Example . $\log_6 9 + \log_6 4 = \log_6 9.4 = \log_6 36 = \log_6 6^2 = 2\log_6 6$
$= 2$.

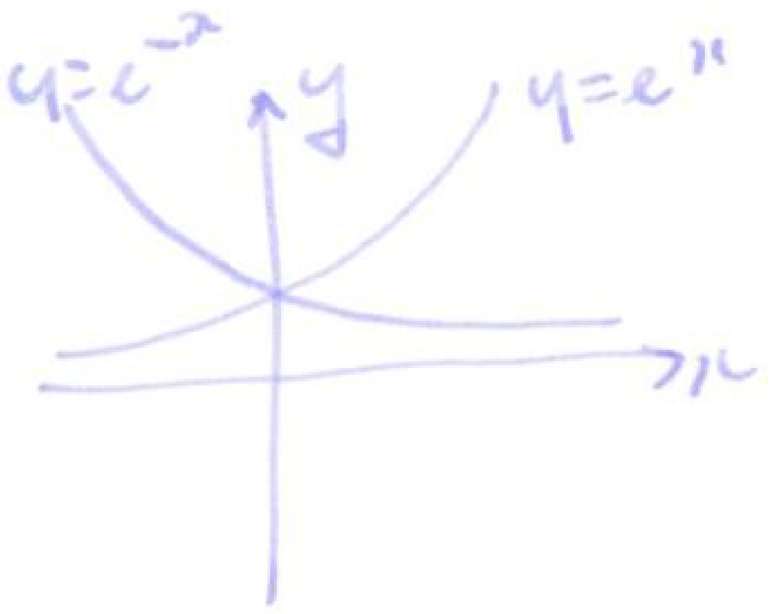
• # bacteria $B(t) = 10.2^t$ t in hours.

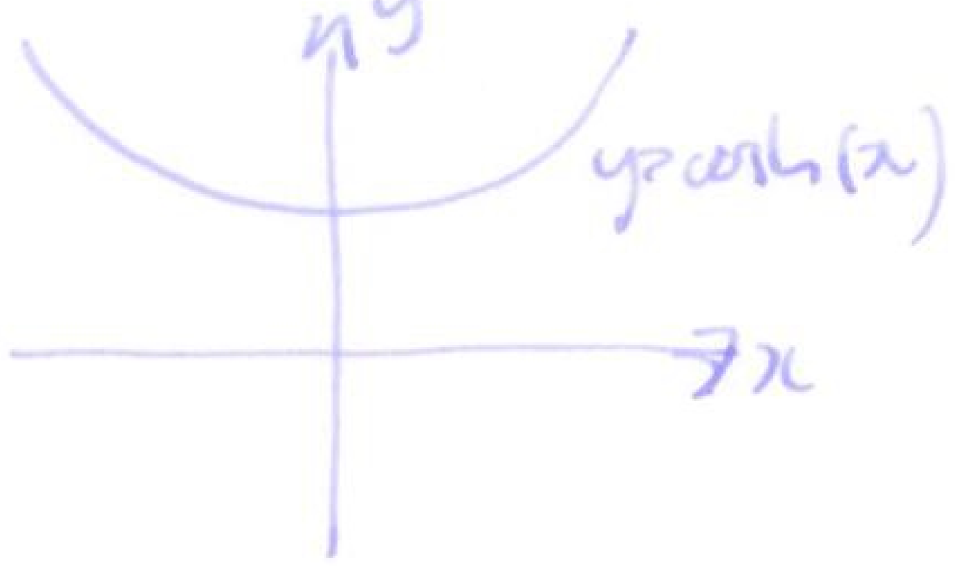
when will there be 1 million bacteria?

