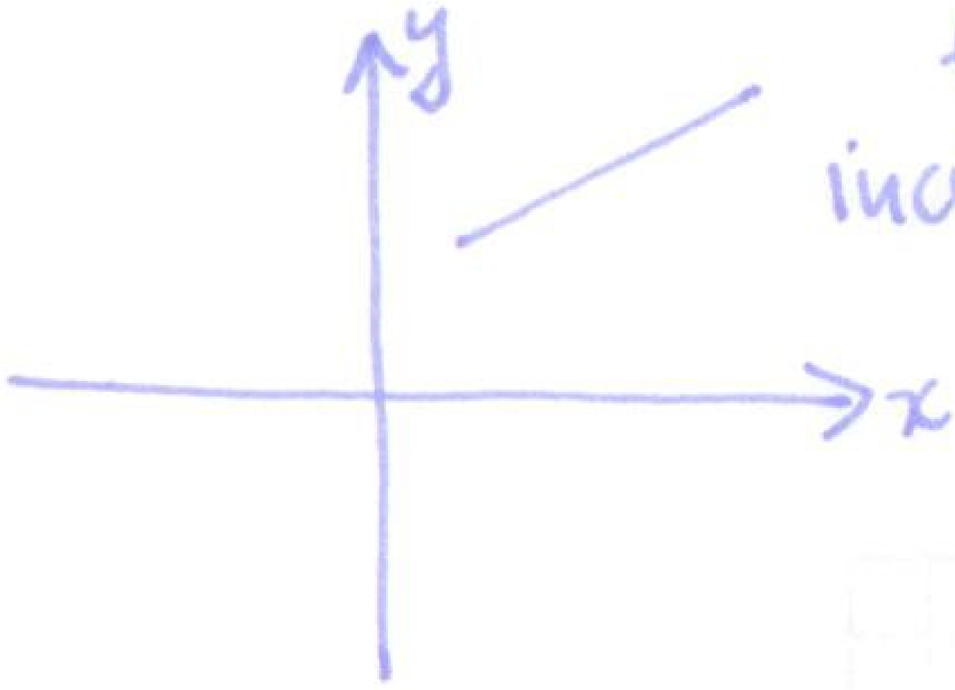
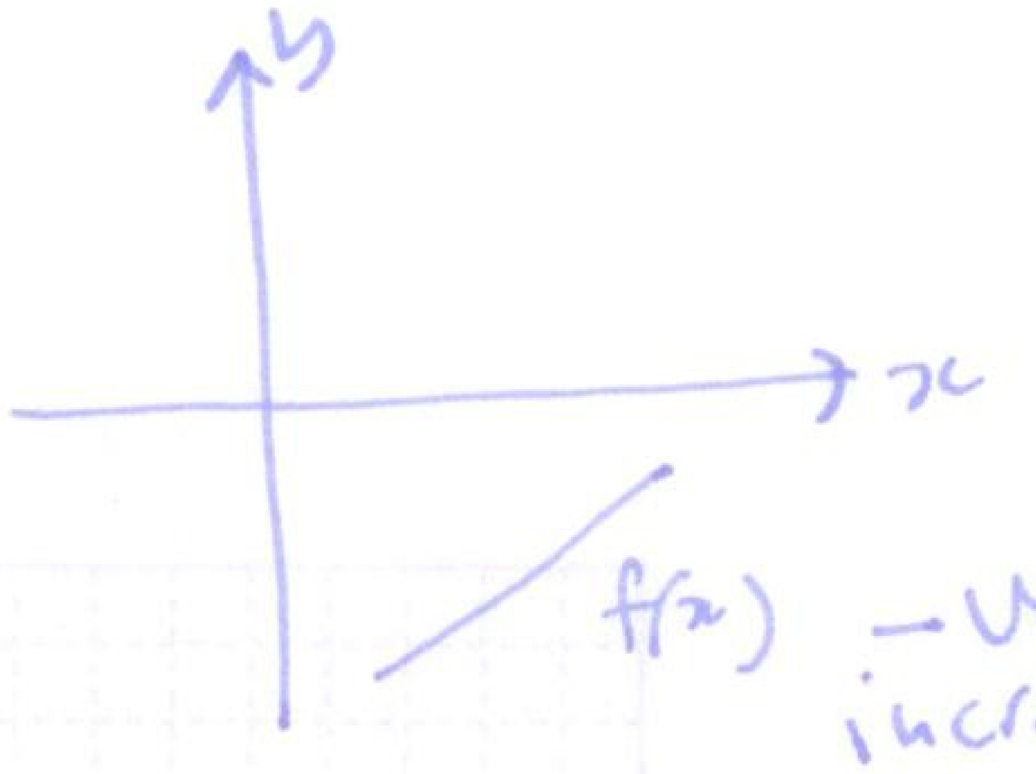
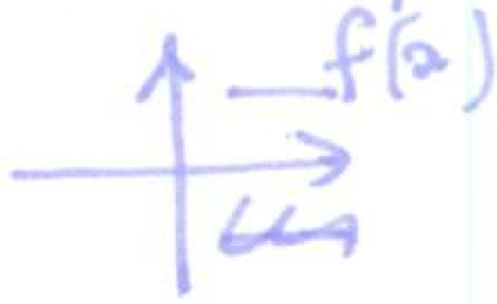


