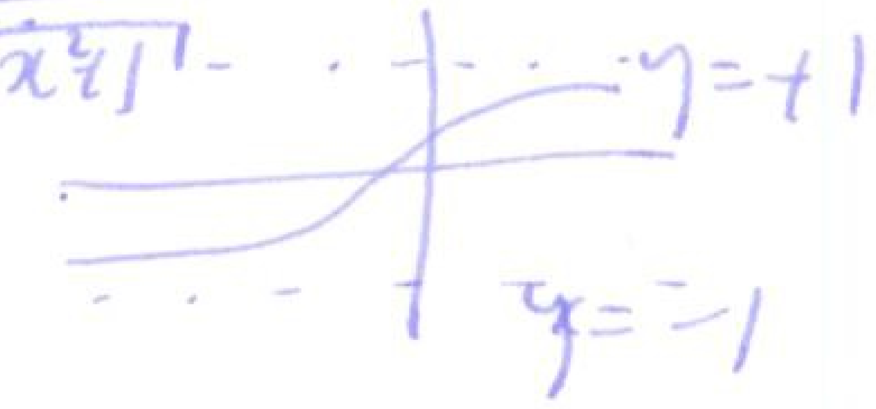
critical point at $x = 0$

first derivative test: $\underset{x=0}{\overset{f'(x)}{+ \mid -}}$ local max. $f(0) = -1$

$f''(x) = \dfrac{(x^2-1)^2 \cdot -2 - (-2x)2(x^2-1) \cdot 2x}{(x^2-1)^4} = \dfrac{6x^2+2}{(x^2-1)^3}$. sign $\underset{-1 \qquad +1}{+ \mid - \mid +}$

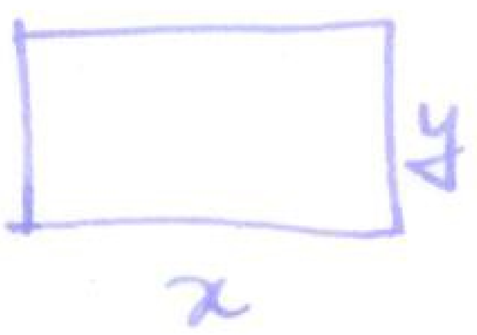
[  $x^2-1$ $\rightsquigarrow$ $\dfrac{1}{x^2-1}$  ]

<u>Example</u> $f(x) = \dfrac{x}{\sqrt{x^2+1}} = \dfrac{1}{\sqrt{1+1/x^2}}$.




## <u>§4.6 Optimization</u>

<u>Example</u> A piece of wire of length L is bent into a rectangle.
What's the largest area rectangle we can make?


$2x + 2y = L$

area $= xy$

i.e. max $A = xy$ such that $2x+2y = L$

$\Rightarrow \quad y = \frac{L}{2} - x$

So max $A = x\left(\frac{L}{2} - x\right) = \frac{L}{2}x - x^2$

find max: $A'(x) = \frac{L}{2} - 2x$ given $0 \le x \le \frac{L}{2}$ (check endpoints)

solve $A'(x) = 0 \Rightarrow x = \frac{L}{4} \Rightarrow y = \frac{L}{4}$. (square)

Example what shape of cylindrical tin minimizes surface area, if you want total volume to be 1 ft$^3$?

$$V = \pi r^2 h = 1 \Rightarrow h = \frac{1}{\pi r^2}$$

$$A = 2\pi r h + 2\pi r^2$$

so $A = 2\pi r\left(\frac{1}{\pi r^2}\right) + 2\pi r^2 = \frac{2}{r} + 2\pi r^2$

$A'(r) = -\frac{2}{r^2} + 4\pi r$ solve $A'(r) = 0$ $-\frac{2}{r^2} + 4\pi r = 0$

$$4\pi r = \frac{2}{r^2}$$

$$r^3 = \frac{1}{2\pi} \quad r = \sqrt[3]{\frac{1}{2\pi}}$$

## §4.7 L'Hôpitals rule

Thm Suppose $f(x)$ and $g(x)$ are differentiable and $f(a) = g(a) = 0$, and $g'(x) \neq 0$ for all $x$ near $a$, then (or $f(a) = g(a) = \pm\infty$) ← [i.e. indeterminate form]

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \quad \text{(provided this limit exists)}$$

Warning: this is not the quotient rule! We are not differentiating $\frac{f(x)}{g(x)}$.

Examples ① $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{3x^2}{1} = 3$