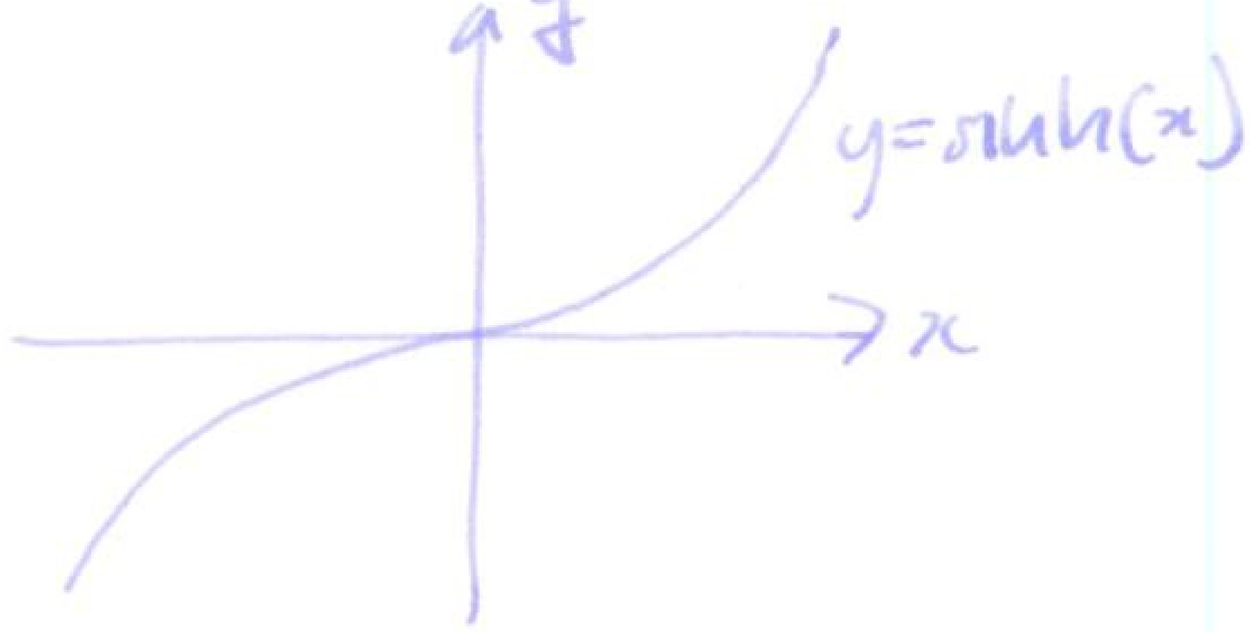
$$10.2^t = 1000\,000$$
$$2^t = 100000$$
$$t = \log_2 100000 = \frac{\ln(100000)}{\ln(2)} \approx$$

Hyperbolic functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}.$$







useful facts • $\sinh(x)$ odd $\cosh(x)$ even

• $\cosh^2 x - \sinh^2 x = 1$

• $\sinh(x+y) = \sinh(x)\cosh y + \cosh x \sinh y$
$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

Check : $\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$$= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} = \frac{4}{4} = 1$$

similarly $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ $\qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)}$

can define inverse functions:

	domain
$\sinh^{-1}(x)$	$(-\infty, \infty)$
$\cosh^{-1}(x)$	$(0, \infty)$
$\tanh^{-1}(x)$	$(-\infty, \infty)$

etc.

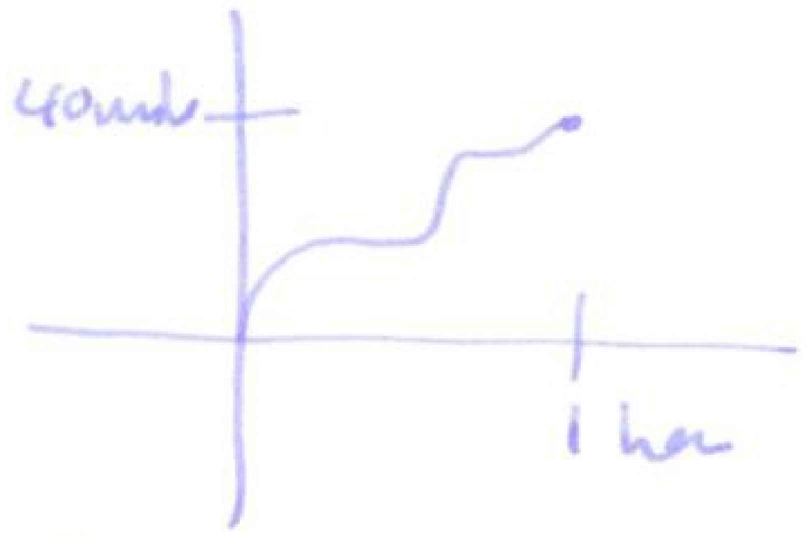
§2.1 Limits, rates of change, tangent lines