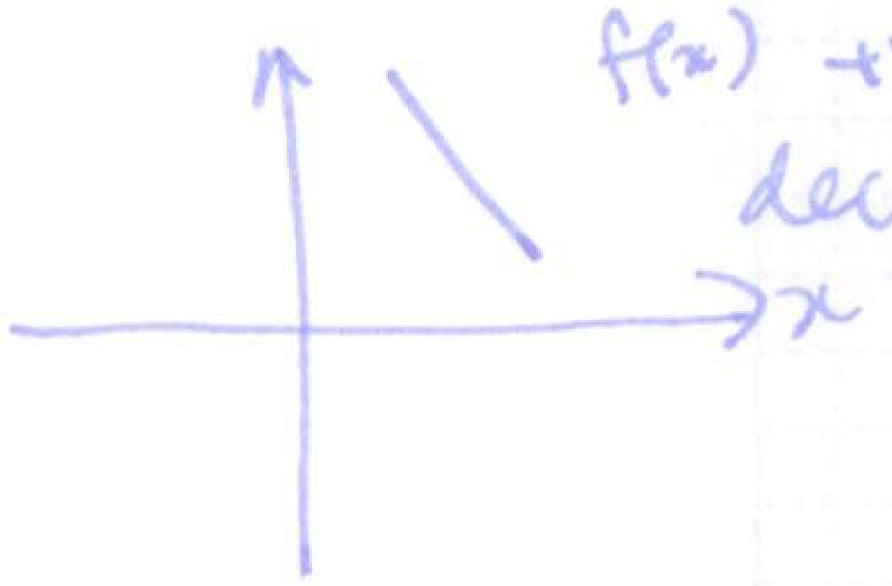
$f(x)$ +ve
increasing



$f(x)$ −ve
increasing

$f'(x)$
positive

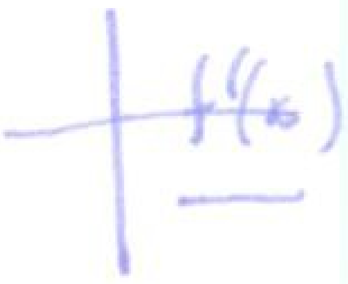


