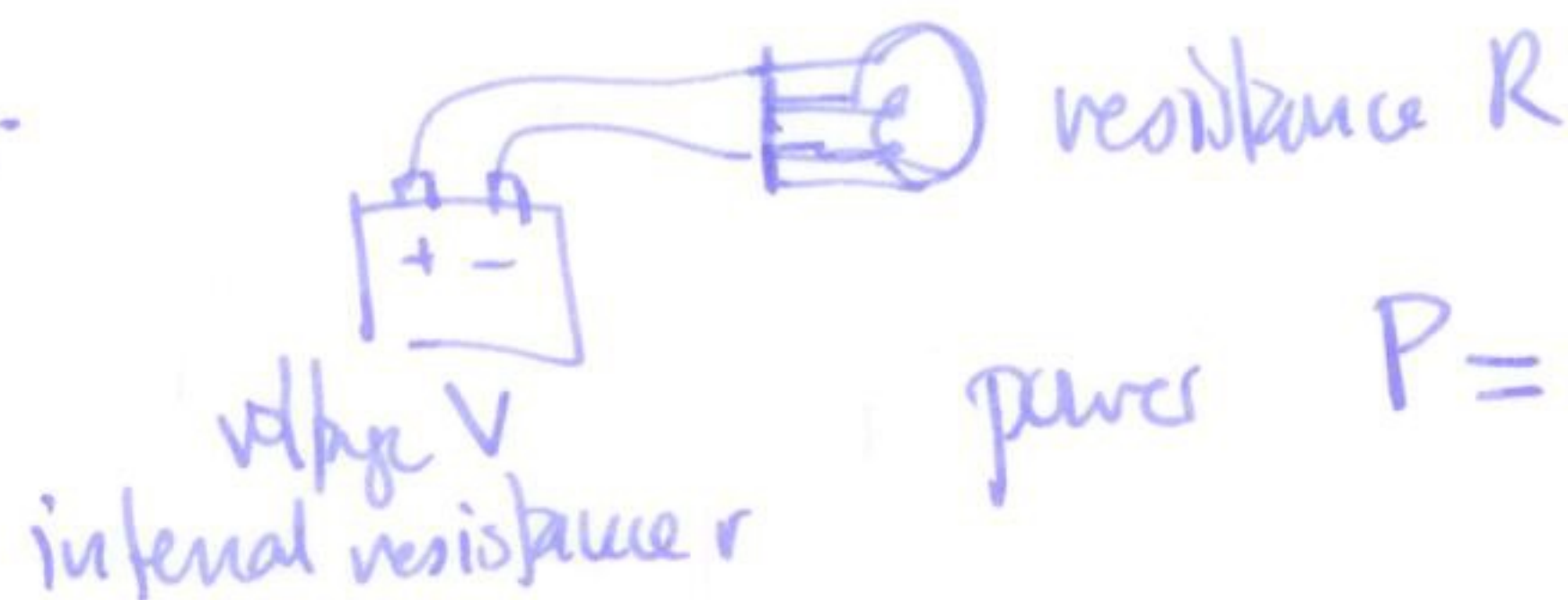


Example battery power



power $P = \frac{V^2 R}{(r+R)^2}$

Q when does the battery give out max power
(assume V, r constant)



find R s.t. $\frac{dP}{dR} = 0$.

$$\frac{dP}{dR} = \frac{d}{dR} \left(\frac{V^2 R}{(r+R)^2} \right) = \frac{(r+R)^2 (V^2 R)' - ((r+R)^2)' V^2 R}{(r+R)^4}$$

$$= \frac{(r+R) V^2 - (1) V^2 R}{(r+R)^2} = \frac{r V^2 + R V^2 - V^2 R}{(r+R)^2}$$

