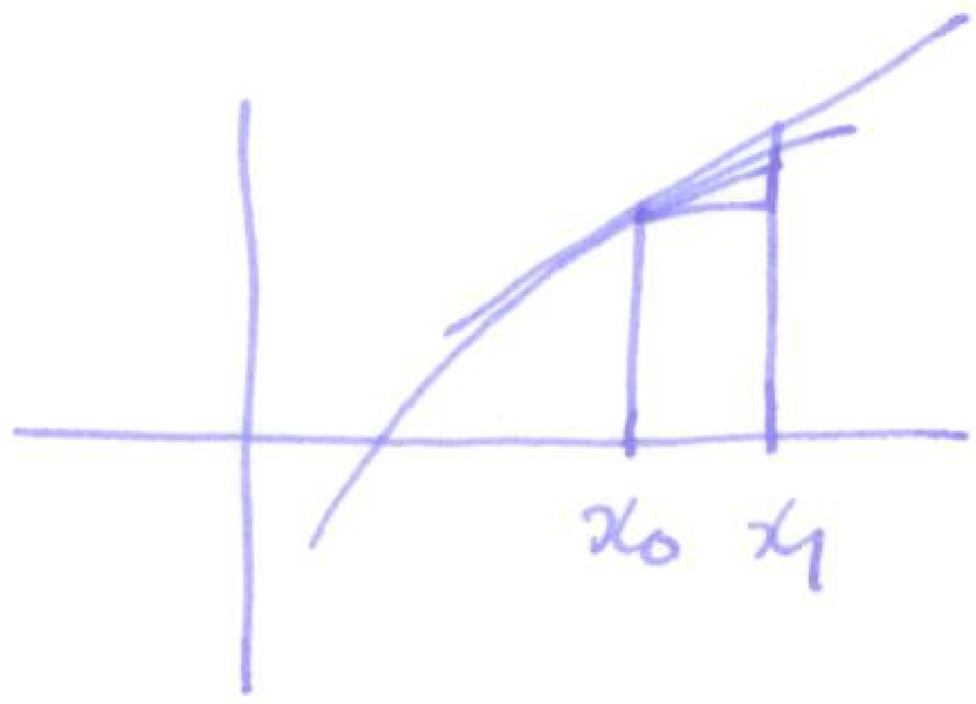
§2.2 Limits



want: to find slopes of tangent lines

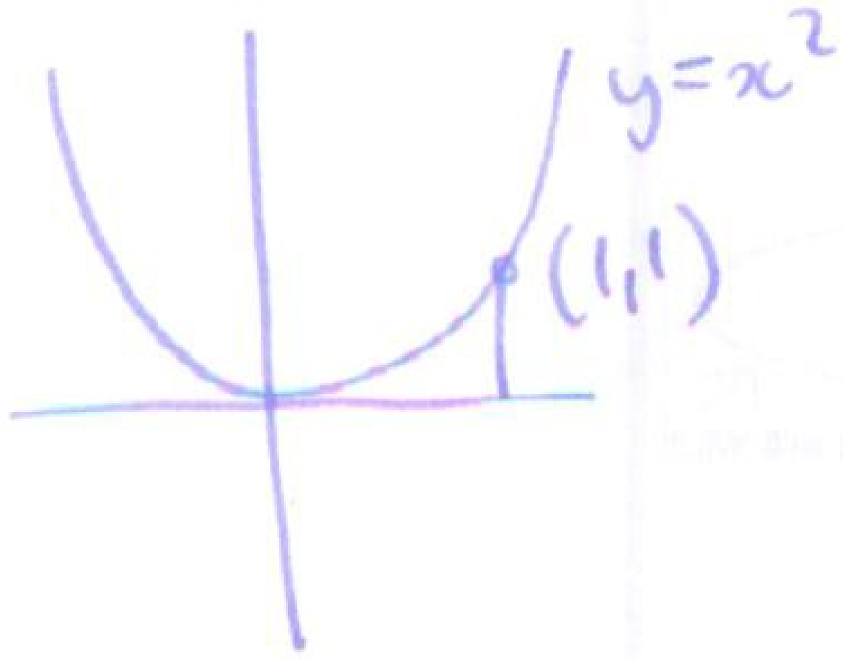
can find: average rate of change from x_0 to x_1

$$\frac{f(x_1)-f(x_0)}{x_1-x_0}$$

Q: why not set $x_1 = x_0$? **A**: $\frac{f(x_0)-f(x_0)}{x_0-x_0} = \frac{0}{0}$ undefined. **NO**.