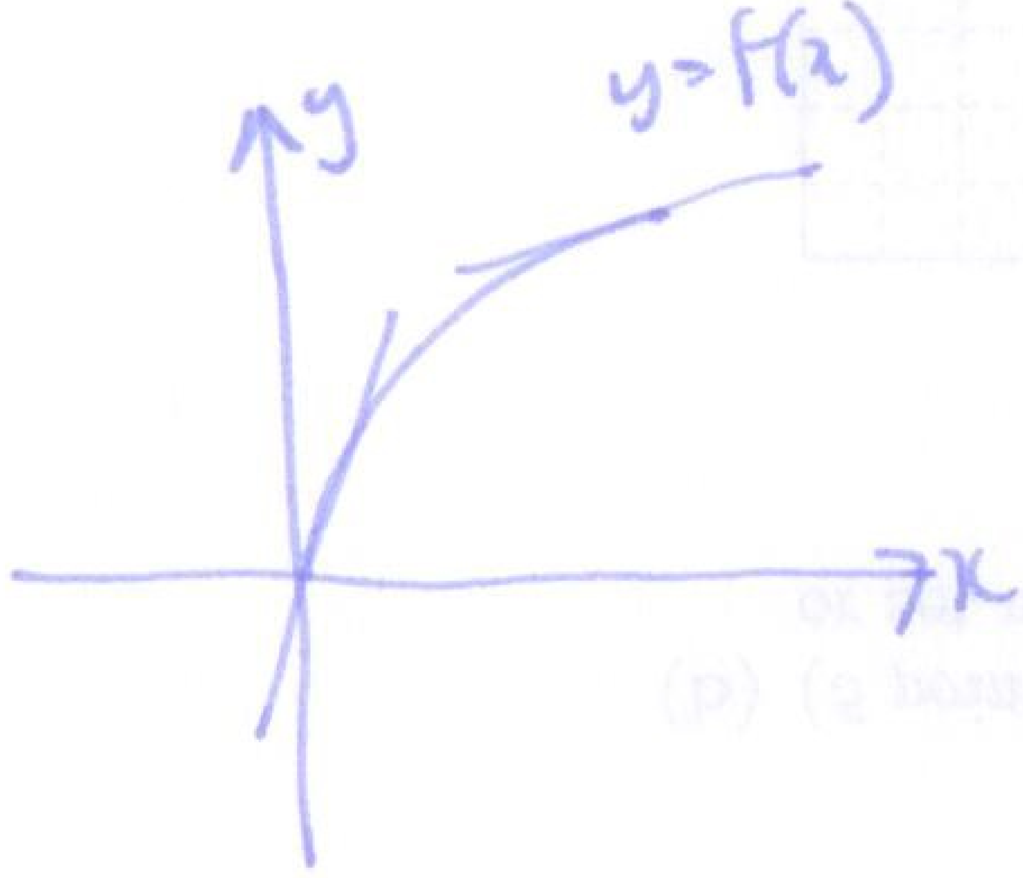


$f(x)$ +ve
decreasing

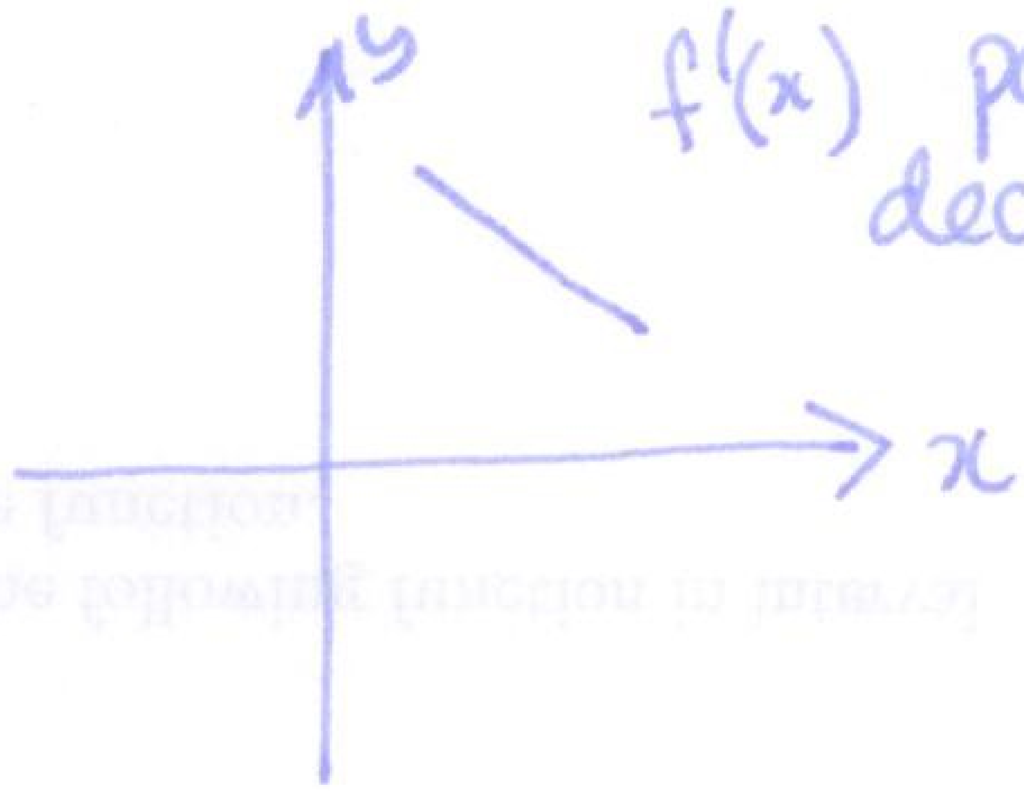
$f(x)$ −ve
decreasing

$f'(x)$