$$= \frac{(r+R)^2 V^2 - (r^2 + 2rR + R^2)' V^2 R}{(r+R)^4}$$

$$= \frac{(r+R)^2 V^2 - (2r + 2R) V^2 R}{(r+R)^4}$$

$$= \frac{V^2 [r^2 + 2rR + R^2 - 2rR - 2R^2]}{(r+R)^4}$$

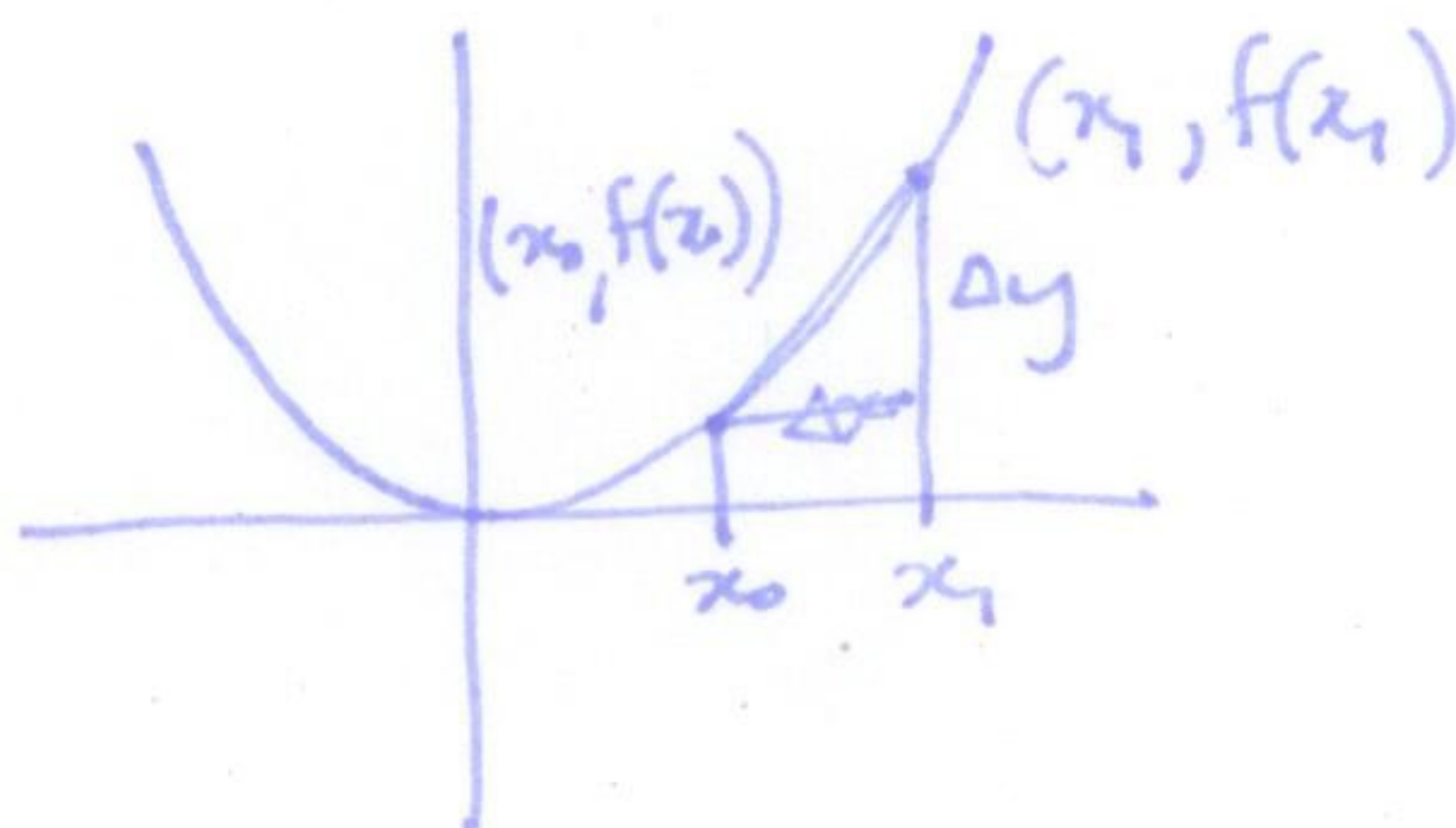
$$= \frac{v^2 [r^2 - R^2]}{(r+R)^4} = \frac{v^2 (r+R)(r-R)}{(r+R)^4} = \frac{v^2 (r-R)}{(r+R)^3} \quad (48)$$

solve $\frac{dP}{dR} = 0$ $\frac{v^2 (r-R)}{(r+R)^3} = 0 \Rightarrow R = r. \square$

§ 3.4 Rates of change

recall average rate of change

$$= \frac{\Delta x}{\Delta y} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



(instantaneous) rate of change

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

If Δx is small, we can use average rate of change to approximate rate of change, and vice-versa!