observations

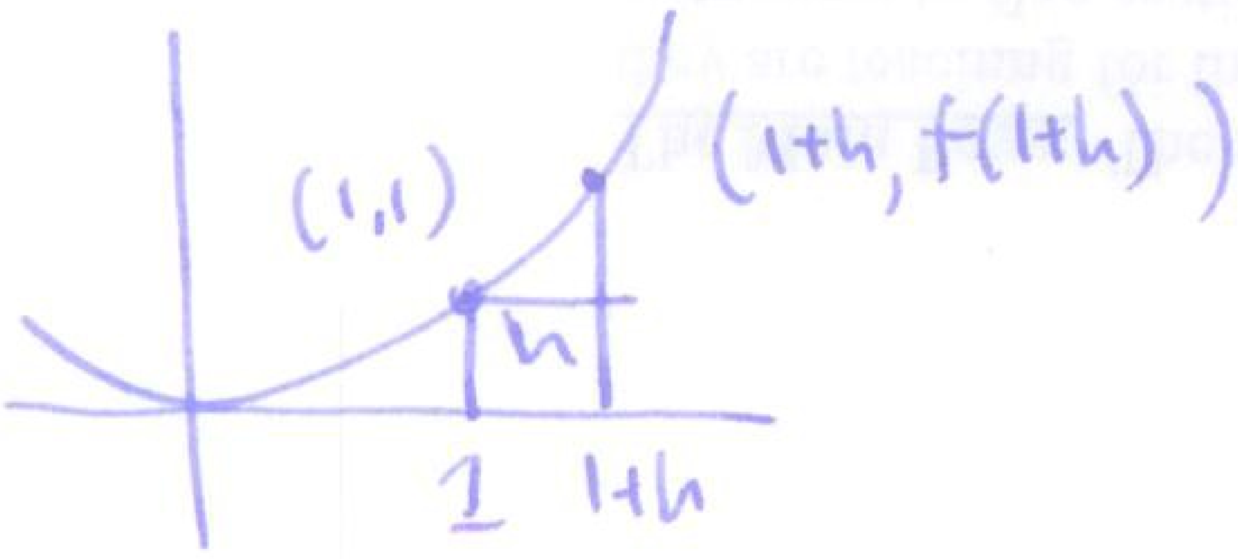
1. if we draw pictures the average slope seems to get closer to the actual slope as the length of the interval gets small
2. seems to work for sample calculations for:



$f(1)=1$

interval	~~slope~~	slope
$[1,2]$	$f(2)=4$	$\frac{4-1}{2-1} = 3.$
$[1,1.5]$	$f(\underset{3/2}{1.5}) = \frac{9}{4}$	$\frac{9/4-1}{1/2} = 5/2 \simeq 2.5$
$[1,1.1]$	$f(1.1) = 1.21$	$\frac{1.21-1}{0.1} \approx 2.1$
$[1,1.01]$	$f(1.01) = 1.0201$	$\frac{1.0201-1}{0.01} \approx 2.01$

3. seems to work algebraically:



interval $[1,1+h]$ $\quad f(1+h) = (1+h)^2$

slope: $\frac{(1+h)^2-1}{h} = \frac{1+2h+h^2-1}{h}$

$= \frac{2h+h^2}{h} \underset{(h\neq 0)}{=} 2+h$

Def^{n} Let f be a function defined on an interval containing c, but not necessarily at c. We say

"the limit of $f(x)$ as x approaches c is equal to L"

written as
$$\lim_{x \to c} f(x) = L$$

if $|f(x) - L|$ becomes arbitrarily small when x is chosen sufficiently close to c.