negative

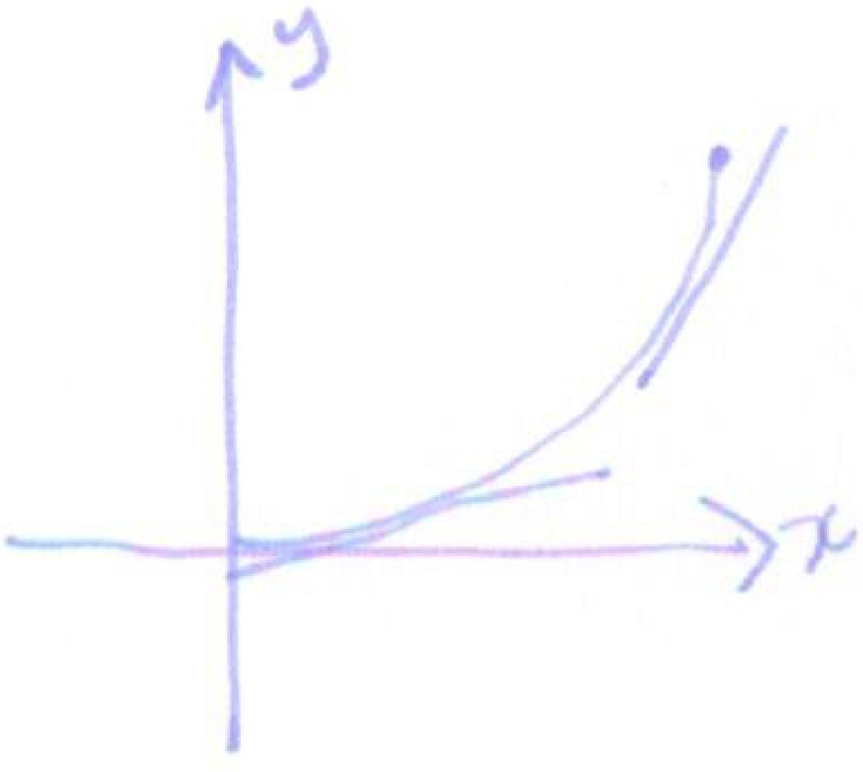




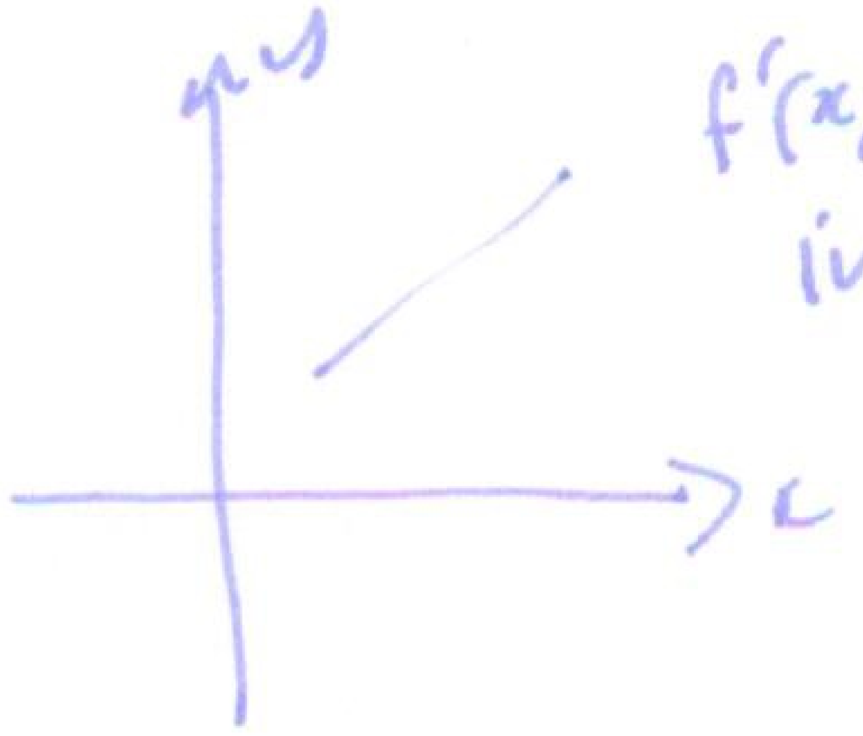
$f(x)$
increasing
rate of increase
decreasing!



