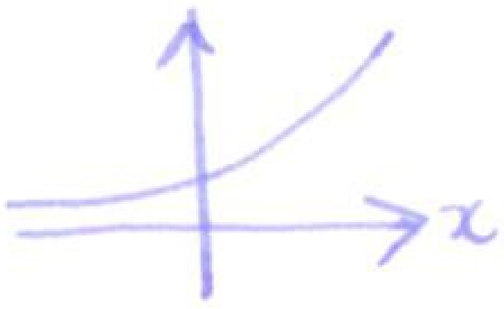
Monotonicity

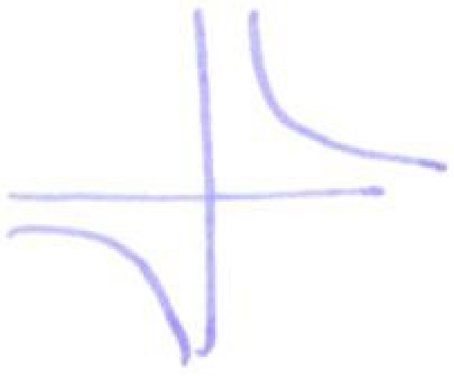
Suppose f is differentiable on (a,b)

If $f'(x) > 0$ for all $x \in (a,b)$ then f is increasing on (a,b)
$f'(x) < 0$ decreasing

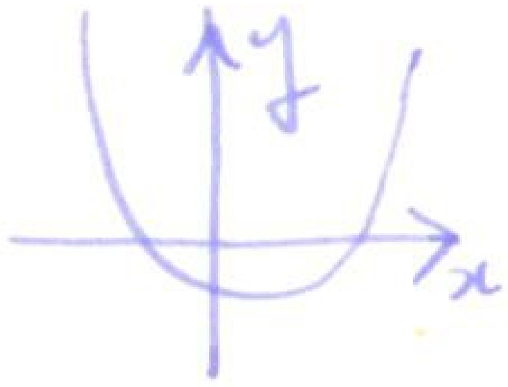
Example $f(x) = e^x$



$f(x) = \frac{1}{x}$



Example where is $f(x) = x^2 - 2x - 3$ increasing?



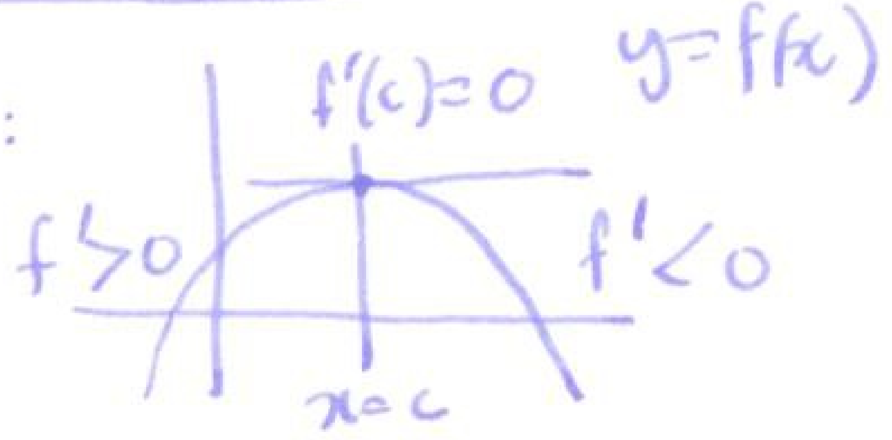
$f'(x) = 2x - 2$ want: x such that $2x - 2 > 0$

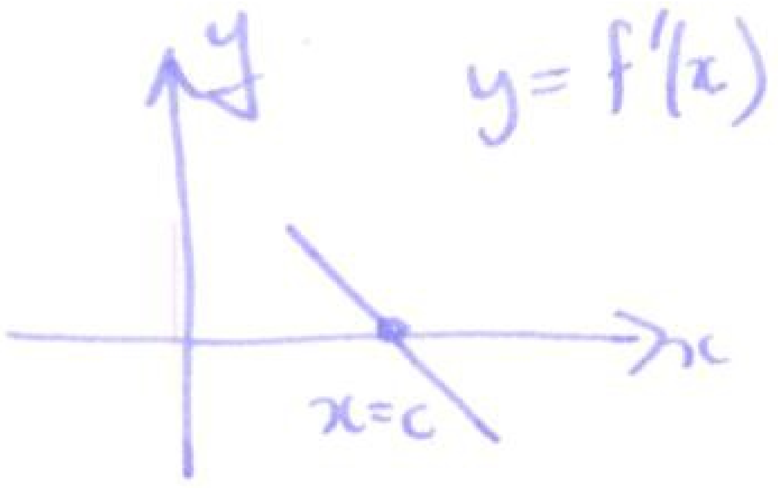
$2x > 2$

$x > 1$ $(1, \infty)$.