motivation: velocity: driving at constant speed

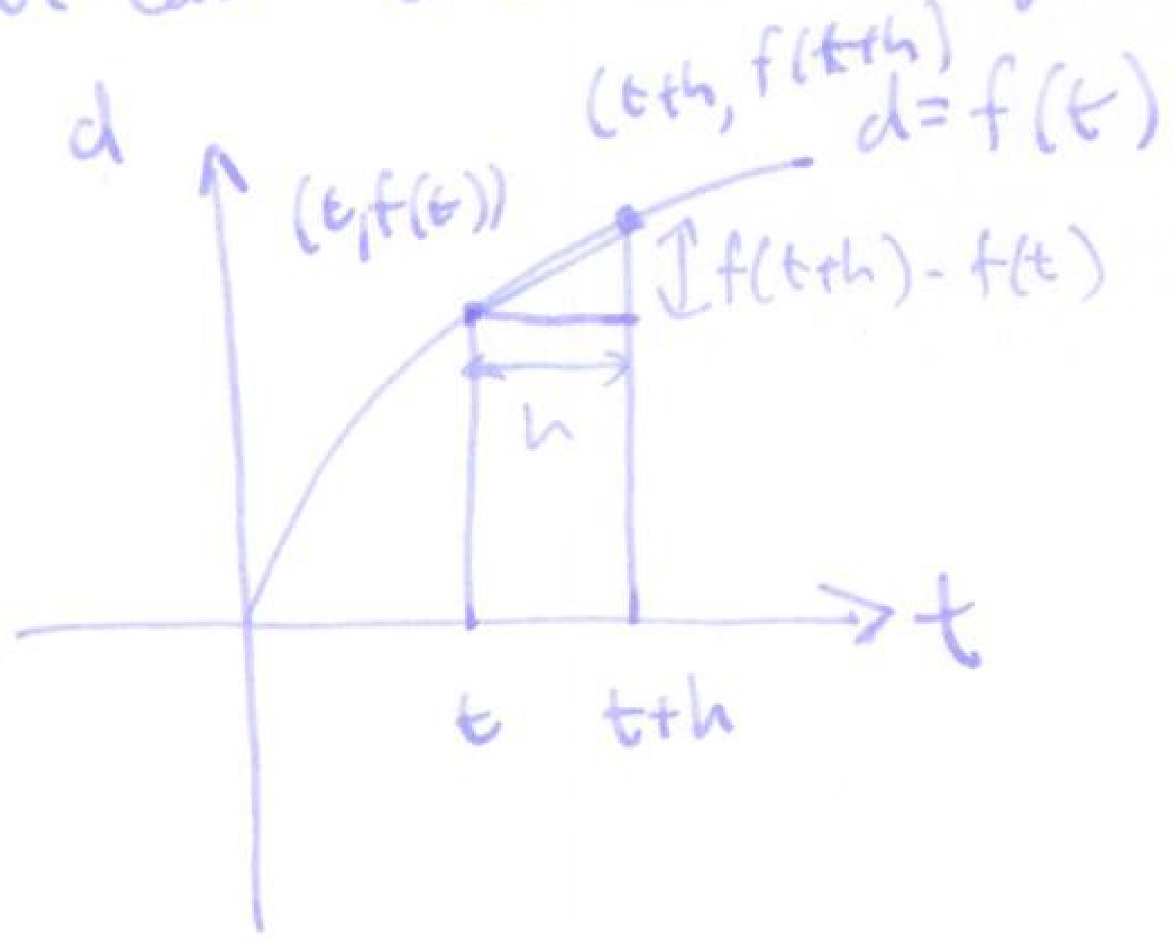
velocity = $\frac{\text{distance}}{\text{time}}$ = slope of graph of distance vs time.

problem: what happens if you don't travel at constant speed?