$f'(x)$ positive
decreasing



$f(x)$
increasing
rate of increase
increasing!



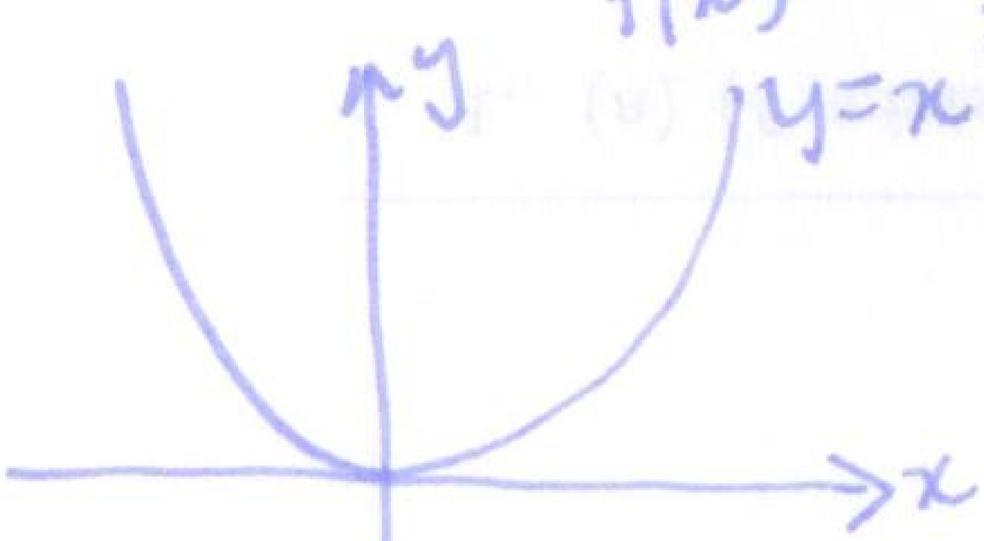
$f'(x)$ positive
increasing

§3.5 Higher derivatives

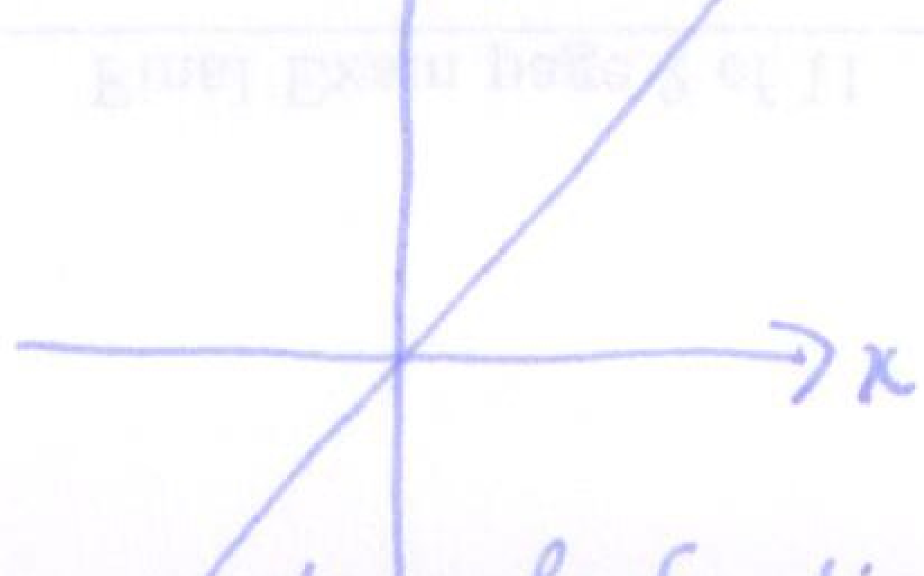
$f(x)=x^2$
$y=x^2$

$f'(x) = 2x$

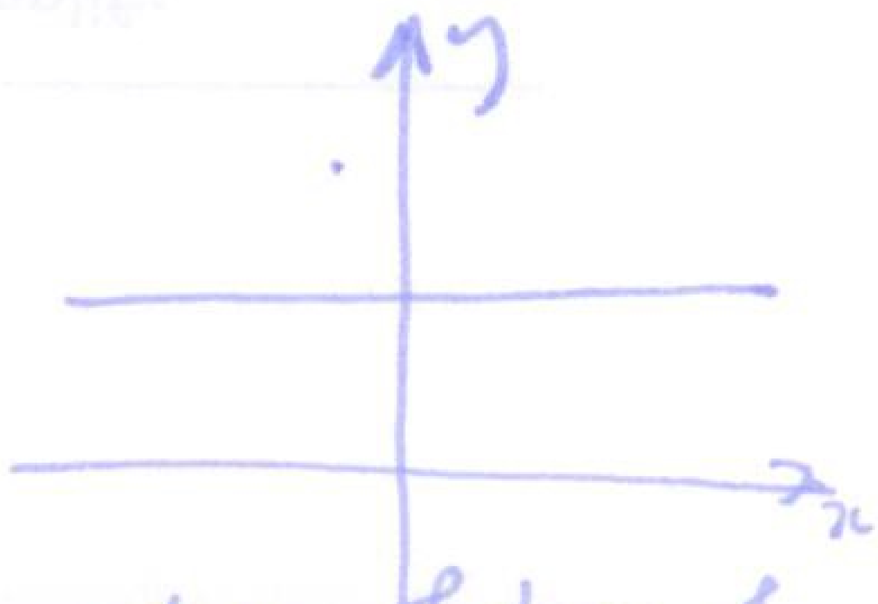
$f''(x) = 2$



graph of function



slope of function
(first derivative)