First derivative test

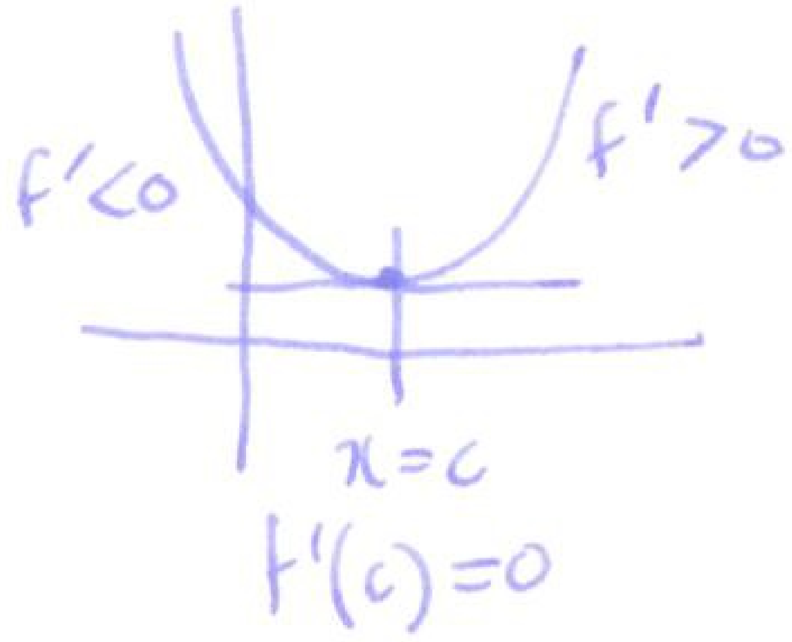
local max:

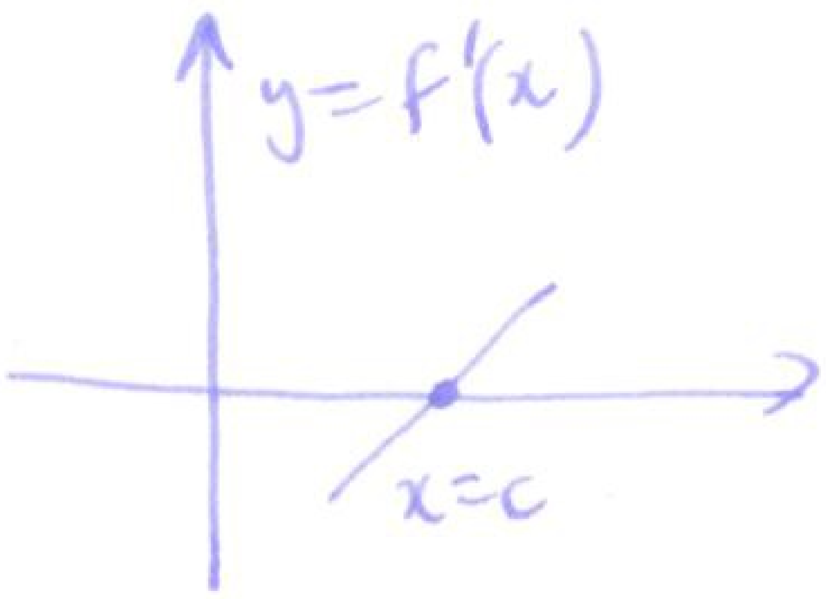




$[f''(c) < 0]$

local min:





$[f''(c) > 0]$

Thm First derivative test

Suppose $f(x)$ is differentiable and c is a critical point, i.e. $f'(c) = 0$

if $f'(x)$ changes from +ve to -ve at c $\Rightarrow$ local max

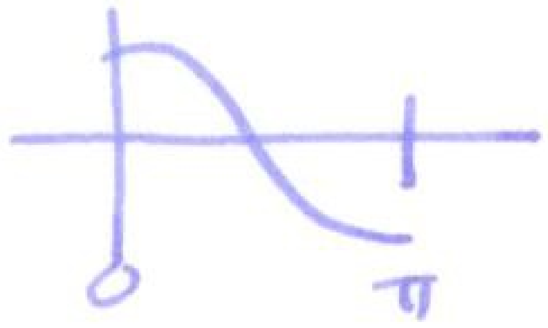
-ve to +ve $\Rightarrow$ local min

<u>Example</u> classify critical points of $f(x) = \cos^2 x + \sin x$ on $[0, \pi]$

find critical points $f'(x) = 2\cos(x) \cdot (-\sin x) + \cos x$

$$= \cos x \, (1 - 2\sin x)$$

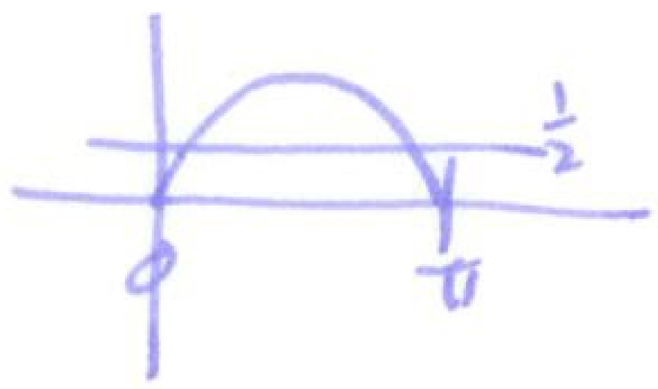
solve $f'(x) = 0$: $\cos x = 0$:



$x = \frac{\pi}{2}$

$1 - 2\sin x = 0$

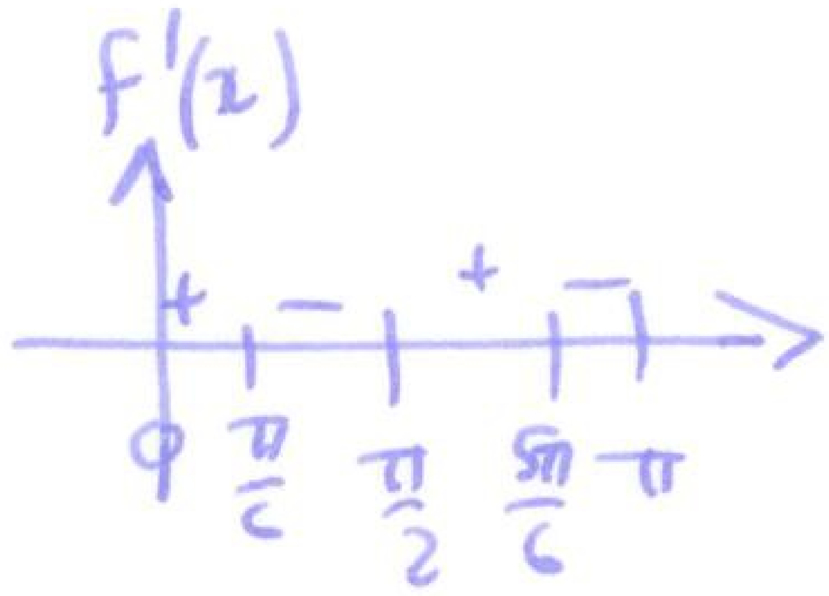
$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

critical points $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

find sign of $f'(x)$



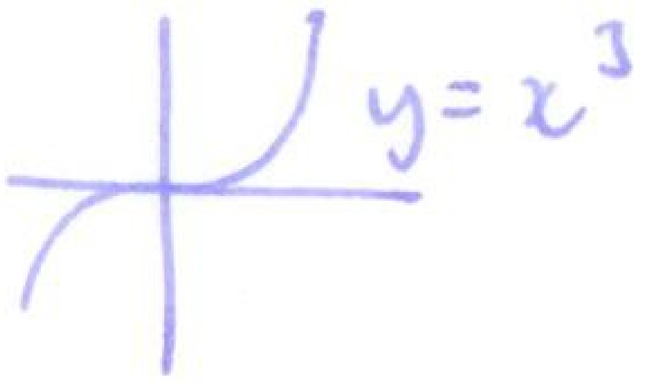
first derivative test :

$\frac{\pi}{6}$ local max

$\frac{\pi}{2}$ local min

$\frac{5\pi}{6}$ local max.