average speed = $\frac{\text{distance travelled in time interval}}{\text{length of time interval}}$



we can look at average speed over any time interval, even very short ones



average speed over an interval $[t, t+h]$

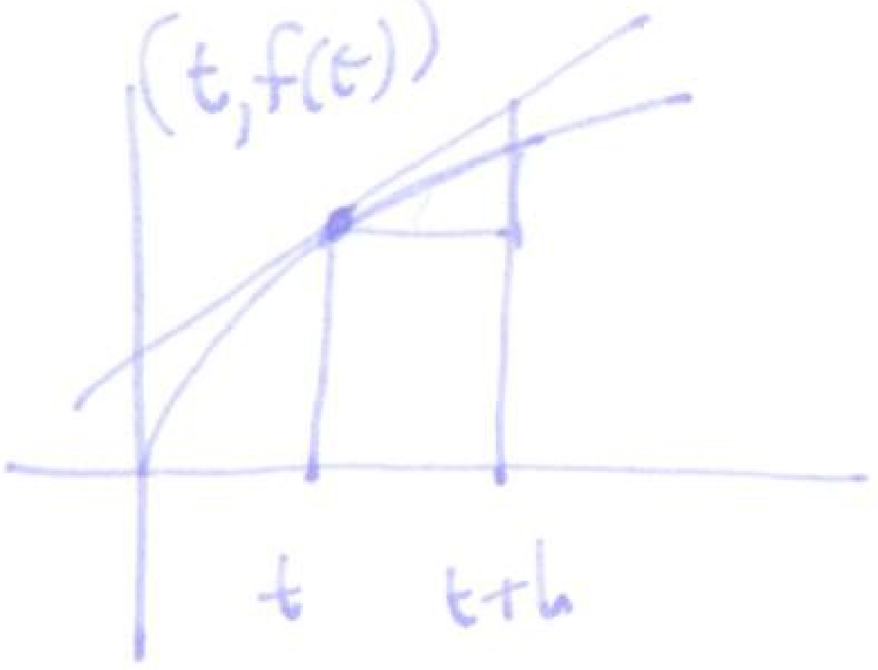
is $\frac{\text{difference in distance}}{\text{difference in time}} = \frac{f(t+h) - f(t)}{t+h-t}$

$= \frac{f(t+h) - f(t)}{h}$

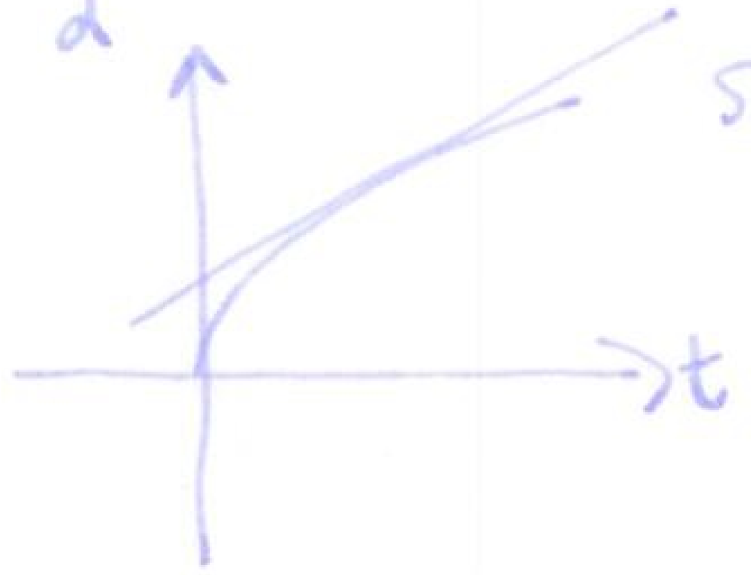
Q: what is the speed at time t?

A. slope of tangent line to graph at $(t, f(t))$

for nice functions, as the length of the interval $[t, t+h]$ gets smaller, the average speed over the interval gets close to the slope of the tangent line.