slope of slope of function
(2nd derivative)

Example ① $f(x) = x^3 + x^2$

$$f'(x) = 3x^2 + 2x$$

$$f''(x) = 6x + 2$$

$$f^{(3)}(x) = f'''(x) = 6$$

$$f^{(4)}(x) = 0$$

② $f(x) = xe^x$

$$f'(x) = xe^x + e^x$$

$$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$$

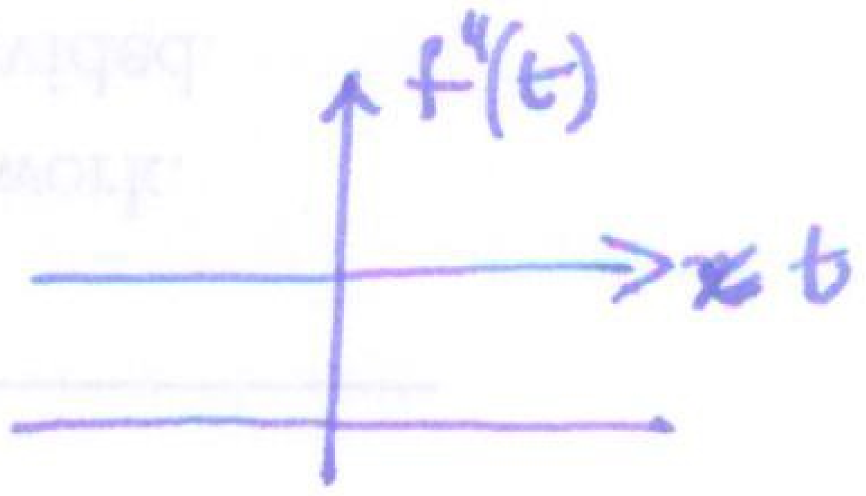
$$f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$$