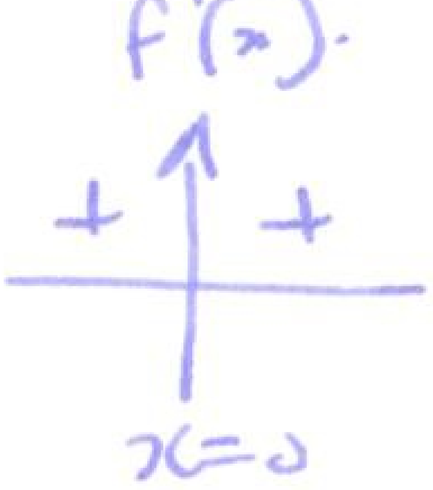
<u>Bad example</u>



$f'(x) = 3x^2$

critical point at $x = 0$

<u>not</u> max or min.

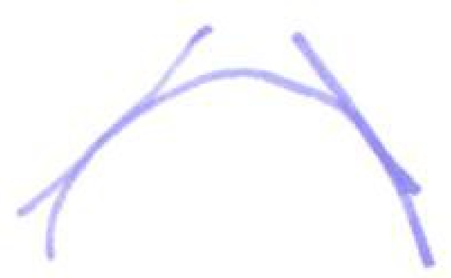


§4.4 Second derivative test

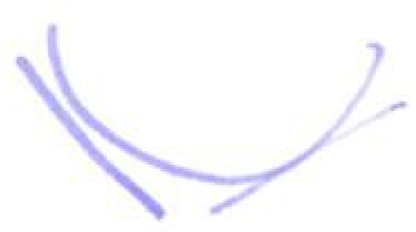
concave down

concave up

how do the slopes change?



