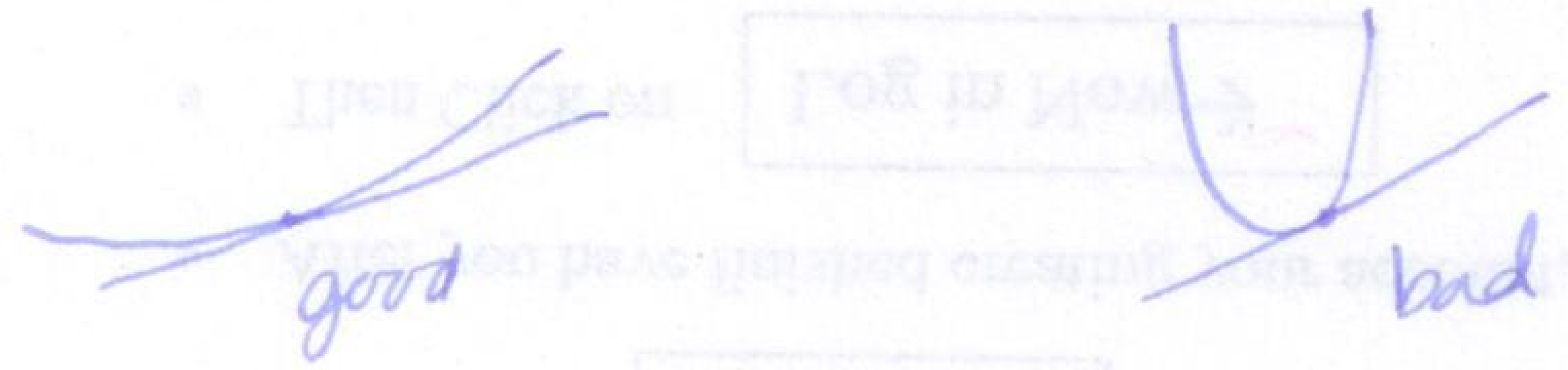
x	$f(x)$	linear approx	error	percentage error
121	11	$\frac{121}{20}+5 = 11.05$	0.05	$\frac{0.05}{11} \times 100$ small
400	20	$\frac{400}{20}+5 = 25$	5	$\frac{5}{20} \times 100$ 25%. big

observation: when is the linear approx a good approximation?



$f''(x)$ small (compared with Δx) $\qquad$ $f''(x)$ big (compared with Δx)

§4.2 Extreme values (max/min)

Absolute max/min

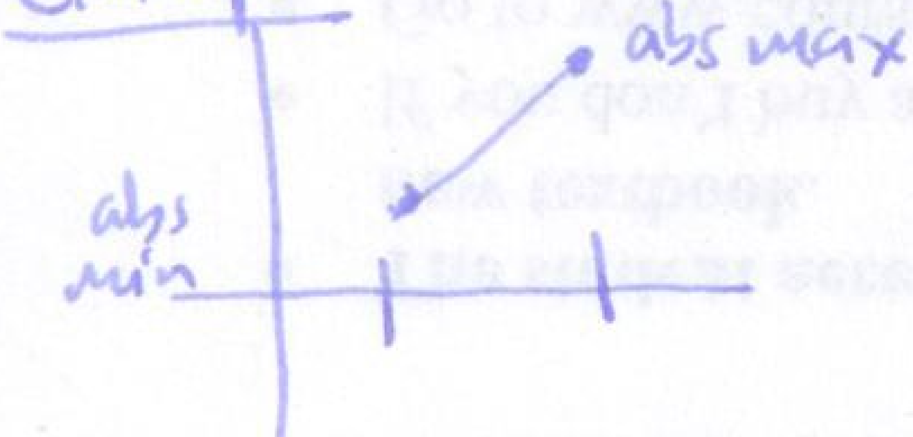
Def^n^ Let $f(x)$ be defined on an interval I. We say