$\vdots$

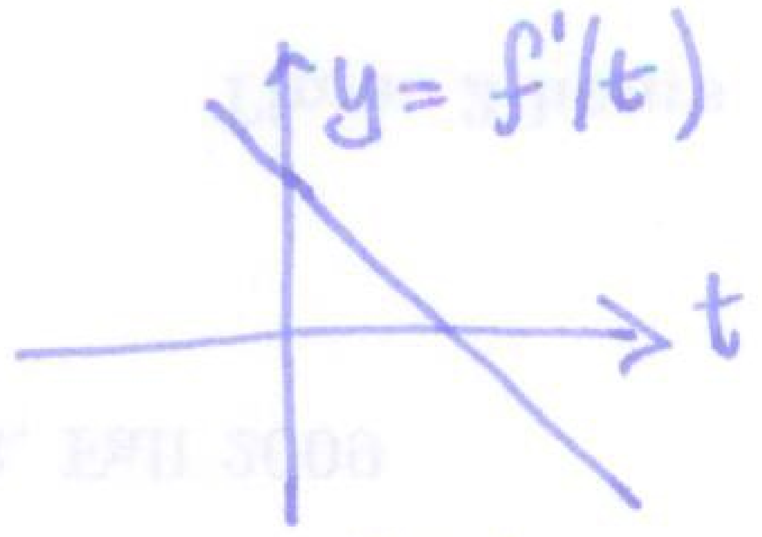
Example moving under acceleration due to gravity: constant $-g$ m/s

acceleration: $f(t)$ distance $f'(t)$ velocity $f''(t)$ acceleration

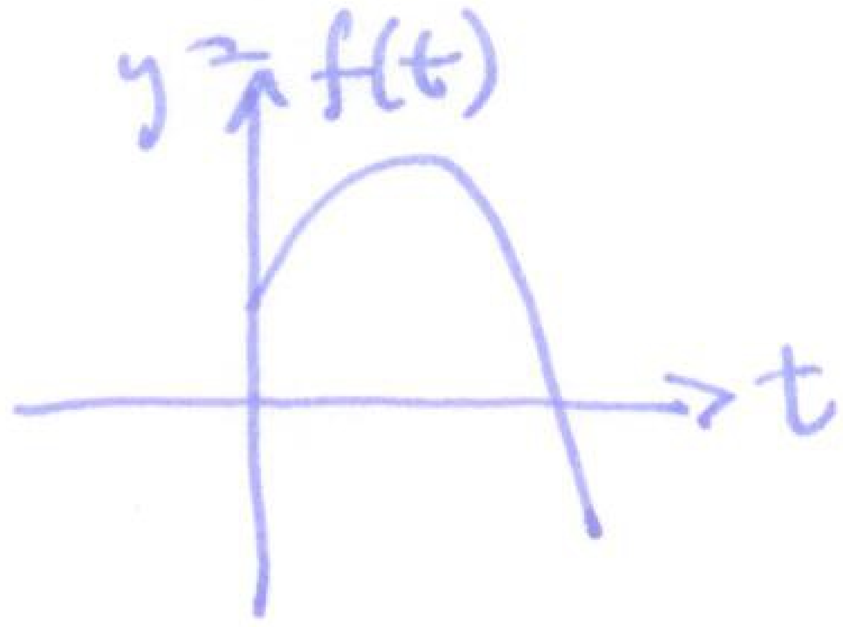
so $f''(t) = -g$ (constant)