concave down $\longleftrightarrow$ slope decreasing

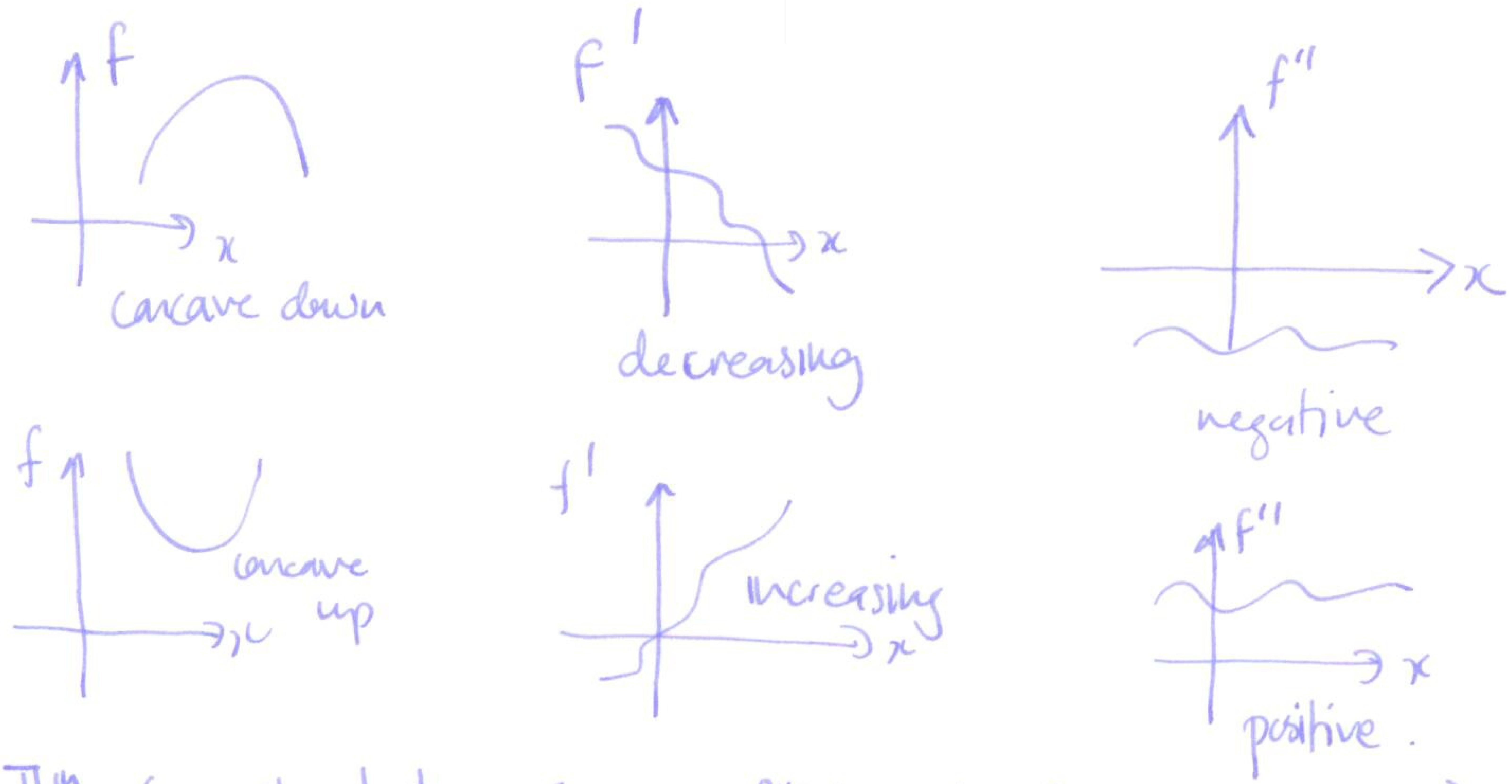


concave up $\longleftrightarrow$ slope increasing

Def^n $f(x)$ differentiable on (a,b)

then f is <u>concave up</u> $\Leftrightarrow$ $f'(x)$ is increasing

f <u>concave down</u> $\Leftrightarrow$ $f'(x)$ is decreasing



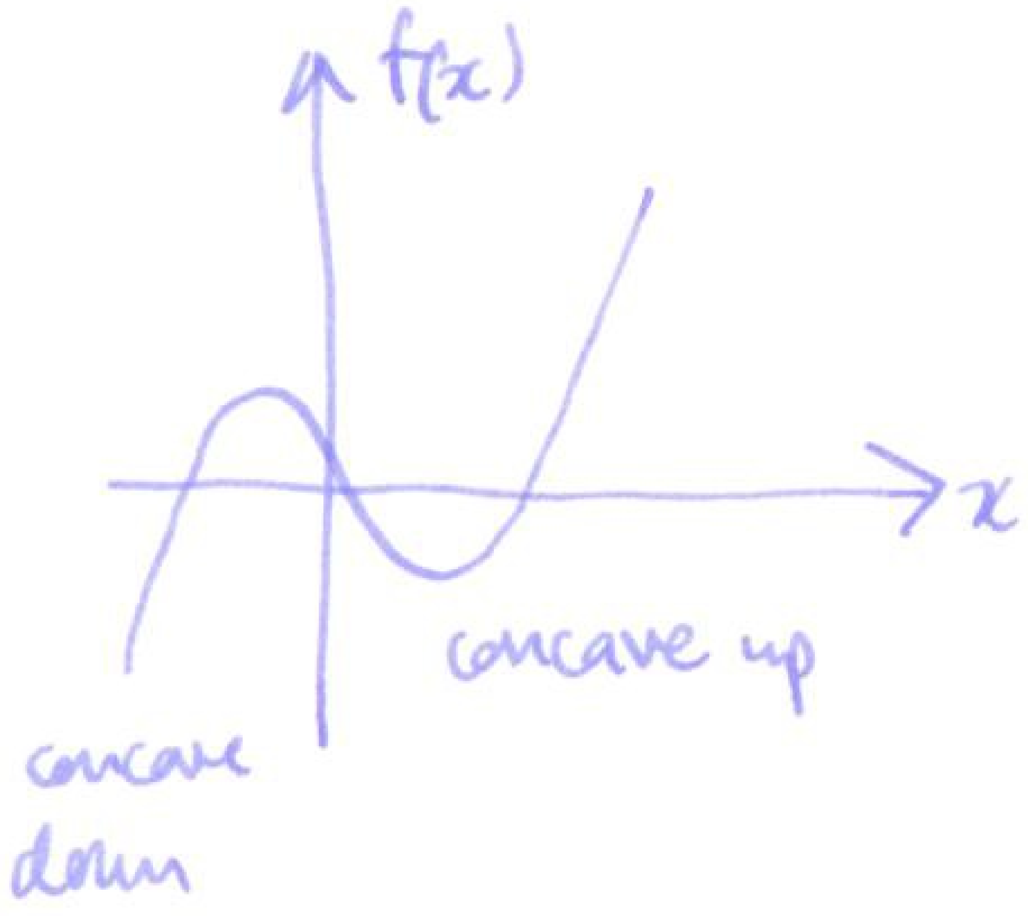
Th^m <u>Concavity test</u> Suppose $f''(x)$ exists for all $x \in (a,b)$