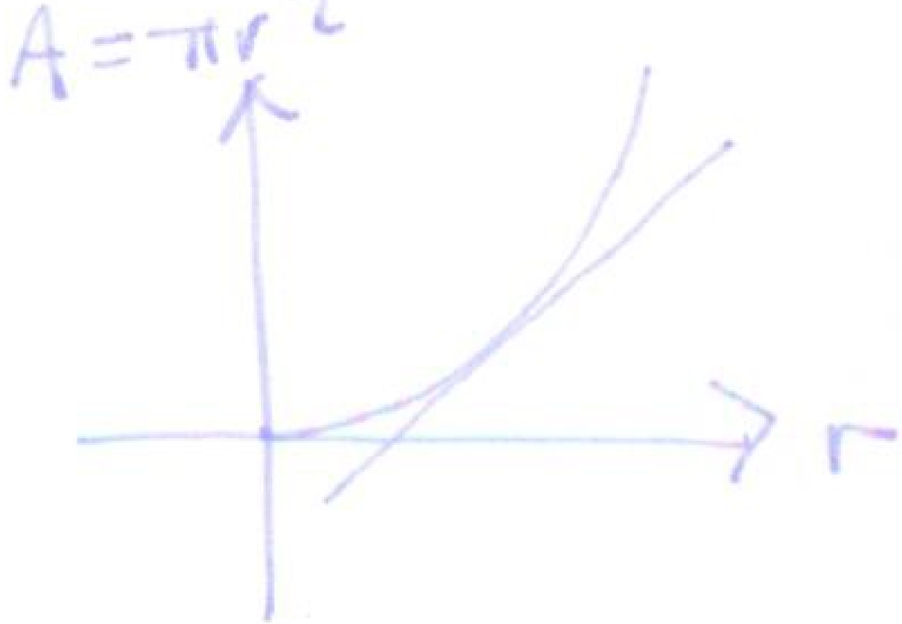


<u>Observation</u>: this works for any function.

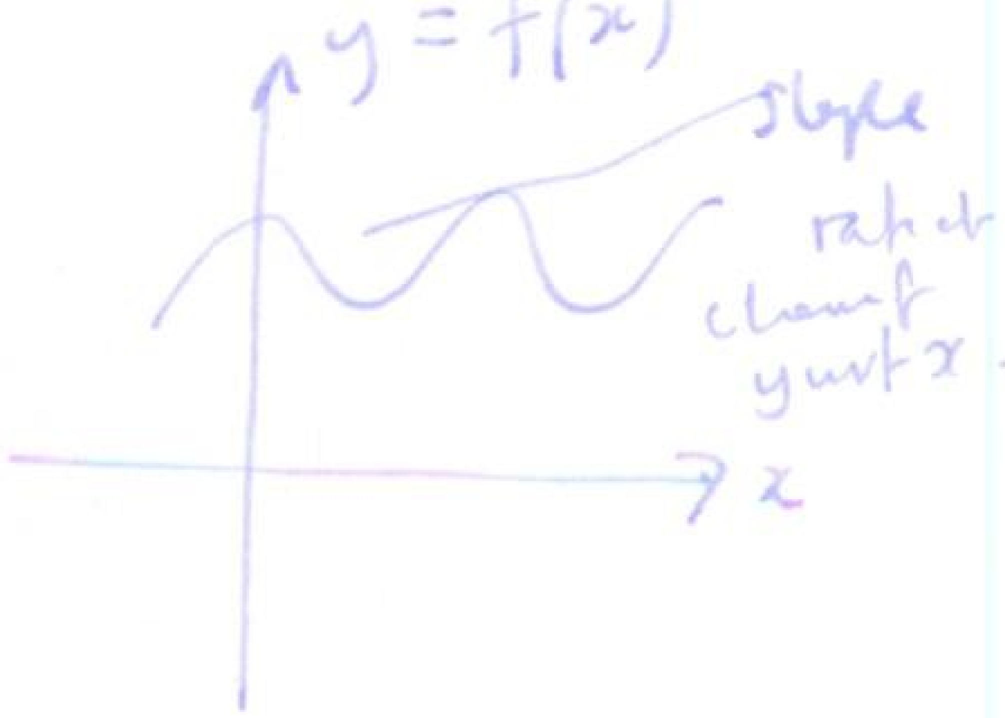


slope = rate of change of distance with time = velocity

slope = rate of change of velocity with time = acceleration



slope = rate of change of area with radius.



<u>summary</u> average rate of change of $f(x)$ on an interval $[x_0, x_1]$

is $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$