$f'(t) = -gt + v_0$ v_0 = initial speed at time $t=0$



$f''(t) = -\frac{1}{2}gt^2 + v_0 t + d_0$ d_0 = initial location



§3.6 Trigonometric functions

Thm $\frac{d}{dx}(\sin(x)) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

Proof $\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$

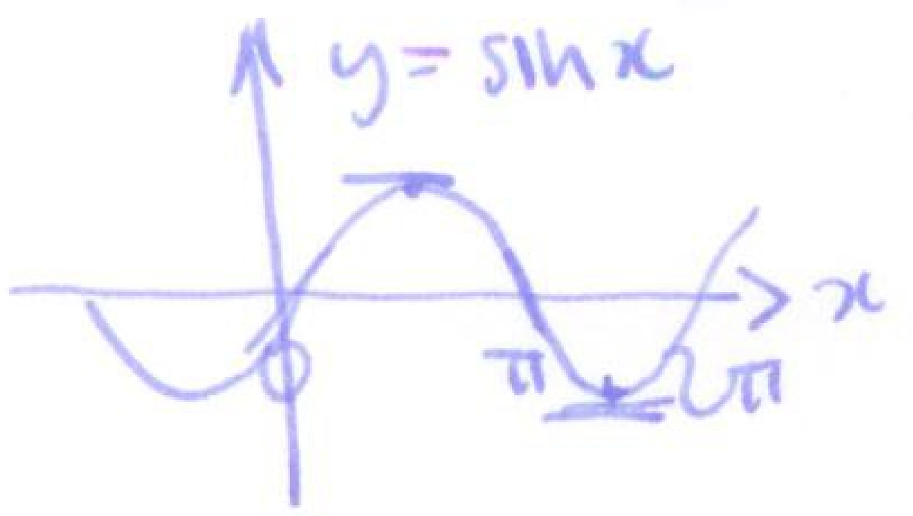
$[\sin(x+h) = \sin x \cos h + \cos x \sin h]$

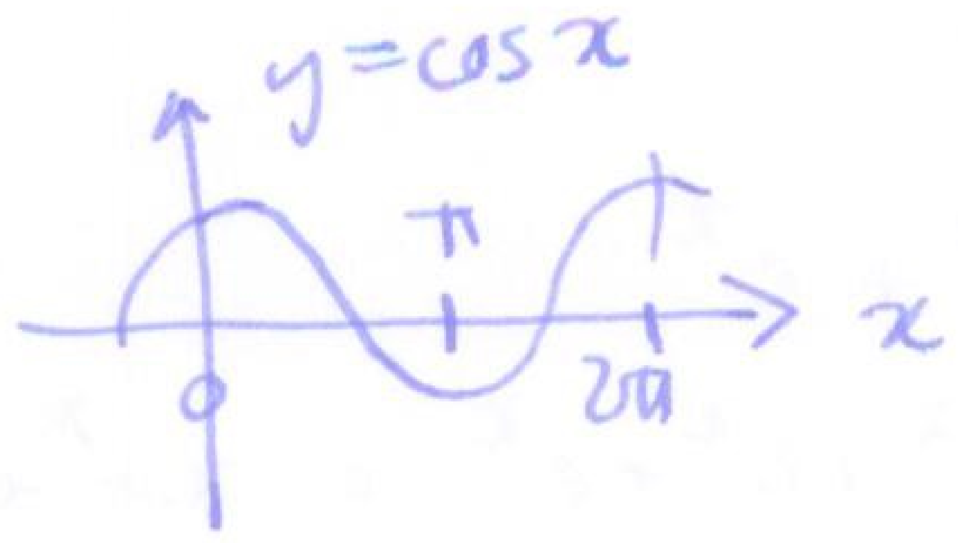
$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \cdot \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \frac{\sin h}{h}$$

$$= \sin x \underbrace{\lim_{h\to 0} \frac{\cos(h)-1}{h}}_{0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{1} = \cos x. \quad \square$$

can this be right?





Example $f(x) = x\sin x$

$f'(x) = x\cos x + \sin x.$

Thm $\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ $\qquad$ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Proof $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x . \cos x - \sin x.(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$ $\square$

§3.7 The chain rule

combining functions: we've looked at $f(x)+g(x)$, $f(x)g(x)$, $f(x)/g(x)$, what about composition? $f\circ g(x) = f(g(x))$.

Examples $\sin(x^2)$, $e^{\sin x}$, $\sqrt{x^2+1}$, etc...

Thm **Chain rule** If f and g are differentiable, then $f\circ g(x) = f(g(x))$ is differentiable, and

$$[f(g(x))]' = f'(g(x)).g'(x)$$

mnemonic: $[f(g(x))]' = \text{outside}'(\text{inside}).\text{inside}'$.

Example $h(x) = \sqrt{x^3+1} = f(g(x))$ where $f(x) = \sqrt{x} = x^{1/2}$

$g(x) = x^3+1$