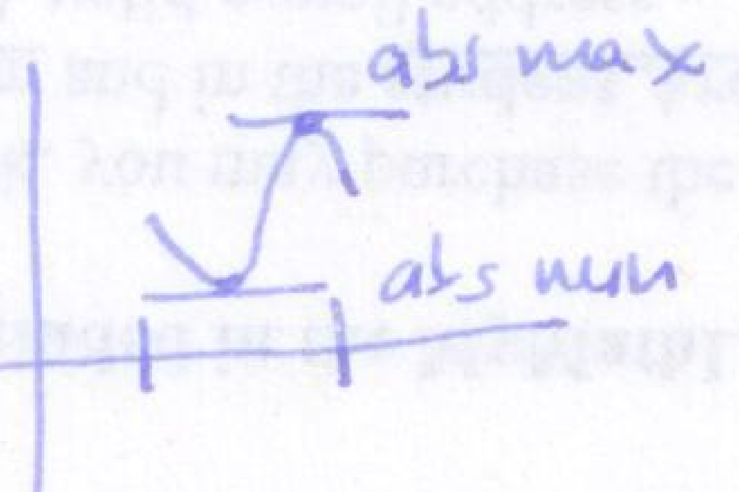
$f(a)$ is the absolute max if $f(x) \leq f(a)$ for all $x \in I$

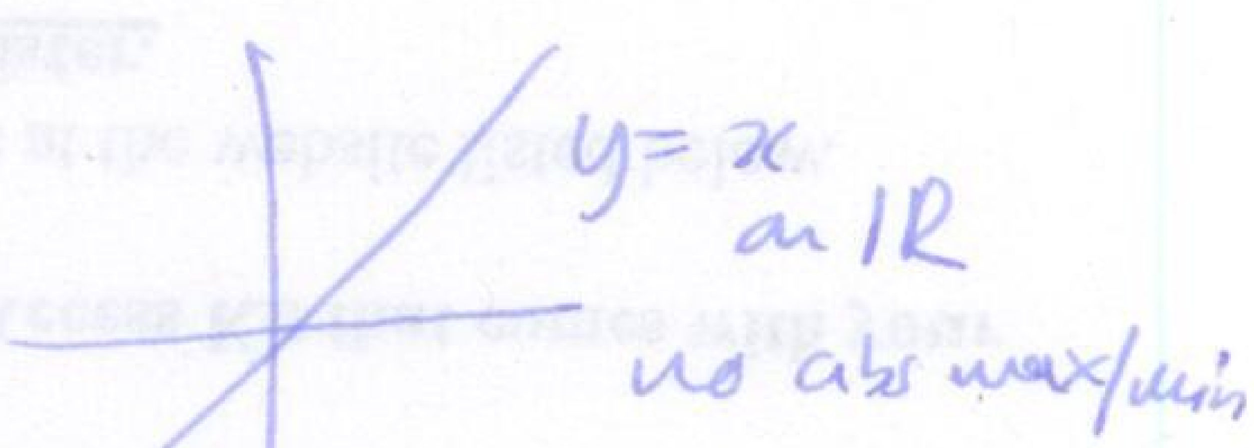
absolute min if $f(x) \geq f(a)$ "