If $f''(x) < 0$ for <u>all</u> $x \in (a,b)$ then $f(x)$ is <u>concave down</u>

$f''(x) > 0$ <u>concave up</u>

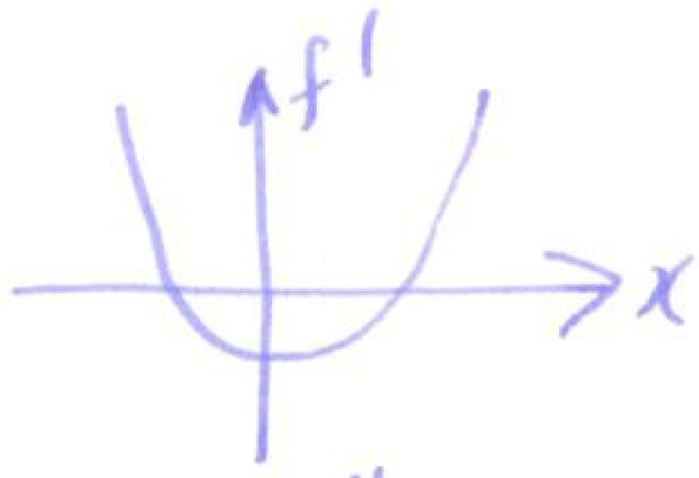
Def^n A point of inflection is where the graph changes from concave up to concave down (or vice versa)