Example area of circle = πr^2
rate of change of area with respect to radius: $\frac{dA}{dr} = 2\pi r$

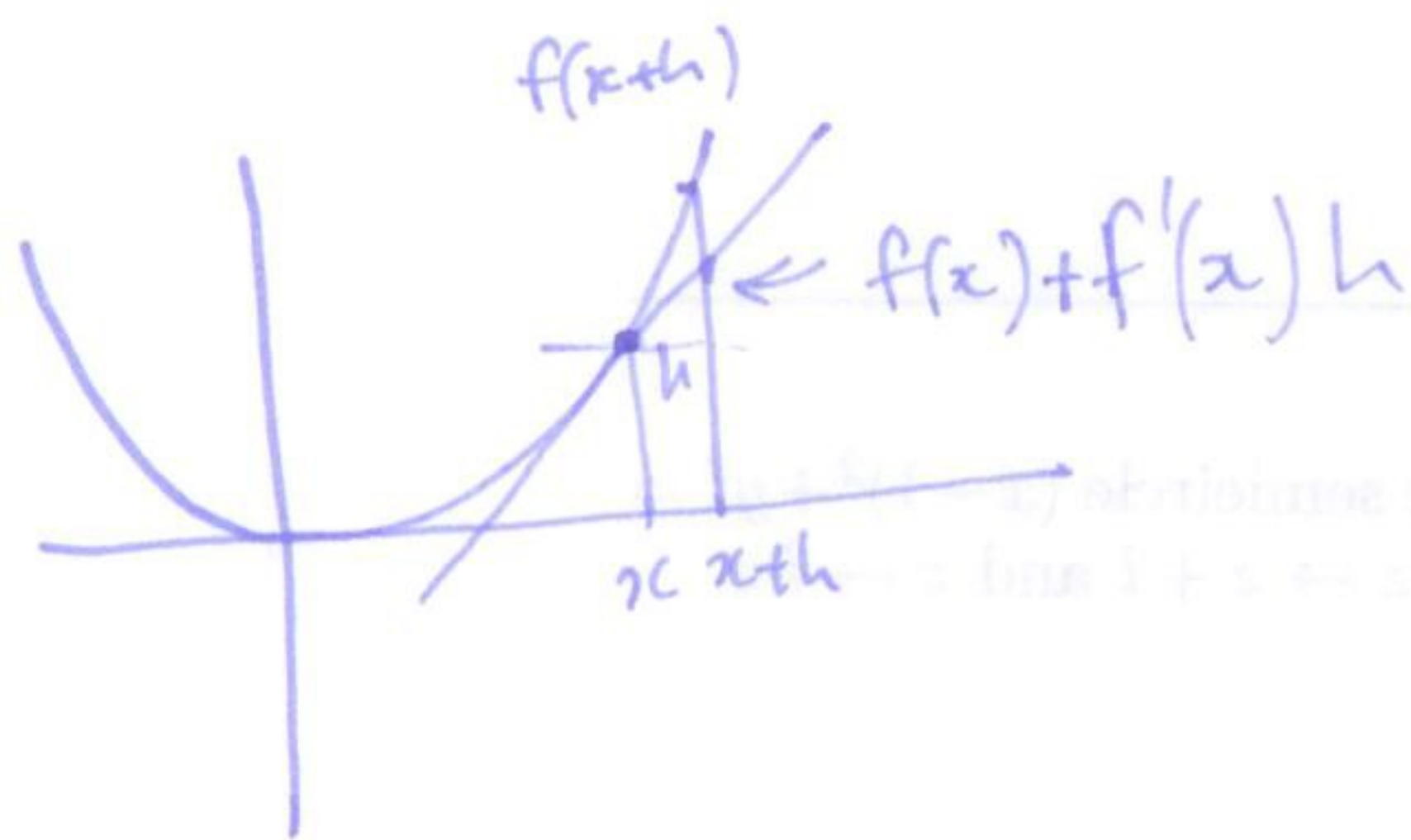
$$\left. \frac{dA}{dr} \right|_{r=2} = 4\pi \quad \left. \frac{dA}{dr} \right|_{r=5} = 10\pi$$

[rate of change of area = perimeter!]

For small h , $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

or

$$f(x+h) \approx f(x) + hf'(x)$$



Example stopping distance in feet $F(s) = 1.1s + 0.05s^2$

(s speed in mph)

calculate stopping distance when $s=30$: $F(30) = 1.1 \cdot 30 + 0.05 \cdot (30)^2$
 $= 78 \text{ ft}$

rate of change of stopping distance

when $s=30$: $F'(30) = 1.1 + 0.1 \cdot 30 = 4.1 \text{ ft/mph}$

use this to estimate stopping distance at $s=31 \text{ mph}$

$$F(s+h) \approx F(s) + F'(s)h$$

$$F(31) \approx F(30) + F'(30) \cdot 1$$

$$\approx 78 + 4.1 \cdot 1 = 82.1$$

On the interpretation of graphs