also say $\quad f(x)$ converges to L as $x \to c$

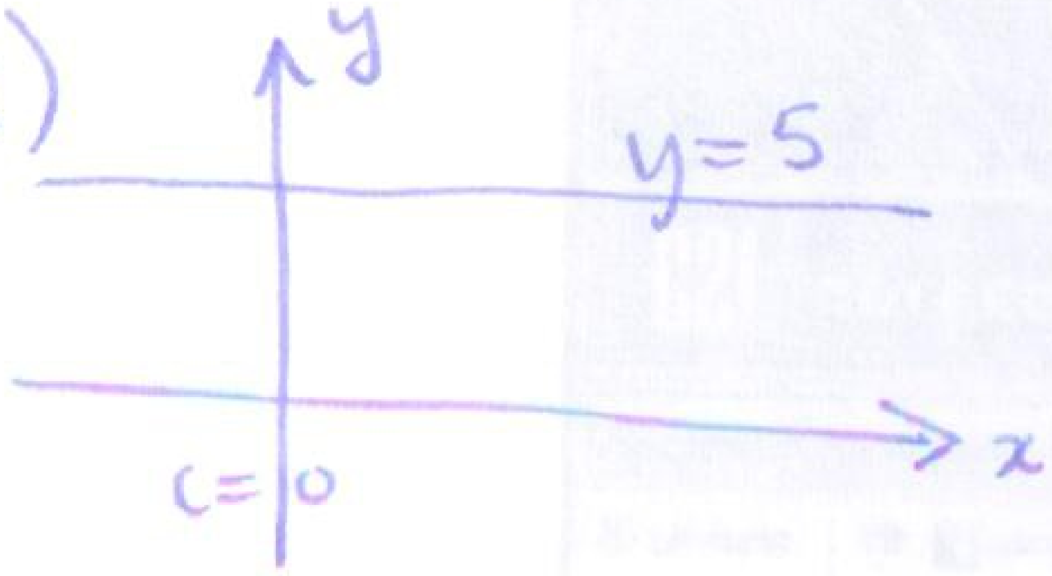
$\qquad f(x) \longrightarrow L$ as $x \longrightarrow c$.

Examples a) $f(x) = 5 \quad c = 0$

b) $f(x) = \dfrac{5x}{x} \quad c = 0$

$(x \neq 0)$

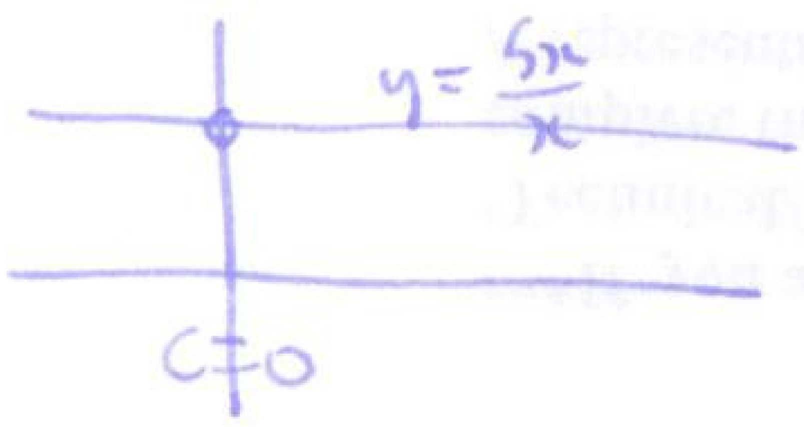
a)



claim $\displaystyle\lim_{x \to 0} 5 = 5$

want to show $|f(x) - 5|$ is close to 0 for x close to 0. but $|f(x) - 5| = |5 - 5| = 0$ always.

b)

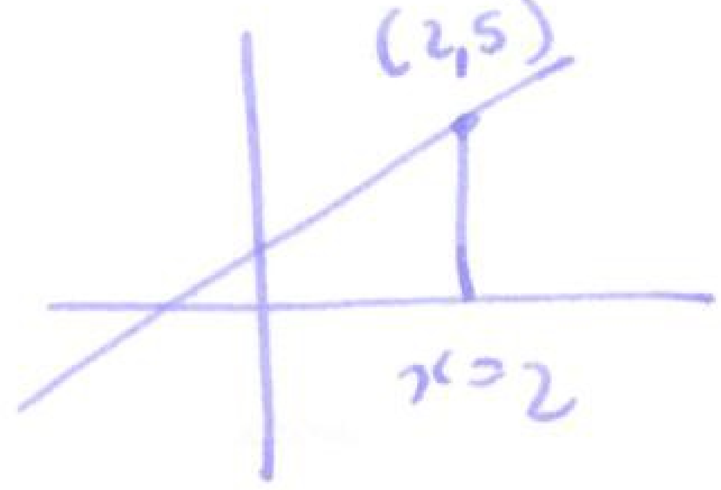


claim $\displaystyle\lim_{x \to 0} \frac{5x}{x} = 5$

want to show $|f(x) - 5|$ close to zero for x close to 0.