Example $f(x) = x^3 - x = x(x^2-1) = x(x+1)(x-1)$



$f'(x) = 3x^2 - 1$

$f''(x) = 6x$



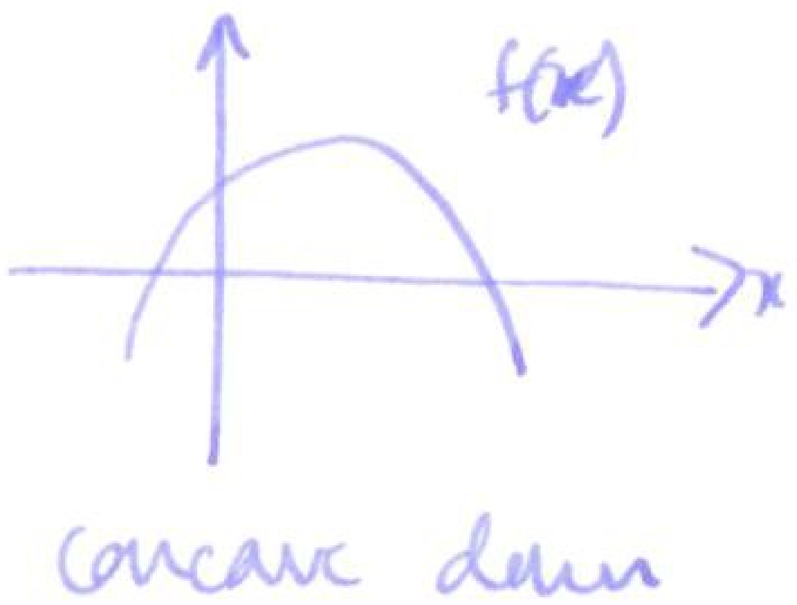
$f''(x) > 0$ for $x > 0$

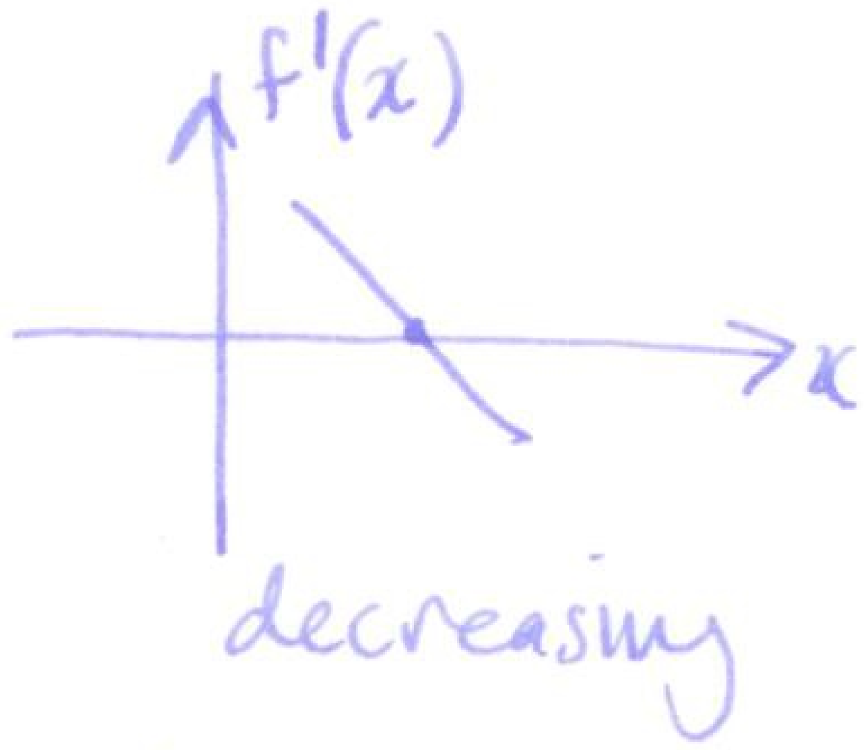
$f''(x) < 0$ for $x < 0$

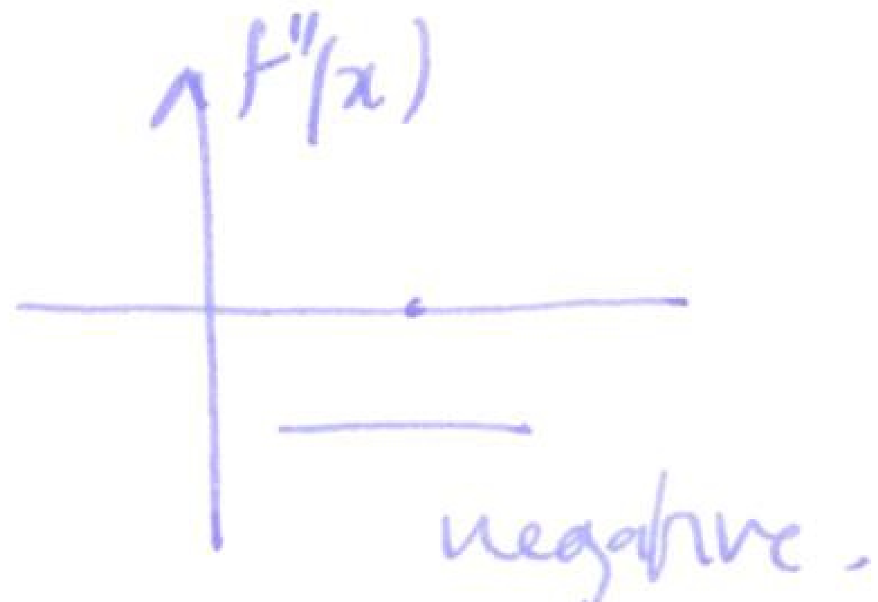
$\therefore$ $x = 0$ is a point of inflection