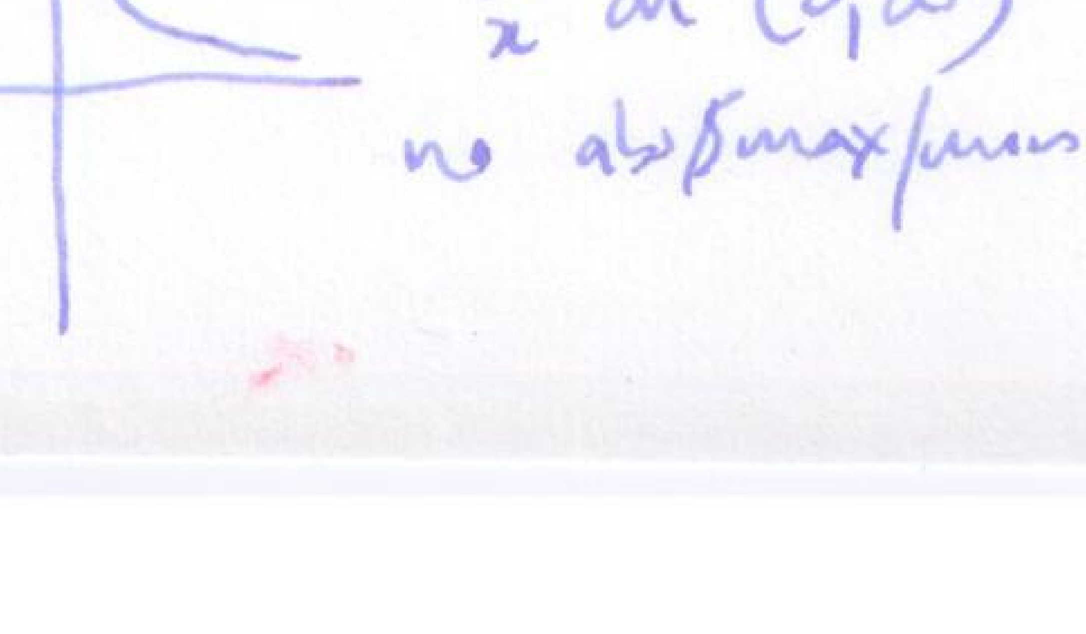
Warning: not all functions have absolute max or min.

Example









Th$^{\underline{m}}$ If $f(x)$ is continuous on a closed bounded interval $I_? = [a,b]$ then $f(x)$ has both an absolute max and an absolute min on I.

Local max/min

Def$^{\underline{n}}$ $f(x)$ has a local $\begin{matrix}\text{min}\\ \text{max}\end{matrix}$ at $x=c$ if $f(c)$ is the $\begin{matrix}\text{minimum}\\ \text{maximum}\end{matrix}$ value of $f(x)$ on some interval containing c.

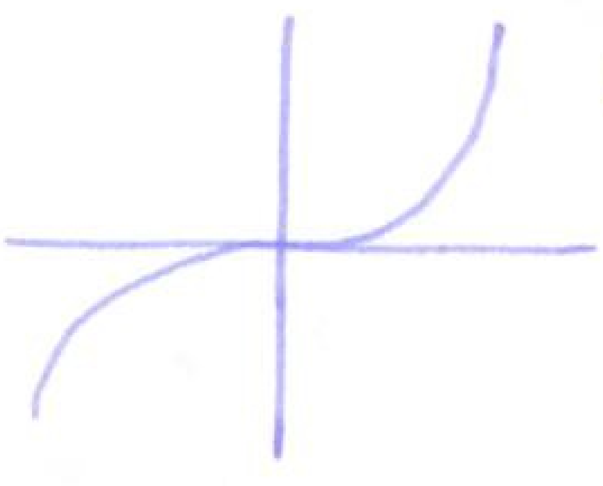
Example

Def$^{\underline{n}}$ $x=c$ is a <u>critical point</u> of f if $f'(c) = 0$

Th$^{\underline{m}}$ If f is differentiable and c is a local max/min, then $f'(c)=0$ i.e. c is a critical point

Warning c critical point $\not\Rightarrow$ c is a local max/min

Example



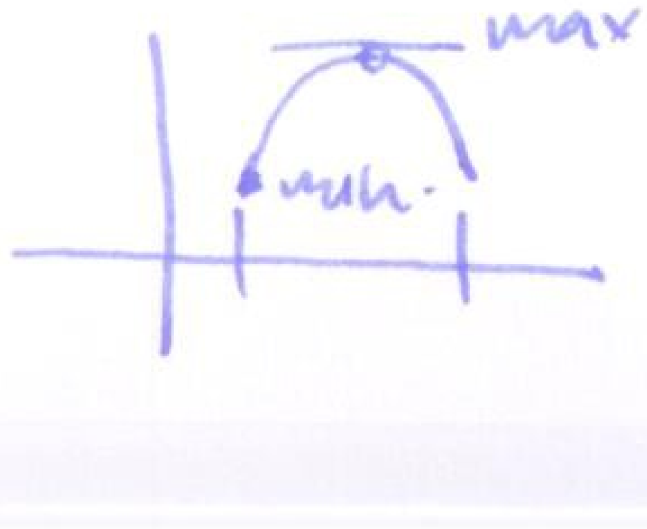
$y = x^3$ $\quad y' = 3x^2$ $\quad x=0$ is a critical point but not a local max/min

Finding absolute max/min on closed intervals.