$$= \left|\frac{5x}{x} - 5\right| \underset{x \neq 0}{=} |5 - 5| = 0$$ for all x close to 0 (but not equal to zero)

c) $\lim_{x \to 2} 2x+1 = 5$



want to show $|f(x)-5|$ close to 0 for x close to 2

$|2x+1-5| = |2x-4| = 2|x-2|$

x close to 2 $\Leftrightarrow$ $|x-2|$ close to 0, so $|f(x)-5|$ close to zero.

<u>Thm</u> for any constants k, c

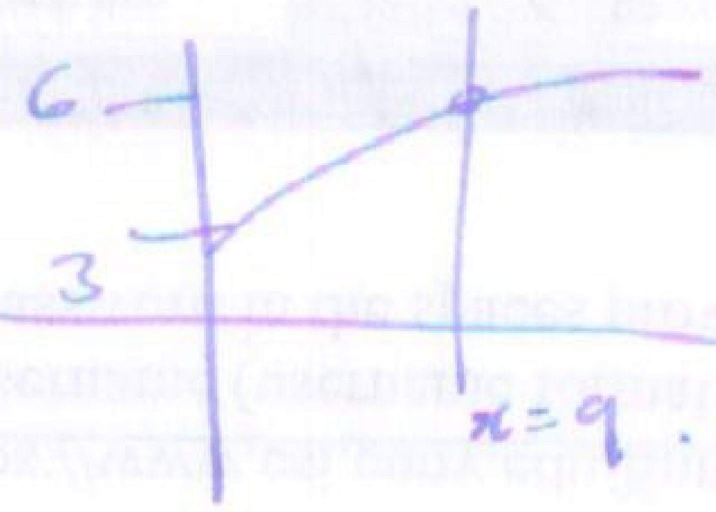
a) $\lim_{x \to c} k = k$ b) $\lim_{x \to c} x = c$

<u>Investigating limits</u>: try · drawing a picture

· estimating/calculating close values.

<u>Example</u> $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$ problem: can't just plug in $x=9$.

<u>try</u>: · drawing a picture:



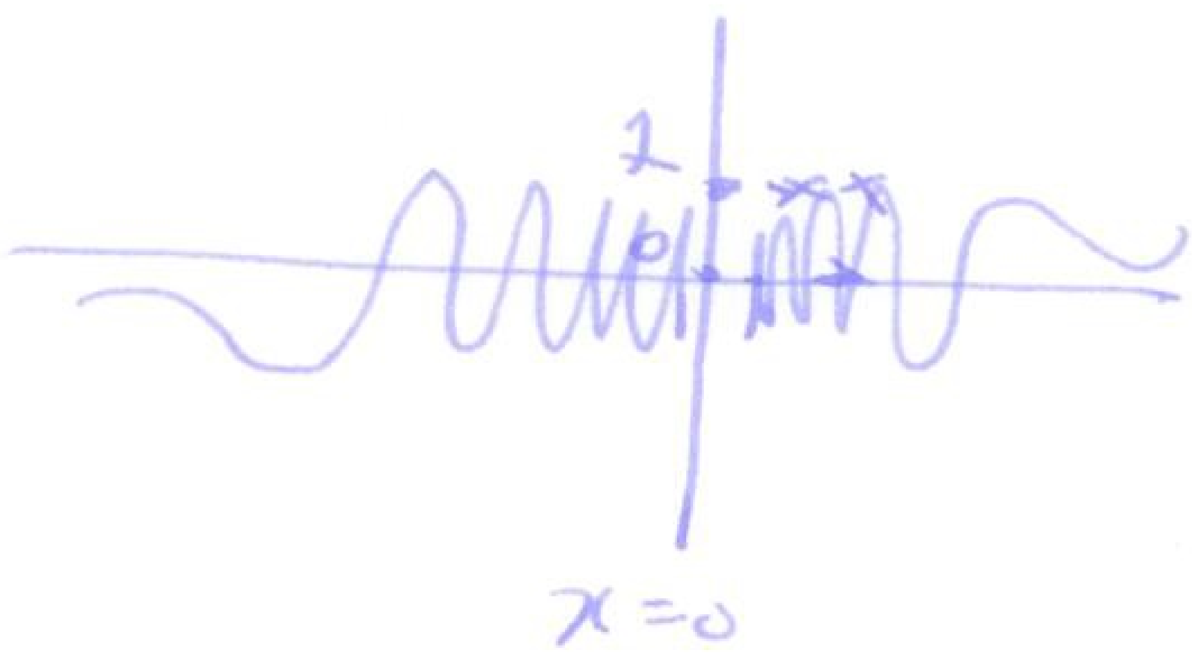
looks like 6.