Second derivative test

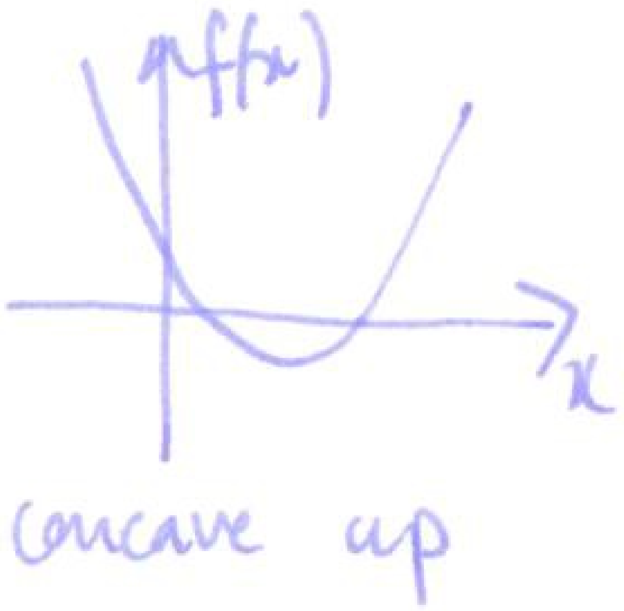
Local max

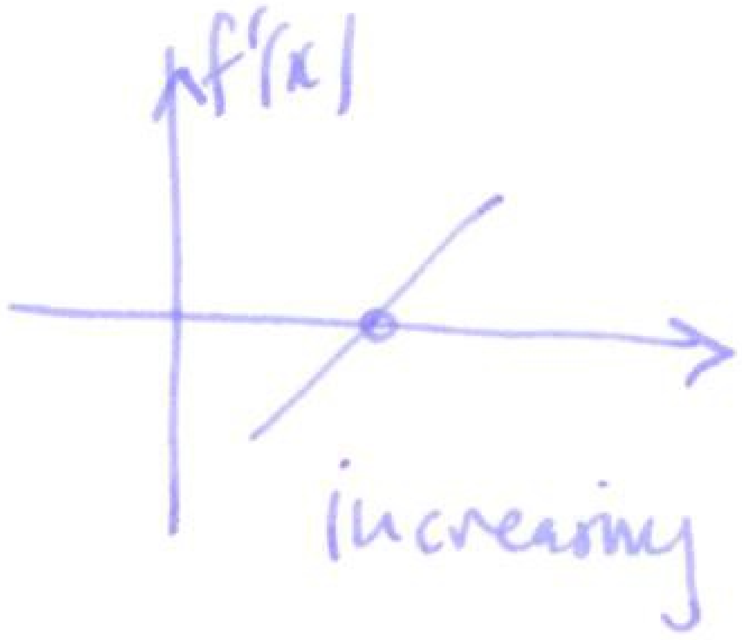


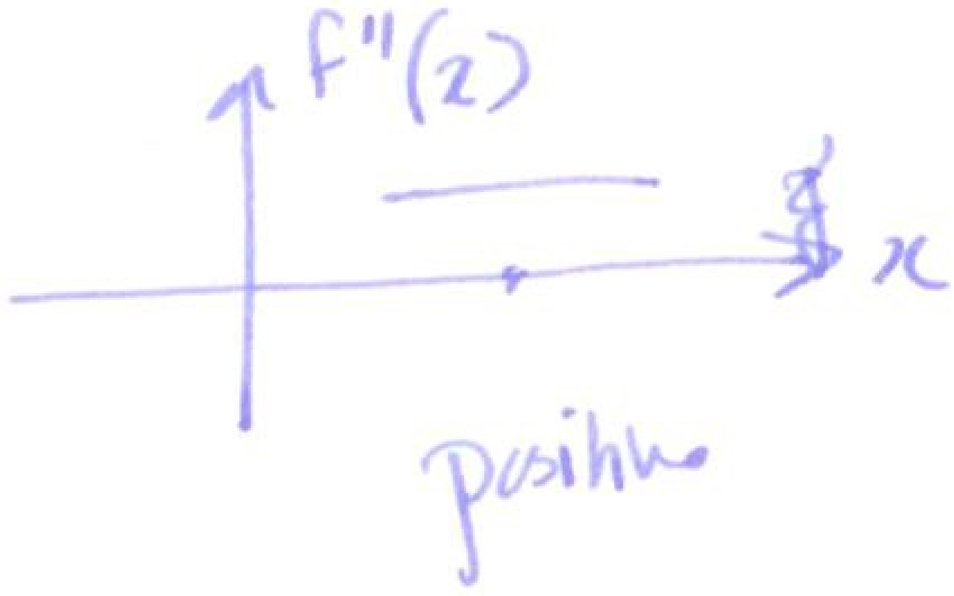




Local min







Th^m Suppose $f(x)$ is differentiable and c is a critical point and $f''(c)$ exists. Then

if $f''(c) > 0 \Rightarrow f(c)$ is a local ~~max~~ min

$f''(c) < 0 \Rightarrow f(c)$ is a local ~~min~~ max

$f''(c) = 0$ NO INFORMATION! (may be local max/min/neither)