① find critical points, i.e. all c s.t. $f'(c) = 0$

② check endpoints.

Examples



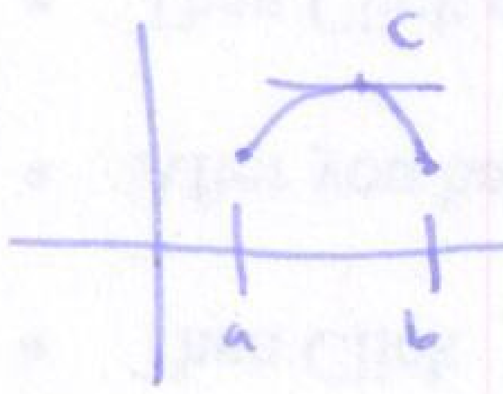
Example find the maximum of $2x^3-15x^2+24x+7$ on $[0,6]$

x^2-8 on $[1,4]$

$\sin x \cos x$ on $[0,\pi]$

Rolle's Th^m — Suppose $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) if $f(a)=f(b)$ then there is $c\in(a,b)$ with $f'(c)=0$.

Proof

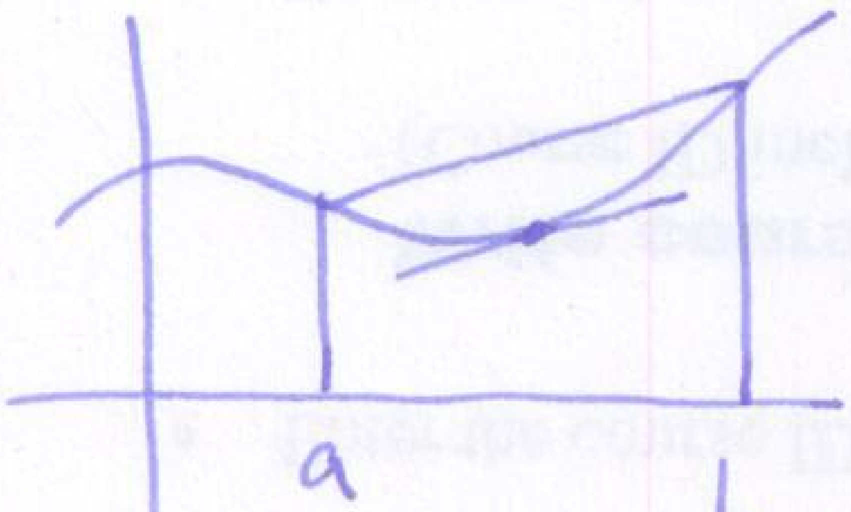


If there is a local max/min in (a,b) then there is c s.t. $f'(c)=0$.

If there is no local max/min then $f(a)=f(b)=\text{max/min}$

$\Rightarrow f(x)$ is constant $\Rightarrow \exists\, c$ s.t. $f'(c)=0$.

§4.3 First derivative test (Mean value theorem, monotonicity)



Th^m (Mean value theorem MVT)

Suppose f is cts on $[a,b]$ and differentiable on (a,b), then there is a $c\in(a,b)$ s.t.

$$f'(c)=\frac{f(b)-f(a)}{b-a} \quad \text{(average RoC)}$$

Corollary If $f(x)$ is differentiable and $f'(x)=0$, then $f(x)=c$ constant function.

Proof (of Corollary) ~~If $f(x)$ is differentiable and $f'(x)=0$, then $f(x)=c$~~

Spose there is a,b s.t. $f(a)\neq f(b)$. Then there is $c\in(a,b)$ s.t. $f'(c)\neq 0$ ⚡

Summary $f'(x)=0 \Rightarrow f$ is constant function.