· compute

x	8.9	9.1	8.99	9.01
$\frac{x-9}{\sqrt{x}-3}$	5.98329	6.016	5.99833	6.00166

· algebraically: we'll do this later.

Bad example : no limit $f(x) = \sin\left(\frac{1}{x}\right)$

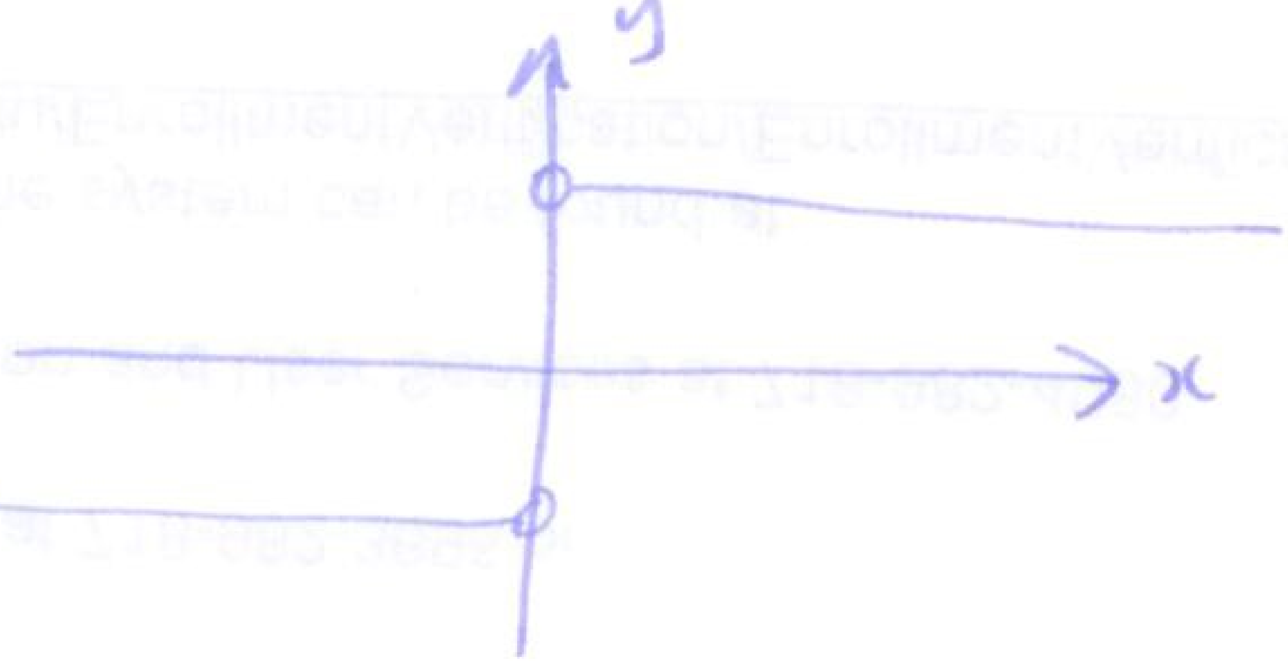


note $f\left(\frac{1}{2\pi n}\right) = \sin(2\pi n) = 0$

$$f\left(\frac{1}{2\pi n + \pi/2}\right) = \sin\left(2\pi n + \frac{\pi}{2}\right) = 1$$

One sided limits

$$f(x) = \frac{x}{|x|}$$



$$= \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

sometimes useful to distinguish left from right

$\lim\limits_{x \to 0+} f(x)$ means right limit

$\lim\limits_{x \to 0-} f(x)$ means left limit

note in order for $\lim\limits_{x \to 0} f(x)$ to exist both $\lim\limits_{x \to 0+} f(x)$ and $\lim\limits_{x \to 0-} f(x)$ must exist and be equal